



Research article

Tripartite stochastic evolutionary game analysis of data sharing in the sports equipment manufacturing industry under the risk of technology supply disruption

Yutong Bai¹, Jian Yang^{2,*}, Jin Dong¹ and Xufeng Lv¹

¹ Physical education institute, Shanxi University, Taiyuan 030006, China

² School of Information, Shanxi University of Finance and Economics, Taiyuan 030006, China

* **Correspondence:** Email: yangj@sxufe.edu.cn.

Abstract: Effective data-sharing in the sports equipment manufacturing industry is central to breaking down data silos, yet multiparty collaboration is still hindered by the risks of technological supply disruptions and exposure of private data. We construct a tripartite stochastic evolutionary game model involving core manufacturers, digital transformation platforms, and third-party software developers, incorporating Gaussian white noise to capture exogenous random disturbances. Evolutionary stability analysis and numerical simulations are used to assess how key parameters and stochastic shocks shape the system's trajectories. The results indicate that supply disruption risks and high privacy exposure costs deter core manufacturers from high-quality data-sharing. Strict platform data governance, strong accountability mechanisms, and the mitigation of external noise, on the other hand, help move the ecosystem toward a stable, system-optimal cooperative state. This study contributes to the stochastic evolutionary game literature on industrial data-sharing and provides a theoretical framework with guidelines for the sports manufacturing sector to mitigate disruption risks and achieve collaborative innovation.

Keywords: sports equipment manufacturing; data silos; technology supply disruption; stochastic evolutionary game; data sharing

Mathematics Subject Classification: Primary: 91A80

1. Introduction

The sports equipment manufacturing industry is undergoing a transformation driven by the rapid development of the Internet of Things (IoT), advanced data analytics, and artificial intelligence technologies [1, 2]. Modern sports equipment increasingly integrates high-precision biomechanical sensors and systems for monitoring human performance, generating large volumes of the users'

physiological and device operational data [3]. These data, when effectively circulated and utilized, enables manufacturers to optimize product design, deliver personalized fitness guidance, and predict equipment failures. The digital and intelligent transformation of this industry therefore depends on breaking down data silos and establishing cross-organizational data-sharing mechanisms [4]. Digital transformation platforms have emerged as central hubs that aggregate hardware-generated data from manufacturers and provide it to downstream third-party software developers, who then build innovative applications such as virtual artificial intelligence (AI) fitness coaches. These platforms, through data governance capabilities including quality audits, privacy oversight, and penalty enforcement, aim to maximize the shared ecosystem's benefits.

However, the current international trade environment introduces a distinctive and understudied risk: Technology supply disruptions [5]. The supply chains for core sensor chips and algorithmic systems can be unexpectedly severed, heightening uncertainty among manufacturers over whether sharing high-quality data may lead to the leakage of trade secrets or the unintended spillover of core technologies. At the same time, third-party developers face risks regarding the quality of data supplied through platforms; purchasing substandard data exposes them to sunk costs and business losses. Platforms must continuously weigh their data governance costs against the broader ecosystem benefits. Because all participants are boundedly rational and operate under incomplete information, they dynamically adjust their strategies according to the expected payoffs, creating a complex multiparty strategic game. Furthermore, real-world business ecosystems are subject to random disturbances, including sudden policy shifts, market volatility, and technological blockades, which traditional deterministic evolutionary models cannot capture [6]. Recent scholarship has therefore begun incorporating stochastic disturbance factors such as Gaussian white noise into evolutionary game frameworks to account for exogenous uncertainty.

Most existing studies on data-sharing games are limited to analyzing bilateral interactions between government and enterprises, or treating digital platforms as passive intermediaries rather than independent governance agents with strategic choices regarding data quality oversight, accountability enforcement, and compensation mechanisms [7]. Moreover, few studies have mathematically operationalized technology supply disruptions, an abrupt supply chain risk, within the game-theoretic framework of data-sharing. Although stochastic evolutionary models have been applied to environmental governance [8], green innovation [9], and port resilience [10], no study has introduced Gaussian white noise to capture the random disturbances arising from technology supply disruptions in a manufacturing data-sharing context. The tripartite strategic interactions among core manufacturers, digital transformation platforms, and third-party software developers under such stochastic conditions remain unexplored.

For this purpose, this study constructs a tripartite stochastic evolutionary game model encompassing core sports equipment manufacturers, digital transformation platforms, and third-party software developers. This study makes three specific theoretical contributions. First, it models digital transformation platforms as independent governance agents with strategic choices regarding data quality oversight, accountability intensity, and compensation mechanisms, a configuration that produces qualitatively different equilibrium conditions compared with the existing models that treat platforms as passive intermediaries [11]. Second, it derives closed-form stability conditions across eight equilibrium scenarios and explicitly identifies the parameter thresholds at which each stakeholder's strategy changes, providing testable boundary conditions that link platforms' governance capability, account-

ability intensity, and privacy risk exposure to system-level outcomes. Third, through systematic numerical simulation under Gaussian white noise perturbations, it characterizes the dynamic impact of stochastic disturbances on evolutionary trajectories and identifies the conditions under which the system converges to the system-optimal cooperative equilibrium.

Methodologically, this study integrates Lyapunov stability theory, the Jacobian matrix method, and Itô stochastic differential equations to analyze both the deterministic equilibrium properties and the stochastic robustness of the data-sharing ecosystem. A distinctive feature of the approach is the explicit incorporation of Gaussian white noise into the replicator dynamics, linking the intensity of technology supply disruption risk directly to the stability conditions of each equilibrium point. The Euler–Maruyama numerical scheme is used to discretize the stochastic differential equations, and systematic parameter sensitivity analyses are conducted to identify critical phase transitions, including the risk exposure threshold beyond which manufacturers withdraw from sharing data and the governance capability threshold above which the platform’s strict oversight effectively stabilizes the cooperative equilibrium.

The remainder of this paper is organized as follows. Section 2 reviews the related literature and provides a comparative positioning. Section 3 constructs the tripartite stochastic evolutionary game model and analyzes the strategy’s stability for each participant. Section 4 derives the stochastic stability conditions, Jacobian matrix, and equilibrium scenarios. Section 5 presents numerical simulations of the initial strategies and key parameters. Section 6 summarizes the conclusions and management implications.

2. Literature review

2.1. Data-sharing and platform governance in sports equipment manufacturing

In the sports equipment manufacturing sector, digital and intelligent transformation has become a widespread trend. Milasi et al. [12] conducted a comprehensive meta-synthesis of IoT applications in the sports industry, identifying smart equipment and performance monitoring as key dimensions of IoT-enabled digital transformation. Traditional manufacturers are shifting their business models, historically supplying consumers with physical hardware but now leveraging IoT technology to collect large volumes of user biomechanical and device operational data. These data assets are valuable for developing next-generation smart sports equipment and delivering personalized exercise guidance. Culot et al. [13] examined the data-sharing conundrum in the age of digital transformation, demonstrating that establishing cross-organizational data-sharing mechanisms has become a central pathway to improving operational efficiency across industrial supply chains.

The sector, however, faces a persistent data silo dilemma, as effective information exchange channels between manufacturers and software developers are still largely absent. Fan et al. [14] analyzed enterprises’ data-sharing from a supply chain perspective using an evolutionary game model, revealing that information asymmetry leads to data bottlenecks and reduced decision-making efficiency. Without unified digital platforms and standardized data protocols, high-quality sports data are frequently left idle or locked within proprietary corporate systems, preventing downstream third-party software developers from accessing the data needed to develop applications. To address these barriers, the industry has initiated the construction of digital sports transformation platforms to integrate hardware data with the upstream software ecosystem. Although these platforms provide the necessary technical infrastructure,

corporate willingness to share data is still low, primarily due to the security risks involved. Amid increasing global supply chain volatility, manufacturers face the threat of technology supply disruptions and the risk of sudden halts in the supply of core sensor chips or algorithmic systems. Wei et al. [15] proposed a blockchain-based government incentive mechanism for manufacturing supply chains' data governance, highlighting that manufacturers who fear sharing high-quality data could cause the leakage of core technical parameters, eroding their market competitiveness. Relying solely on technical platforms cannot resolve the data silo problem; it is therefore important to investigate the sharing motivations of all stakeholders through the lenses of benefit distribution and risk aversion.

From the perspective of supply chain resilience, Hosseini et al. [16] and Queiroz et al. [17] reviewed quantitative methods for exploring supply chain resilience and epidemic outbreaks' impacts, establishing that technology supply disruptions have emerged as a major external shock confronting manufacturing supply chains. Ivanov et al. [18] and Dubey et al. [19] demonstrated that digital technologies and the IoT can enhance supply chains' visibility and agility, but their effectiveness is hindered by the data silo problem, as companies are still reluctant to share the underlying operational data. Kusi-Sarpong et al. [20] proposed a supply chain sustainability innovation framework, establishing that cross-organizational data-sharing platforms have become a consensus for dismantling these silos. Core manufacturers, however, face a strategic paradox: Data-sharing improves the ecosystem's resilience, yet exposes them to the leakage of proprietary technologies. Drawing on the resource-based view, Brandon-Jones et al. [21] found that firms regard core data as proprietary strategic assets and enforce strict protection. Tang et al. [22] developed an evolutionary game model for indirect data-sharing in manufacturing big data consortia, showing that when firms rely on exogenous core technologies, the threats associated with technology spillovers and confidentiality breaches are amplified, and privacy and security risks constitute a barrier shaping cooperation strategies. This risk-averse tendency provides a theoretical justification for introducing the excess privacy risk cost (C_{risk}) into the game model of this paper, quantitatively characterizing the consequences of technology supply disruptions.

Regarding policy and governance, the sustainable evolution of a data-sharing ecosystem depends on sound policy formulation and platform governance. Acemoglu et al. [23] demonstrated that unregulated data markets are susceptible to pricing failures and resource misallocation, while Choi et al. [24] argued that platforms offering equitable financial incentives can subsidize enterprises' data access costs, thereby increasing their willingness to share. Over-reliance on financial incentives, however, is insufficient; without stringent regulation, enterprises exhibit opportunistic behaviors such as free-riding and superficial sharing. Wang et al. [25] used evolutionary game theory to demonstrate that penalty mechanisms for compliance violations help curb such behaviors. Spiekermann and Korunovska [26] developed a value theory for personal data, highlighting that the platform's data governance capabilities, including strict data quality review standards and a strong privacy protection architecture, are equally important. This dual approach reduces the security concerns of core manufacturers and strengthens developers' confidence in the data ecosystem.

2.2. Stochastic evolutionary game theory and random disturbances

Data-sharing is a dynamic iterative process involving multiple stakeholders with deeply intertwined interests. Given that participants exhibit bounded rationality and opportunistic tendencies, evolutionary game theory is an effective tool for analyzing data-sharing mechanisms. Static frameworks cannot capture the temporal process by which participants continuously learn, adapt, and optimize their

strategies through prolonged interactions, and evolutionary game theory has therefore been progressively applied to model diverse data-sharing scenarios [27]. This theory has been widely used to address coordination and regulatory complexities among multiple stakeholders, with scholars examining the roles of benefit allocation and penalty mechanisms. Liu et al. [28] investigated cooperative mechanisms within platform-enabled data ecosystems in the context of smart logistics, while Li and Mei [29] formulated a tripartite game model for corporate data-sharing under government oversight, demonstrating that when the benefits of sharing outweigh the losses from technology spillovers, the system converges toward a stable cooperative equilibrium. More recently, scholars have applied these models to specialized manufacturing contexts, including manufacturing big data consortia [30] and digital service ecosystems [31], defining stability conditions for multiagent data resource-sharing and value co-creation. These findings support the applicability of tripartite game models in analyzing digital manufacturing supply chains and provide the theoretical foundation for this paper's investigation of cross-organizational data collaboration in sports manufacturing.

However, traditional evolutionary models operate under the assumption of a deterministic, static external environment, a premise with clear empirical limitations. In real business ecosystems, the participants' decision-making is subject to various random disturbances, including policy shifts, market volatility, and technological blockades. Fudenberg and Harris [32] advanced the notion that integrating aggregate shocks into macro-level evolutionary dynamics is conceptually important, and Tanimoto [33] emphasized the influence of noise and stochastic environments on population-level evolutionary dynamics. Mao [34] elaborated on the mathematical framework of stochastic differential equations, providing the theoretical basis for constructing stochastic dynamic models. In applied contexts, Sun and Lei [35] integrated Gaussian white noise into tripartite evolutionary models, demonstrating that stochastic volatility changes the participants' strategic evolution paths and that the system's stability under stochastic perturbations differs from its deterministic counterpart. Regarding data-sharing in sports equipment manufacturing, the risk of technology supply disruptions carries high volatility and unpredictability; superimposing Gaussian white noise onto the replicator dynamics therefore provides a more accurate simulation of the system's evolution under uncertainty.

Stochastic evolutionary game theory is recognized as an effective analytical tool for examining dynamic decision-making under uncertainty, with important applications in supply chain management [36, 37]. Recent research from 2024–2025 has further established this methodology's value in complex supply chain governance. Xu and Zheng [38] demonstrated that under Gaussian white noise, historically stable deterministic equilibrium points become fragile, requiring tighter parameter constraints to retain stability. Studies on prefabricated building supply chains have shown that stochastic disturbance terms influence both the speed of a strategy's evolution and the convergence trajectory of each agent [39]. These findings indicate that when analyzing volatile ecosystems such as the sports equipment manufacturing sector studied in this paper, stochastic differential equations offer stronger explanatory power than classical deterministic architectures.

Despite the progress made in data-sharing game theory, important gaps remain. The existing literature largely overlooks the structural characteristics of the sports equipment manufacturing industry, where hardware data generation and software application development are organizationally separate. Moreover, few studies have mathematically operationalized technology supply disruptions within the game-theoretic framework of data-sharing. Although stochastic evolutionary models have produced insights across various disciplines, their application to the sports equipment sector remains limited.

This paper addresses these gaps by constructing a tripartite stochastic evolutionary game model that incorporates both the hardware-software structural dichotomy and Gaussian white noise to capture supply disruption risks. The contribution is twofold: Theoretically, it extends stochastic evolutionary game theory to the hardware-software cross-domain data context; practically, it provides quantitative decision-making frameworks through simulations of platform governance and penalty mechanisms.

2.3. Literature synthesis and research positioning

Integrating the preceding research streams reveals three cross-cutting gaps that motivate this study. First, the existing data-sharing game literature has not modeled the sports equipment manufacturing industry as a distinct context with its own structural characteristics. Most studies address data-sharing in generic supply chains [40], healthcare [41], or government [42], where the hardware-software dichotomy and the specific risks of technology supply disruptions are absent. Second, while stochastic evolutionary game models have been applied to environmental governance [43], green innovation [44], and port resilience [45], no study has introduced Gaussian white noise to capture the random disturbances arising from technology supply disruptions in a manufacturing data-sharing context. Third, the existing tripartite models typically treat the platform as a passive intermediary rather than an active governance agent with independent strategic choices regarding data quality oversight, accountability enforcement, and compensation mechanisms.

Table 1 provides a systematic comparison of this study with related tripartite evolutionary game models across five dimensions: Stakeholders, data-sharing context, stochastic disturbances, platform governance role, and key focus. This comparison shows that the present study is, to our knowledge, the first to model technology supply disruption risks through Gaussian white noise in a sports equipment manufacturing data-sharing ecosystem, with the platform as an independent governance agent whose strategic decisions regarding data quality and accountability interact with the choices of the manufacturers and developers.

Table 1. Comparison of this study with relevant literature.

Study	Stakeholders	Context	Stochastic	Platform Role	Key Focus
Dong et al. [1]	Government, providers, demanders	Digital government	Yes	Passive	Cross-department data-sharing
Liu et al. [28]	Platform, logistics, users	Smart logistics	No	Passive	Platform data cooperation
Li and Mei [29]	Government, enterprises, platforms	Corporate data	No	Passive	Government oversight
Zhao and Yang [7]	Providers, platform, demanders	Supply chain	No	Active	Data quality and trust
Tang et al. [22]	Manufacturers, consortium, users	Manufacturing big data	No	Coordinator	Indirect data-sharing
Zhao et al. [10]	Shipping companies, port, government	Port resilience	Yes	Operator	Extreme weather
This study	Manufacturers, platform, developers	Sports equipment manufacturing	Yes (supply disruption)	Active governance	Data-sharing under disruption risk; platform penalty mechanisms

3. Tripartite stochastic evolutionary game model

3.1. Stakeholder interactions and game structure

The data-sharing scenario within the sports equipment manufacturing industry primarily involves three core entities: The core sports equipment manufacturer (CSM), the digital transformation platform (SIDTP), and the third-party software developer (TPSD). Figure 1 presents the decision tree of this tripartite evolutionary game, mapping the eight possible strategy combinations generated by the binary choices of each participant (high-quality sharing or perfunctory sharing for manufacturers, strict governance or lax governance for platforms, and normative use or illegal abuse for developers), with each terminal node corresponding to a distinct equilibrium outcome. Figure 2 depicts the overarching system architecture, spanning the entire lifecycle from data generation through platform governance to data circulation: The flow of hardware-generated data from manufacturers to the platform, the platform's quality audit and privacy oversight mechanisms, the provisioning of governed data to downstream developers, the incentive and penalty relationships among the three parties, and the exogenous stochastic disturbances that drive the system's evolutionary dynamics.

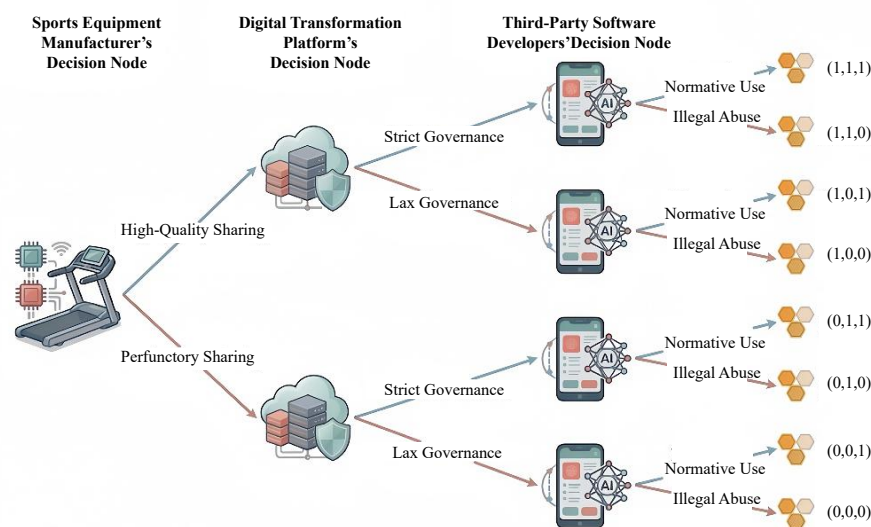


Figure 1. Tripartite evolutionary game decision tree.

Driven by their respective economic interests, these entities engage in strategic interactions. Manufacturers provide the underlying hardware data and decide whether to supply high-quality data based on a trade-off between the reputational benefits of ecosystem synergy and the security costs associated with potential technology supply disruptions. Developers use this data for application development; their compliance decisions depend on their trust in the platform's governance capabilities. The platform is the central hub for data governance and matching, aiming to maximize shared ecosystem benefits and dismantle data silos. To achieve this, the platform institutes a management framework encompassing quality audits, privacy oversight, and penalties for violations to mitigate risks. To account for the volatility of real-world business environments, this paper introduces Gaussian white noise to analyze the

impact of external stochastic disturbances on the system's evolution and the dynamic decision-making trajectories of all participants.

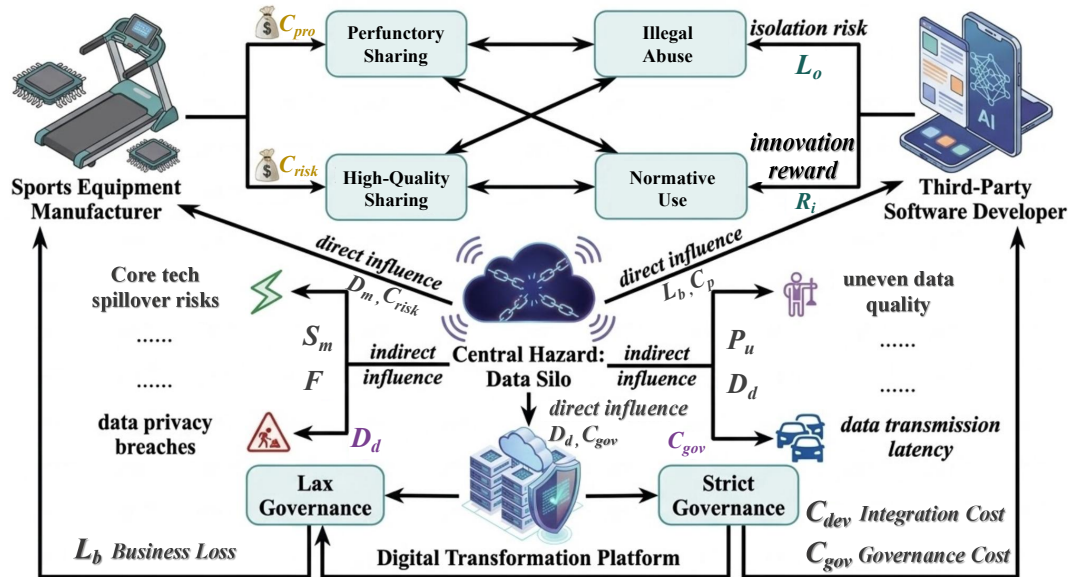


Figure 2. Tripartite evolutionary game framework.

3.2. Model assumptions

To quantitatively model the decision-making behaviors of the aforementioned entities, the relevant assumptions and parameters (as summarized in Table 2) are defined as follows.

Assumption 1. Strategy space and game timing. The CSM, SIDTP, and TPSD choose strategies high-quality sharing (x) or perfunctory sharing ($1 - x$), strict governance (y) or lax governance ($1 - y$), and normative use (z) or illegal abuse ($1 - z$), respectively, with $x, y, z \in [0, 1]$ evolving dynamically over time t . The strategic interaction unfolds in five sequential stages per evolutionary time step dt : (i) The SIDTP announces the governance parameters (a, i); (ii) the CSM chooses the data quality; (iii) the SIDTP audits with detection probability a ; (iv) the TPSD realizes returns or losses; and (v) the SIDTP executes rewards and penalties. Payoff matrix entries (Table 3) are computed as expected values over Stage (iii), with random perturbations arriving continuously throughout.

Assumption 2. Manufacturer (CSM) payoffs. The CSM receives a platform incentive subsidy S_m . Under perfunctory sharing, the CSM bears only basic data processing costs. Strict governance detects low-quality data with probability a (a serves as both the detection accuracy and trust dividend allocation proportion, both endogenously determined by the platform's governance infrastructure), exacting penalties D_m and iF (iF denotes the expected fine borne by the manufacturer under perfunctory sharing, i.e., the nominal fine F discounted by the platform's accountability and enforcement intensity i ($0 < i < 1$)). Conversely, high-quality sharing incurs the risk cost C_{risk} ($C_{risk} = c_1 + c_2 + c_3 + c_4$, where c_1 is de-identification and cleansing; c_2 is the privacy leakage premium; c_3 is the technology spillover exposure; c_4 is the supply disruption-induced risk premium). If the TPSD uses data normatively, the CSM gains a reputation R_r and, under strict governance, receives trust dividend share aG_t .

Table 2. Parameter symbols and their economic definitions.

Category	Symbol	Definition and explanation
Decision Variables	x	Probability of the core manufacturer choosing high-quality sharing ($0 \leq x \leq 1$).
	y	Probability of the digital platform choosing strict governance ($0 \leq y \leq 1$).
	z	Probability of the software developer choosing normative use ($0 \leq z \leq 1$).
Benefits	R_r	Reputational and ecological synergy benefits for hardware sales due to normative use.
	R_i	Long-term commercial innovation and realization revenue for developers.
	S_m	Digital transformation incentive subsidy provided by the platform to the manufacturer.
	S_g	Regulatory and construction subsidies provided by the government to the platform.
	G_t	Systemic trust dividends generated by effective data governance.
Costs and losses	C_{pro}	Data cost incurred by the manufacturer for perfunctory sharing.
	C_{risk}	Extra risk cost for manufacturer during high-quality sharing (e.g., technical spillover).
	C_{gov}	Technical and management costs for the platform to implement strict governance.
	C_{dev}	Commercial development and server maintenance costs for the software developer.
	C_p	Quality inspection and legal recourse costs for normative developers.
	L_b	Business losses suffered by developers due to perfunctory/low-quality data.
	L_o	Implicit reputational loss and ecological exclusion cost for illegal abuse.
Transfers and penalties	P_u	Data service/usage fees paid by the developer to the platform.
	F	Fine imposed by the platform on the noncompliant manufacturer.
	D_m	Default compensation claimed by the platform from the manufacturer.
	D_d	Directed ecological protection and damage compensation paid by the platform.
Parameters	a	Platform data governance capability coefficient ($0 < a < 1$).
	i	Platform accountability and enforcement intensity coefficient ($0 < i < 1$).
	$\sigma_1, \sigma_2, \sigma_3$	Intensity coefficients of random disturbances for the three respective agents.
	$dw(t)$	Standard Brownian motion (Wiener process).

Assumption 3. Platform (SIDTP) payoffs. An SIDTP opting for strict governance incurs computational and audit costs C_{gov} . Effective governance generates the trust dividend G_t . When CSMs share high-quality data and TPSDs utilize it normatively, the platform receives government subsidies S_g . Under lax governance, none of these costs or benefits accrue. If a TPSD obtains poor-quality data and suffers losses due to governance failure (probability $1 - a$), the platform pays compensation D_d .

Assumption 4. Developer (TPSD) payoffs. TPSDs opting for normative use must pay data service fees P_u and bear the integration and development costs C_{dev} . If the CSM provides high-quality data, compliant TPSDs achieve market innovation returns R_i and the trust dividend aG_t . However, if the CSM engages in perfunctory sharing, the resulting poor-quality underlying data trigger business losses L_b due to application failures and legal recourse costs C_p , while the platform bears the protective compensation D_d . Conversely, if a TPSD chooses illegal abuse (e.g., illegal scraping or data resale), it faces long-term market losses due to ecosystem exclusion and the implicit reputational costs L_o . Simultaneously, under strict platform governance with high-quality underlying data, violators suffer the confiscation of illicit gains and account suspension through targeted enforcement (i.e., a negative trust dividend $-aG_t$; this

bidirectional structure, with $+aG_t$ rewarding normative use and $-aG_t$ penalizing abuse, generates the $2xy(aG_t)$ term in the replicator dynamics, as detailed in Section 3.3). Notably, if the underlying data are perfunctory and of poor quality, violators simply discard the data due to a lack of speculative value, incurring neither research and development (R&D) investments nor claims for damages, resulting in net gains and losses of 0.

Assumption 5. Stochastic disturbances. The ecosystem is subject to external random disturbances (policy shifts, technological shocks) following a Gaussian distribution with a state-dependent intensity. The term $\sigma_i x_i(1 - x_i)dw_i(t)$ captures this, with the impact diminishing as the strategies approach consensus ($x_i \rightarrow 0$ or 1).

3.3. Model construction

Drawing upon the assumptions and analysis mentioned earlier, we develop a benefit matrix illustrating the interactions among the core sports equipment manufacturer (CSM), the digital transformation platform (SIDTP), and the third-party software developers (TPSD) as presented in Table 3.

3.3.1. Strategy stability analysis for the core sports equipment manufacturer

According to Table 3, the expected benefit U_{11} for the high-quality sharing strategy, the expected benefit U_{12} for the perfunctory sharing strategy, and the average expected benefit $\bar{U}_1$ of core sports equipment manufacturers are as follows:

$$U_{11} = yz(aG_t + R_r + S_m - C_{pro} - C_{risk}) + y(1 - z)(aG_t + S_m - C_{pro} - C_{risk}) + z(1 - y)(R_r + S_m - C_{pro} - C_{risk}) + (1 - y)(1 - z)(S_m - C_{pro} - C_{risk}) \quad (3.1)$$

$$U_{12} = yz(S_m - C_{pro} - iF - aD_m) + y(1 - z)(S_m - C_{pro} - aD_m) + z(1 - y)(S_m - C_{pro} - iF) + (1 - y)(1 - z)(S_m - C_{pro}) \quad (3.2)$$

$$\bar{U}_1 = xU_{11} + (1 - x)U_{12} \quad (3.3)$$

To reveal the evolutionary trajectory and steady state of the strategic decisions made by the three game subjects during the sharing process, we first solve the replication dynamics equation for the core sports equipment manufacturers as follows:

$$\begin{aligned} F(x) &= \frac{dx}{dt} = x(U_{11} - \bar{U}_1) = x(1 - x)(U_{11} - U_{12}) \\ &= x(1 - x)[y(aG_t + aD_m) + z(R_r + iF) - C_{risk}] \end{aligned} \quad (3.4)$$

The derivative of $F(x)$ is

$$\frac{dF(x)}{dx} = (1 - 2x)[y(aG_t + aD_m) + z(R_r + iF) - C_{risk}] \quad (3.5)$$

The theorem of the stability of differential equations states that the core sports equipment manufacturer's strategy must satisfy $F(x) = 0$ and $\frac{dF(x)}{dx} < 0$. Let $y_0 = \frac{C_{risk} - (R_r + iF)z}{aG_t + aD_m}$. When $y = y_0$, the replication kinetic equation $F(x) = 0$. At this point, the strategy of the core sports equipment manufacturers is in a steady state regardless of the value of x , whereas when $z = 1$, two cases will occur.

When $y < y_0$, we can obtain $\frac{dF(x)}{dx}|_{x=0} < 0$ and $\frac{dF(x)}{dx}|_{x=1} > 0$. Therefore, when $x = 0$, it is a stable state. This indicates that the strategy of the core sports equipment manufacturers is closely related to the strategy of the digital transformation platform, and when the platform deviates from a certain level of strict governance strategy, manufacturers will tend to share perfunctorily.

When $y > y_0$, we can obtain $\frac{dF(x)}{dx}|_{x=1} < 0$ and $\frac{dF(x)}{dx}|_{x=0} > 0$. Therefore, when $x = 1$, it represents a stable state. This implies that manufacturers will opt for the high-quality sharing strategy if the likelihood of the platform selecting the strict governance strategy surpasses a certain threshold.

According to the analysis above, the core sports equipment manufacturer's replication dynamic phase diagram can be obtained, as shown in Figure 3.

Table 3. Tripartite payoff matrix of the data-sharing game.

CSM strategy	SIDTP strategy	TPSD strategy	
		Normative use (z)	Illegal abuse ($1 - z$)
High-quality sharing (x)	Strict governance (y)	$aG_t + R_r + S_m - C_{pro} - C_{risk}$	$aG_t + S_m - C_{pro} - C_{risk}$
		$S_g - C_{gov} - S_m + P_u$	$-C_{gov} - S_m$
		$aG_t + R_i - C_{dev} - P_u$	$-aG_t - L_o$
	Lax governance ($1 - y$)	$R_r + S_m - C_{pro} - C_{risk}$	$S_m - C_{pro} - C_{risk}$
		$-S_m + P_u$	$-S_m$
		$R_i - C_{dev} - P_u$	$-L_o$
Perfunctory sharing ($1 - x$)	Strict governance (y)	$S_m - C_{pro} - iF - aD_m$	$S_m - C_{pro} - aD_m$
		$-C_{gov} - (1 - a)D_d + aD_m - S_m + P_u$	$-C_{gov} + aD_m - S_m$
		$-(1 - a)(C_{dev} + L_b - D_d) - C_p - P_u$	0
	Lax governance ($1 - y$)	$S_m - C_{pro} - iF$	$S_m - C_{pro}$
		$-D_d - S_m + P_u$	$-S_m$
		$-(C_{dev} + L_b) - C_p + D_d - P_u$	0

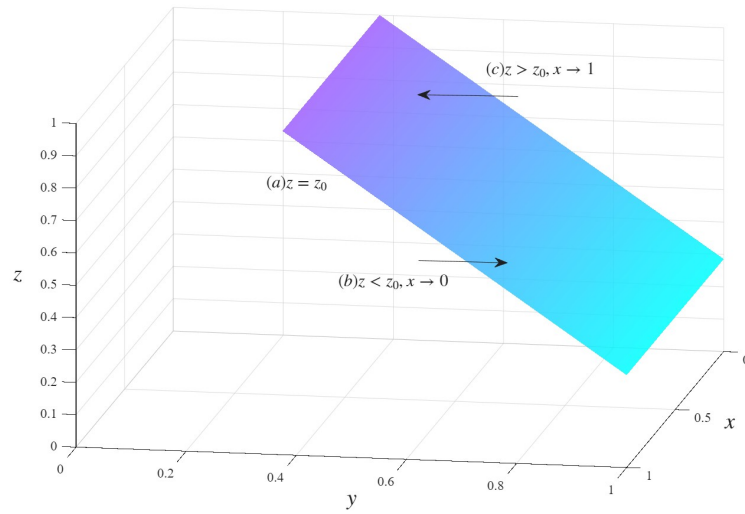


Figure 3. Replication dynamic phase diagram of the core sports equipment manufacturer.

3.3.2. Strategy stability analysis for the digital transformation platform

For the digital transformation platform, the expected benefit U_{21} of the strict governance strategy, the expected benefit U_{22} of the lax governance strategy, and the average expected benefit $\bar{U}_2$ are, respectively, as follows:

$$\begin{aligned}
 U_{21} = & xz(S_g - C_{gov} - S_m + P_u) + x(1 - z)(-C_{gov} - S_m) \\
 & + (1 - x)z(-C_{gov} - (1 - a)D_d + aD_m - S_m + P_u) \\
 & + (1 - x)(1 - z)(-C_{gov} + aD_m - S_m)
 \end{aligned} \quad (3.6)$$

$$\begin{aligned}
 U_{22} = & xz(-S_m + P_u) + x(1 - z)(-S_m) \\
 & + (1 - x)z(-D_d - S_m + P_u) + (1 - x)(1 - z)(-S_m)
 \end{aligned} \quad (3.7)$$

$$\bar{U}_2 = yU_{21} + (1 - y)U_{22} \quad (3.8)$$

The replication dynamic equation for the digital transformation platform is as follows:

$$\begin{aligned}
 F(y) = \frac{dy}{dt} &= y(U_{21} - \bar{U}_2) = y(1 - y)(U_{21} - U_{22}) \\
 &= y(1 - y)[-(aD_m)x + (aD_d)z + (S_g - aD_d)xz + aD_m - C_{gov}]
 \end{aligned} \quad (3.9)$$

The derivative of $F(y)$ is

$$\frac{dF(y)}{dy} = (1 - 2y)[-(aD_m)x + (aD_d)z + (S_g - aD_d)xz + aD_m - C_{gov}] \quad (3.10)$$

Let $z_0 = \frac{C_{gov} - aD_m + aD_mx}{aD_d + (S_g - aD_d)x}$. When $z = z_0$, the replication kinetic equation $F(y) = 0$. At this point, the strategy of the digital transformation platform is in a steady state regardless of the value of y , whereas when $z = z_0$, two cases will occur.

When $z < z_0$, we can obtain $\frac{dF(y)}{dy}|_{y=1} > 0$ and $\frac{dF(y)}{dy}|_{y=0} < 0$. Therefore, when $y = 0$, it is a stable state. This indicates that the digital transformation platform will choose the lax governance strategy when the

likelihood that the third-party software developers will select the normative use strategy is lower than a specific threshold.

When $z > z_0$, we can obtain $\frac{dF(y)}{dy}|_{y=0} > 0$ and $\frac{dF(y)}{dy}|_{y=1} < 0$. Therefore, when $y = 1$, it is a stable state. This means that if the likelihood of the third-party software developers selecting the normative use strategy exceeds a particular threshold, the digital transformation platform will select the strict governance strategy.

According to the analysis above, the digital transformation platform's replication dynamic phase diagram can be obtained, as shown in Figure 4.

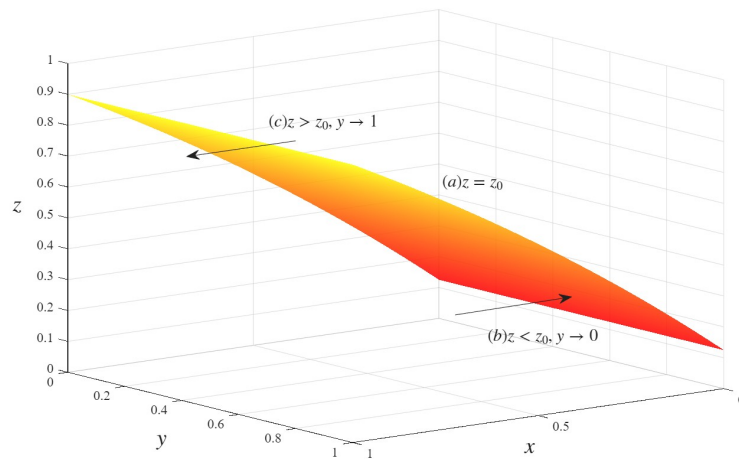


Figure 4. Replication dynamic phase diagram of digital transformation platform.

3.3.3. Strategy stability analysis for the third-party software developer

For the third-party software developers, the expected benefit U_{31} of the normative use strategy, the expected benefit U_{32} of the illegal abuse strategy, and the average expected benefit $\bar{U}_3$ are, respectively, as follows:

$$\begin{aligned} U_{31} = & xy(aG_t + R_i - C_{dev} - P_u) + x(1-y)(R_i - C_{dev} - P_u) \\ & + (1-x)y[-(1-a)(C_{dev} + L_b - D_d) - C_p - P_u] \\ & + (1-x)(1-y)[-(C_{dev} + L_b) - C_p + D_d - P_u] \end{aligned} \quad (3.11)$$

$$U_{32} = xy(-aG_t - L_o) + x(1-y)(-L_o) \quad (3.12)$$

$$\bar{U}_3 = zU_{31} + (1-z)U_{32} \quad (3.13)$$

The third-party software developer's replication dynamic equation is

$$\begin{aligned}
F(z) &= \frac{dz}{dt} = z(U_{31} - \bar{U}_3) = z(1-z)(U_{31} - U_{32}) \\
&= z(1-z)[2aG_txy + (R_i + L_o + L_b + C_p - D_d)x \\
&\quad + a(C_{dev} + L_b - D_d)y + a(D_d - C_{dev} - L_b)xy \\
&\quad + D_d - C_{dev} - L_b - C_p - P_u]
\end{aligned} \tag{3.14}$$

The derivative of $F(z)$ is

$$\begin{aligned}
\frac{dF(z)}{dz} &= (1-2z)[2aG_txy + (R_i + L_o + L_b + C_p - D_d)x \\
&\quad + a(C_{dev} + L_b - D_d)y + a(D_d - C_{dev} - L_b)xy \\
&\quad + D_d - C_{dev} - L_b - C_p - P_u]
\end{aligned} \tag{3.15}$$

Let $x_0 = \frac{P_u + C_{dev} + L_b + C_p - D_d - a(C_{dev} + L_b - D_d)y}{R_i + L_o + L_b + C_p - D_d + 2aG_txy + a(D_d - C_{dev} - L_b)y}$. When $x = x_0$, the replication kinetic equation $F(z) = 0$. At this point, the strategy of the third-party software developers is in a steady state regardless of the value of z , whereas when $x = x_0$, two cases will occur.

When $x < x_0$, we can obtain $\frac{dF(z)}{dz}|_{z=0} < 0$ and $\frac{dF(z)}{dz}|_{z=1} > 0$. The equilibrium point is thus $z = 0$. Accordingly, the third-party software developer will select the illegal abuse strategy if the likelihood of the core sports equipment manufacturers choosing the high-quality sharing strategy is lower than a specific threshold.

When $x > x_0$, we can obtain $\frac{dF(z)}{dz}|_{z=1} < 0$ and $\frac{dF(z)}{dz}|_{z=0} > 0$. The equilibrium point is thus $z = 1$. This indicates that if the probability of core sports equipment manufacturers opting for the high-quality sharing strategy exceeds a particular threshold, the third-party software developers will choose the normative use strategy.

According to the analysis above, the third-party software developer's replication dynamic phase diagram can be obtained, as shown in Figure 5.

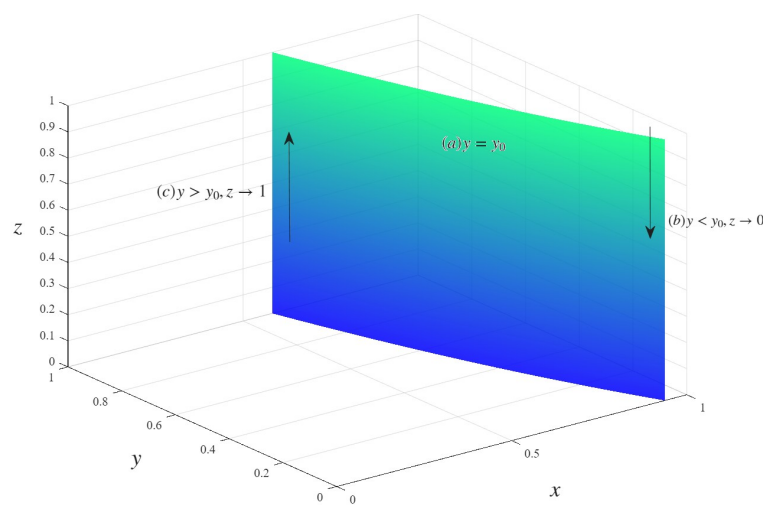


Figure 5. Replication dynamic phase diagram of the third-party software developer.

4. Stochastic stability analysis and equilibrium solutions

4.1. Stochastic disturbances to the evolutionary game

While the evolutionary game framework blends game-theoretic logic with dynamical systems, it is ill-suited to capturing strategic choice in volatile, uncertain settings. Technology supply disruptions cause direct revenue losses for the three parties and introduce indirect turbulence through abrupt policy shifts, market swings, and unforeseen technological blockades. To better account for the uncertainty inherent in strategy co-evolution under such shocks, this study introduces Gaussian stochastic perturbations into the standard evolutionary game model. Specifically, Gaussian white noise enters each replicator's dynamic as a random disturbance term [46]. The stochastic evolutionary dynamics of the core manufacturer, the digital platform, and the third-party developer are, respectively, governed by Equations (4.1) through (4.3).

$$dx(t) = x(t)(1 - x(t))[y(aG_t + aD_m) + z(R_r + iF) - C_{risk}]dt + \sigma_1 x(t)(1 - x(t))dw(t) \quad (4.1)$$

$$dy(t) = y(t)(1 - y(t))[xzS_g + (1 - x)aD_m + (1 - x)zaD_d - C_{gov}]dt + \sigma_2 y(t)(1 - y(t))dw(t) \quad (4.2)$$

$$\begin{aligned} dz(t) = z(t)(1 - z(t)) & \left[x(R_i + L_o - C_{dev}) + 2xy(aG_t) - P_u \right. \\ & \left. - (1 - x)C_p - (1 - x)(1 - ay)(C_{dev} + L_b - D_d) \right] dt + \sigma_3 z(t)(1 - z(t))dw(t) \end{aligned} \quad (4.3)$$

Here, $x(t)$, $y(t)$, and $z(t)$ respectively denote the population shares adopting high-quality sharing, strict governance, and normative use within the manufacturer, platform, and developer groups. The term $w(t)$ denotes a one-dimensional standard Brownian motion, i.e., a continuous-time stochastic process with irregular, nondifferentiable sample paths. Its increment $dw(t)$ captures Gaussian white noise satisfying $\Delta w = w(t + h) - w(t) \sim N(0, h)$ for $t > 0$ and $h > 0$. The parameters σ_1 , σ_2 , and σ_3 measure the stochastic perturbation intensity for the three agents.

We clarify that C_{risk} and σ_i are not independent uncertainties but two complementary dimensions of the same technology supply disruption risk, with C_{risk} capturing its first-moment deterministic cost that enters the drift term and σ_i capturing its second-moment volatility that enters the diffusion term; consequently, although the common Brownian motion $w(t)$ perturbs all three participants, only the manufacturer bears the direct disruption risk through C_{risk} (notably its component c_4), whereas the platform and developer bear the indirect risk transmitted through reduced data availability, transaction interruption, and ecosystem instability.

4.2. Stability analysis of equilibrium points

According to the stability theory for stochastic differential equations, the parameter space can be further characterized by examining the three-party system at the initial time $t = 0$ with $x(0) = 0$, $y(0) = 0$, $z(0) = 0$. The trivial solution of the foregoing equations implies that, without white noise, the system would remain stationary indefinitely. In practice, however, this ideal noise-free state is rarely attainable given the stochastic risks posed by technology supply disruptions. Using established stability criteria for

stochastic differential equations [47, 48], we conduct a stability analysis on the evolutionary dynamics of the core manufacturer (4.1), digital platform (4.2), and third-party developer (4.3).

Given a stochastic differential equation of the general form

$$dx(t) = f(t, x(t))dt + g(t, x(t))dw(t), \quad x(t_0) = x_0 \quad (4.4)$$

where $x(t)$, $f(t, x)$, and $g(t, x)$ are scalar-valued. Suppose there are a function $V(t, x)$ and positive constants c_1, c_2 such that $c_1|x|^p \leq V(t, x) \leq c_2|x|^p$ for all $t \geq 0$. If a constant $\gamma > 0$ satisfies $LV(t, x) \leq -\gamma V(t, x)$ for $t \geq 0$, where the infinitesimal generator is $LV(t, x) = V_t(t, x) + V_x(t, x)f(t, x) + \frac{1}{2}g^2(t, x)V_{xx}(t, x)$, then the trivial solution achieves p -th moment exponential stability, with the bound $E|x(t, x_0)|^p < \frac{c_2}{c_1}|x_0|^p e^{-\gamma t}$ for all $t \geq 0$. These conditions yield the stability criteria for $dx(t)$, $dy(t)$, and $dz(t)$.

Setting $V(t, x) = x$, $V(t, y) = y$, and $V(t, z) = z$ with $c_1 = c_2 = 1$, $p = 1$, and $\gamma = 1$ yields $LV(t, x) = f(t, x)$, $LV(t, y) = f(t, y)$, and $LV(t, z) = f(t, z)$. For the trivial solutions of Equations (4.1)–(4.3) to be p -th moment exponentially stable and constitute a unique evolutionarily stable strategy (ESS), the following must hold:

$$y(aG_t + aD_m) + z(R_r + iF) - C_{risk} + 1 < 0 \quad (4.5)$$

$$xzS_g + (1 - x)aD_m + (1 - x)zaD_d - C_{gov} + 1 < 0 \quad (4.6)$$

$$x(R_i + L_o - C_{dev}) + 2xy(aG_t) - P_u - (1 - x)C_p - (1 - x)(1 - ay)(C_{dev} + L_b - D_d) + 1 < 0 \quad (4.7)$$

Under this regime, the manufacturer settles on perfunctory sharing, the platform on lax governance, and the developer on illegal abuse. Conversely, if the trivial solutions of Equations (4.1)–(4.3) are p -th moment exponentially unstable while a unique ESS exists, the requisite inequalities are:

$$y(aG_t + aD_m) + z(R_r + iF) - C_{risk} > 1 \quad (4.8)$$

$$xzS_g + (1 - x)aD_m + (1 - x)zaD_d - C_{gov} > 1 \quad (4.9)$$

$$x(R_i + L_o - C_{dev}) + 2xy(aG_t) - P_u - (1 - x)C_p - (1 - x)(1 - ay)(C_{dev} + L_b - D_d) > 1 \quad (4.10)$$

Under this regime, the manufacturer embraces high-quality sharing, the platform enforces strict governance, and the developer complies with normative use.

4.3. Stochastic evolutionary equilibrium solution

Consider the one-dimensional Itô equation $dx(t) = f(t, x(t))dt + g(t, x(t))dw(t)$ with $x(t_0) = x_0$. We use the Euler–Maruyama method for stochastic numerical approximation [49, 50]. The interval $t \in [t_0, T]$ is partitioned into N equal subintervals: $t_0 < t_1 < t_2 < \dots < t_{N-1} < t_N = T$, with a step size

$h = (T - t_0)/N$. Applying the Euler scheme to Equations (4.1)–(4.3) yields the discretized recurrence formulas below, from which numerical solutions are obtained by simulation.

$$x(t_{n+1}) = x(t_n) + hf(t_n, x(t_n)) + \Delta w_n g(t_n, x(t_n)) \quad (4.11)$$

$$\begin{aligned} x(t_{n+1}) = x(t_n) + h \cdot x(t_n)(1 - x(t_n)) & [y(t_n)(aG_t + aD_m) + z(t_n)(R_r + iF) - C_{risk}] \\ & + \sigma_1 x(t_n)(1 - x(t_n))\Delta w_n \end{aligned} \quad (4.12)$$

$$\begin{aligned} y(t_{n+1}) = y(t_n) + h \cdot y(t_n)(1 - y(t_n)) & [x(t_n)z(t_n)S_g + (1 - x(t_n))aD_m \\ & + (1 - x(t_n))z(t_n)aD_d - C_{gov}] + \sigma_2 y(t_n)(1 - y(t_n))\Delta w_n \end{aligned} \quad (4.13)$$

$$\begin{aligned} z(t_{n+1}) = z(t_n) + h \cdot z(t_n)(1 - z(t_n)) & [x(t_n)(R_i + L_o - C_{dev}) + 2x(t_n)y(t_n)(aG_t) \\ & - P_u - (1 - x(t_n))C_p - (1 - x(t_n))(1 - ay(t_n))(C_{dev} + L_b - D_d)] \\ & + \sigma_3 z(t_n)(1 - z(t_n))\Delta w_n \end{aligned} \quad (4.14)$$

4.4. Analysis of the stochastic evolutionary game model

The Strategy Equilibrium Point(SEPs) would be $E_1(0, 0, 0)$, $E_2(0, 0, 1)$, $E_3(0, 1, 0)$, $E_4(1, 0, 0)$, $E_5(0, 1, 1)$, $E_6(1, 0, 1)$, $E_7(1, 1, 0)$, and $E_8(1, 1, 1)$ when $F(x) = 0$, $F(y) = 0$, and $F(z) = 0$ are satisfied, where $F(x)$, $F(y)$, and $F(z)$ are the replicator dynamics equations established in Section 3.3. The initial step to validate the SEP involves computation of the Jacobian matrix of the tripartite evolution system.

$$J = \begin{bmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{bmatrix} = \begin{bmatrix} \frac{\partial F(x)}{\partial x} & \frac{\partial F(x)}{\partial y} & \frac{\partial F(x)}{\partial z} \\ \frac{\partial F(y)}{\partial x} & \frac{\partial F(y)}{\partial y} & \frac{\partial F(y)}{\partial z} \\ \frac{\partial F(z)}{\partial x} & \frac{\partial F(z)}{\partial y} & \frac{\partial F(z)}{\partial z} \end{bmatrix} \quad (4.15)$$

The derivations of the core elements within the matrix are as follows:

$$\begin{cases} F_{11} = (1 - 2x)[y(aG_t + aD_m) + z(R_r + iF) - C_{risk}] \\ F_{12} = x(1 - x)(aG_t + aD_m) \\ F_{13} = x(1 - x)(R_r + iF) \\ F_{21} = y(1 - y)[zS_g - aD_m - zaD_d] \\ F_{22} = (1 - 2y)[xzS_g + (1 - x)aD_m + (1 - x)zaD_d - C_{gov}] \\ F_{23} = y(1 - y)[xS_g + (1 - x)aD_d] \\ F_{31} = z(1 - z)[(R_i + L_o + C_p + L_b - D_d) + ay(2G_t - C_{dev} - L_b + D_d)] \\ F_{32} = z(1 - z)[2xaG_t + a(1 - x)(C_{dev} + L_b - D_d)] \\ F_{33} = (1 - 2z)[x(R_i + L_o - C_{dev}) + 2xy(aG_t) - P_u \\ \quad - (1 - x)C_p - (1 - x)(1 - ay)(C_{dev} + L_b - D_d)] \end{cases} \quad (4.16)$$

The stability of the strategy combination can be evaluated using Lyapunov's first method. An ESS is achieved when all eigenvalues of the Jacobian matrix are negative, indicating asymptotic stability of

the SEP. Conversely, if any eigenvalue possesses a positive real part, the SEP becomes unstable. By substituting the spatial coordinates of each pure strategy equilibrium into the Jacobian matrix J , we obtain the corresponding eigenvalues ($\lambda_1, \lambda_2, \lambda_3$) for the tripartite game system, which are summarized in Table 4.

Table 4. System equilibrium points and their corresponding eigenvalues.

Equilibrium point	λ_1	λ_2	λ_3
$E_1(0, 0, 0)$	$-C_{risk}$	$aD_m - C_{gov}$	$D_d - C_p - P_u - L_b - C_{dev}$
$E_2(0, 0, 1)$	$R_r + iF - C_{risk}$	$aD_d - C_{gov} + aD_m$	$C_{dev} + L_b + C_p + P_u - D_d$
$E_3(0, 1, 0)$	$aG_t + aD_m - C_{risk}$	$C_{gov} - aD_m$	$(1 - a)(D_d - L_b - C_{dev}) - C_p - P_u$
$E_4(1, 0, 0)$	C_{risk}	$-C_{gov}$	$R_i + L_o - C_{dev} - P_u$
$E_5(0, 1, 1)$	$aG_t + aD_m + R_r + iF - C_{risk}$	$C_{gov} - aD_m - aD_d$	$C_p + P_u - (1 - a)(D_d - L_b - C_{dev})$
$E_6(1, 0, 1)$	$C_{risk} - R_r - iF$	$S_g - C_{gov}$	$C_{dev} + P_u - R_i - L_o$
$E_7(1, 1, 0)$	$C_{risk} - aG_t - aD_m$	C_{gov}	$R_i + L_o - C_{dev} - P_u + 2aG_t$
$E_8(1, 1, 1)$	$C_{risk} - aG_t - aD_m - R_r - iF$	$C_{gov} - S_g$	$C_{dev} + P_u - R_i - L_o - 2aG_t$

Table 5. Stability analysis of equilibrium points and conditions for ESS.

Equilibrium point	Eigenvalue signs	Stability	ESS conditions
$E_1(0, 0, 0)$	$-, *, -$	ESS (conditional)	$C_{gov} > aD_m$
$E_2(0, 0, 1)$	$*, +, *$	Unstable	—
$E_3(0, 1, 0)$	$*, *, -$	ESS (conditional)	$aD_m > C_{gov}$ and $C_{risk} > aD_m + aG_t$
$E_4(1, 0, 0)$	$+, -, +$	Unstable	—
$E_5(0, 1, 1)$	$*, *, +$	Unstable	—
$E_6(1, 0, 1)$	$*, *, -$	ESS (conditional)	$S_g < C_{gov}$, $R_r + iF > C_{risk}$, and $R_i + L_o > C_{dev} + P_u$
$E_7(1, 1, 0)$	$+, *, +$	Unstable	—
$E_8(1, 1, 1)$	$*, -, *$	ESS (conditional)	$aG_t + aD_m + R_r + iF > C_{risk}$, $S_g > C_{gov}$, and $R_i + L_o + 2aG_t > C_{dev} + P_u$

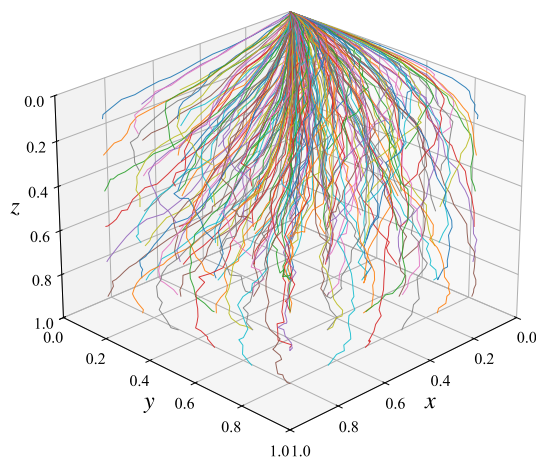
In the current data-sharing ecosystem of the sports equipment manufacturing industry, the lack of unified data standards and an incomplete compliance framework often subject data consumers (third-party software developers) to high legal recourse and evidence collection costs (C_p) when dealing with inferior data. In real-world business environments, these legal recourse costs typically far exceed the direct compensation (D_d) obtainable from the platform. Consequently, based on industry realities, this paper establishes a fundamental constraint: Legal recourse costs exceed compensation (i.e., $C_p > D_d$).

By introducing this constraint into the eigenvalue expressions derived in Table 4, it becomes evident

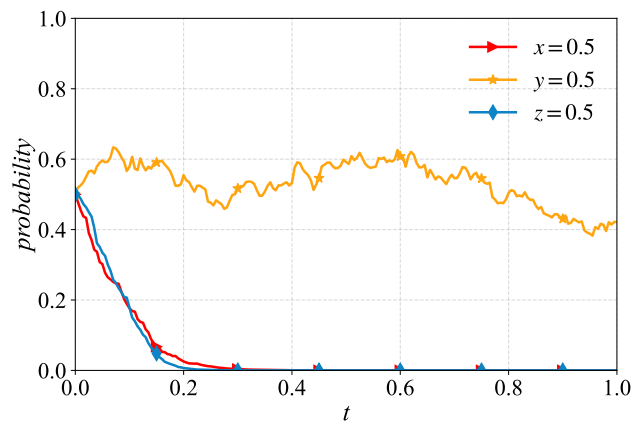
that $\lambda_3 > 0$ for $E_2(0, 0, 1)$, $\lambda_1 > 0$ for $E_4(1, 0, 0)$, $\lambda_3 > 0$ for $E_5(0, 1, 1)$, and $\lambda_1 > 0$ for $E_7(1, 1, 0)$. Because at least one eigenvalue is strictly positive, these four equilibrium points are unconditionally unstable. After eliminating these unstable points, the system retains only four potential locally asymptotically stable points under the deterministic setting (i.e., Scenarios 1 through 4). The specific parameter conditions required for these remaining points to achieve an evolutionarily stable strategy (ESS) are summarized in Table 5.

4.4.1. Scenario 1: Complete collapse $E_1(0, 0, 0)$

As illustrated in Figure 6, the three-dimensional evolutionary trajectory and the two-dimensional equilibrium cross-section are presented for Scenario 1. The stability condition for this equilibrium point is $C_{gov} > aD_m$ in Table 5. Under this condition, the ESS of the system is $(0, 0, 0)$, indicating that the core manufacturer chooses perfunctory sharing, the digital platform chooses lax governance, and the third-party software developer chooses illegal abuse. In this scenario, the data-sharing ecosystem lacks fundamental endogenous driving forces. The cost incurred by the digital platform to implement strict data governance (C_{gov}) far exceeds the potential penal compensation recoverable from violators (aD_m), prompting the platform to completely abandon quality control. Due to the absence of underlying security safeguards and systemic trust mechanisms, the manufacturer's acute privacy risks are not offset by reciprocal benefits, and the software developers confront high adverse selection risks. Consequently, the optimal rational decision for all three tripartite agents is to avert risk and withdraw from the collaboration, culminating in a complete stagnation of the data trading ecosystem. In this situation, the platform reaches a stable state first (Figure 6(b), curve y).



(a) Three-dimensional evolutionary trajectory.



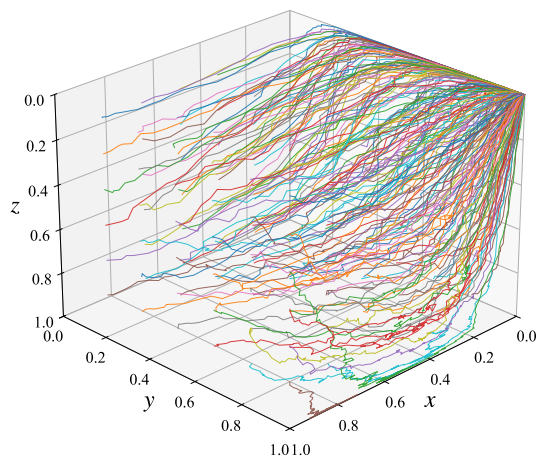
(b) Equilibrium state under $E_1(0, 0, 0)$.

Figure 6. Evolutionary dynamics under Scenario 1 ($E_1(0, 0, 0)$).

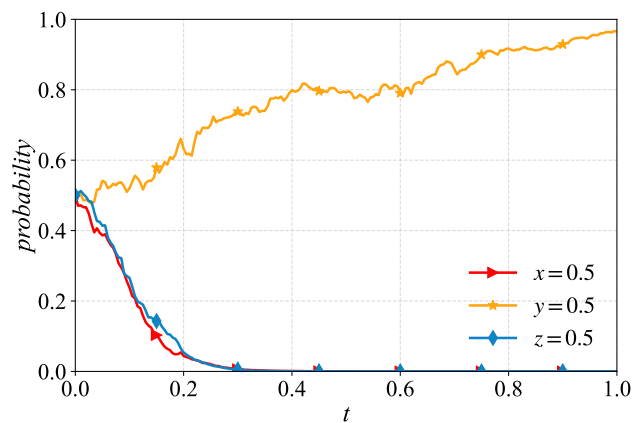
4.4.2. Scenario 2: Unilateral governance $E_3(0, 1, 0)$

As illustrated in Figure 7, the three-dimensional evolutionary trajectory and the two-dimensional equilibrium cross-section are presented for Scenario 2. The stability conditions for this equilibrium point are $aD_m > C_{gov}$ and $C_{risk} > aD_m + aG_t$ in Table 5. Under these conditions, the ESS of the system is $(0, 1, 0)$, indicating that the core manufacturer chooses perfunctory sharing, the digital platform adopts

strict governance, and the third-party software developer engages in illegal abuse. The evolutionary trajectory y representing the platform's strategy ascends rapidly after an initial period of oscillation, ultimately stabilizing at 1. Conversely, the trajectories x and z , representing the supply and demand sides, respectively, exhibit an accelerated downward trend, eventually converging to 0. In this scenario, the digital transformation platform demonstrates a proactive willingness to maintain ecological order and enacts stringent data governance. However, due to an inherent imbalance in the underlying cost-benefit structure, high compliance risks (C_{risk}) and high legal recourse costs (C_p) compel both the manufacturer and the developer to circumvent transactions. This dynamic results in a state of unilateral systemic failure, characterized by idle platform resources and a completely interrupted data flow. In this situation, the platform reaches a stable state first (Figure 7(b), curve y).



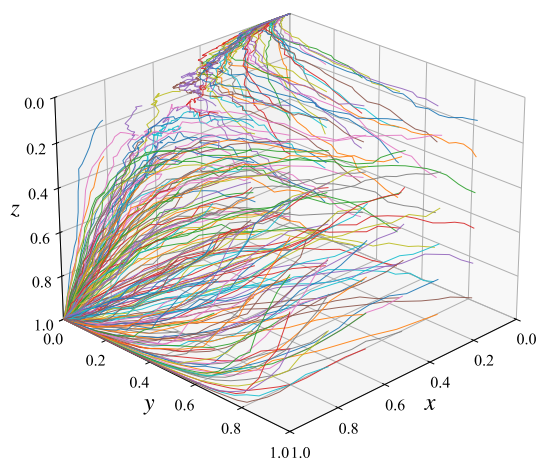
(a) Three-dimensional evolutionary trajectory.



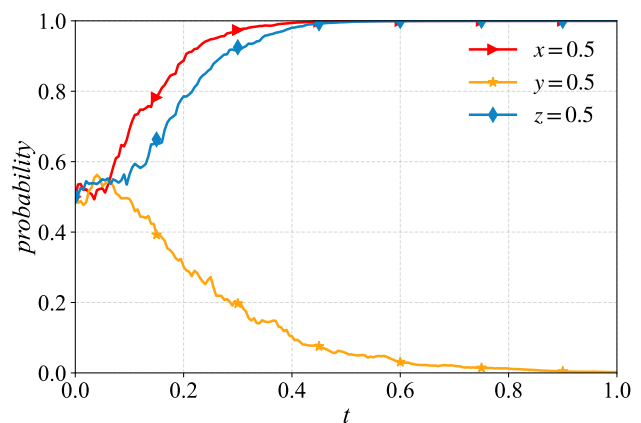
(b) Equilibrium state under $E_3(0, 1, 0)$.

Figure 7. Evolutionary dynamics under Scenario 2 ($E_3(0, 1, 0)$).

4.4.3. Scenario 3: Spontaneous cooperation $E_6(1, 0, 1)$



(a) Three-dimensional evolutionary trajectory.

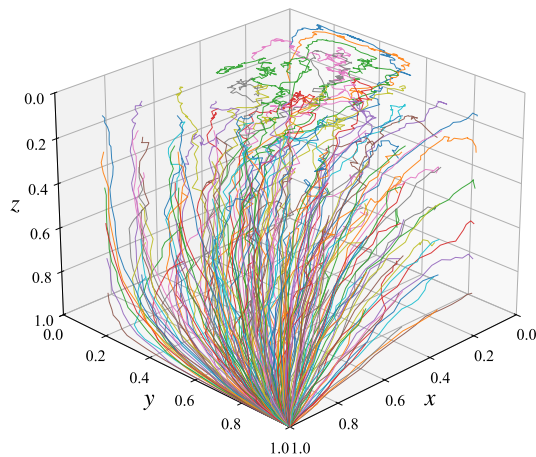


(b) Equilibrium state under $E_6(1, 0, 1)$.

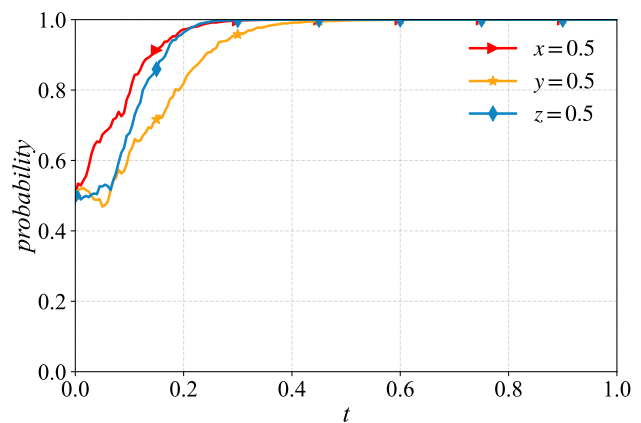
Figure 8. Evolutionary dynamics under Scenario 3 ($E_6(1, 0, 1)$).

As illustrated in Figure 8, the three-dimensional evolutionary trajectory and the two-dimensional equilibrium cross-section are presented for Scenario 3. The stability conditions for this equilibrium point are $S_g < C_{gov}$, $R_r + iF > C_{risk}$, and $R_i + L_o > C_{dev} + P_u$ in Table 5. Under these conditions, the ESS of the system converges to $(1, 0, 1)$, indicating that the core sports equipment manufacturer chooses high-quality sharing, the digital transformation platform adopts lax governance, and the third-party software developer adheres to normative use. The evolutionary trajectories x and z , representing the supply and demand sides, respectively, ascend rapidly and converge to 1.0 after absorbing severe initial random fluctuations. Conversely, the trajectory y , representing the platform's strategy, exhibits a pronounced downward trend, eventually dissipating to 0. This configuration reflects a spontaneous, market-driven alignment between data supply and demand. However, constrained by insufficient government subsidies (S_g), the digital platform lacks the necessary economic incentives to internalize the costs of strict governance (C_{gov}), leading to a strategic regulatory vacuum. In the presence of exogenous random disturbances, the lack of a verified quality control mechanism means that inferior data will trigger a rapid collapse of this spontaneous cooperative alignment, exposing the systemic vulnerability of the ecosystem. In this situation, the manufacturer reaches a stable state first (Figure 8(b), curve x).

4.4.4. Scenario 4: System-optimal cooperative equilibrium $E_8(1, 1, 1)$



(a) Three-dimensional evolutionary trajectory.



(b) Equilibrium state under $E_8(1, 1, 1)$.

Figure 9. Evolutionary dynamics under Scenario 4 ($E_8(1, 1, 1)$).

As illustrated in Figure 9, the three-dimensional evolutionary trajectory and the two-dimensional equilibrium cross-section are presented for Scenario 4. The stability conditions for this equilibrium point are $aG_t + aD_m + R_r + iF > C_{risk}$, $S_g > C_{gov}$, and $R_i + L_o + 2aG_t > C_{dev} + P_u$ in Table 5. Under these conditions, the ESS of the system converges to $(1, 1, 1)$, indicating that the core sports equipment manufacturer chooses high-quality sharing, the digital transformation platform enforces strict governance, and the third-party software developer adheres to normative use. After overcoming initial Gaussian white noise interference, the evolutionary trajectories x , y , and z , representing the tripartite agents, exhibit a clear upward trend and stabilize at the system-optimal cooperative equilibrium boundary of 1.0. This configuration represents the ideal optimal state sought by this theoretical model. In this scenario, governmental incentives (S_g) and the platform's efficient governance mechanism

create a synergistic effect. The platform's strict oversight effectively mitigates the developer's sunk costs, while a well-designed reputation and profit-sharing mechanism ($R_r + iF$) fully compensates the manufacturer for its privacy exposure risks (C_{risk}). Consequently, the expected returns for all three stakeholders surpass their respective input costs, creating a positive feedback loop within the ecosystem. This incentive-compatible structure effectively resists exogenous random disturbances, thereby realizing the highly efficient circulation and value maximization of data resources across the sports manufacturing industry. In this situation, the platform reaches a stable state first (Figure 9(b), curve y).

5. Numerical simulations and parameter sensitivity analysis

5.1. Parameter selection and calibration

To study the dynamic evolution trajectory of the tripartite game system and verify the stability properties derived in Section 4 under parameter variations, we conducted numerical simulation experiments. In practical scenarios, multiple parameters are difficult to measure precisely, and the setting of the simulation parameters therefore follows the principles described below.

First, the assignment of baseline parameters draws on multiple sources: (a) Published data-sharing and evolutionary game literature, which provides value ranges and setting conventions for parameters such as reputation benefits, trust dividends, and governance capability coefficients [2, 18]; (b) industry reports on sports platform economics and digital transformation incentives [3]; (c) expert interviews conducted with specialists in relevant fields, in which the interviewees included two senior operations managers from sports equipment manufacturers, a technical director of an industrial internet platform serving the sports manufacturing sector, a government official from the Provincial Department of Industry and Information Technology, and an academic researcher specializing in industrial data governance, whose professional insights informed the identification of reasonable parameter ranges; and (d) the conditional stability thresholds derived in Table 5, which provide theoretical bounds for the feasible parameter space. For parameters that cannot be determined from a single empirical source, calibrated values are obtained through iterative simulation tests. Sensitivity analysis is conducted on the key parameters to confirm the robustness of the qualitative results. For the expert interviews, the interviewees were selected by purposive sampling targeting practitioners with direct experience in data governance within sports equipment manufacturing. Each interview lasted 45 to 60 minutes and followed a semi-structured protocol covering three themes: (i) Data-sharing cost structures, (ii) platform governance parameters, and (iii) perceived technology supply disruption risks. Transcripts were independently coded by two researchers, with disagreements resolved through discussion to consensus. Qualitative judgments were converted into parameter values by anchoring each parameter to the consensus estimate, then cross-checking the resulting range against the literature and the theoretical stability thresholds in Table 5 to fix the baseline values.

Second, all monetary parameters in the model are expressed in consistent relative units. Since the evolutionary game analysis focuses on the comparative magnitudes and threshold conditions among parameters rather than their absolute levels, the simulation results depend on the relative relationships among parameters, not on the specific choice of monetary unit. Non-monetary parameters (probabilities, coefficients, and proportions) are dimensionless. All monetary parameters are expressed in consistent relative units because the stability conditions in Table 5 involve only relative magnitudes, and each value in Table 6 is anchored to an observable quantity, with the sensitivity analysis confirming that

our qualitative results are robust and that the reported thresholds remain illustrative rather than precise predictions (Section 6.2).

Table 6. Simulation parameter settings and calibration rationale.

Parameter	Baseline value	Source and calibration rationale
R_r	22	Reputation benefit in evolutionary games [2]; sensitivity: {7, 18, 22, 48}.
R_i	35	Annual innovation revenue for sports platform developers [3].
S_m	8	China's smart manufacturing pilot subsidy rates ($\approx 8\text{--}12\%$ of equipment cost).
S_g	15	The government's digital transformation fund for platform operators.
G_t	10	Ecosystem trust dividend [23].
C_{pro}	5	Annualized data collection and processing cost for medium-scale manufacturer.
C_{risk}	36	Privacy de-identification and technology spillover risk; sensitivity: {16, 36, 52, 72}.
C_{gov}	12	Platform data governance operational cost ($\approx 8\text{--}12\%$ of operating budget).
C_{dev}	20	Annualized unit development cost (3% discount, 7-year horizon).
C_p	38	Legal and evidence costs for data quality disputes; sensitivity: {18, 38, 55, 85}.
L_b	25	Annual business loss from application failure (low-quality data).
L_o	30	Long-term market exclusion and reputational cost ($\approx 120\%$ of L_b).
P_u	10	Annual data service fee for manufacturing data application programming interface(API) access.
F	15	Standard penalty clauses in platform data-sharing agreements.
D_m	12	Default compensation in manufacturing data-sharing contracts ($\approx 150\%$ of P_u).
D_d	8	Platform compensation to affected developers ($\approx P_u$ less administrative deduction).
a	0.55	Platform data governance capability; mid-range detection accuracy; sensitivity: {0.18, 0.38, 0.55, 0.78}.
i	0.65	Platform accountability and enforcement; moderate to high; sensitivity: {0.15, 0.35, 0.65, 0.85}.
σ_1	0.5	Stochastic disturbance intensity; moderate baseline; robust per sensitivity analysis.
σ_2	0.5	$= \sigma_1$; symmetric baseline for cross-stakeholder comparison.
σ_3	0.5	$= \sigma_1$; symmetric baseline for cross-stakeholder comparison.

Third, the controlled variable principle is adopted in the parameter sensitivity analyses. When specific parameter changes occur, other variables are held constant to minimize linkage interference. The deterministic ESS conditions in Section 4 serve as theoretical benchmarks for interpreting the simulation outcomes.

Fourth, for each parameter configuration under stochastic disturbances, multiple independent simulation runs (a minimum of 10 per configuration) were performed. The evolutionary trajectories shown in the figures are representative trajectories selected from these runs, chosen to best reflect the general evolutionary pattern observed across all runs under that configuration [51, 52].

Through the abovementioned design, the dynamic evolution trends of the tripartite strategy equilibrium can be systematically explored. The specific parameter settings are detailed in Table 6.

To keep the parameterization specific to the sports equipment manufacturing industry, the key monetary parameters in Table 6 are anchored to industry-specific sources, reflecting its hardware-software structural dichotomy.

5.2. Impact of initial strategy probabilities on the system's evolution

To investigate how the initial strategic choices of each stakeholder influence the evolutionary trajectory of the tripartite system, the numerical simulations in this section adopt the baseline parameter settings from Table 6. Under this configuration, the ESS of the system converges to $(0, 1, 0)$, meaning that the manufacturer chooses perfunctory sharing, the platform adopts strict governance, and the software developer engages in illegal abuse. With this baseline, we adjust the initial probabilities of the three agents to observe the dynamic evolution of the system under stochastic disturbances.

5.2.1. Impact of the manufacturer's initial strategy (x)

Figure 10(a) and Figure 10(b) illustrate the evolutionary impact of varying the manufacturer's initial willingness to participate ($x \in \{0.1, 0.3, 0.5, 0.7\}$) while holding the initial probabilities of the digital platform and the software developer constant ($y = 0.5, z = 0.5$).

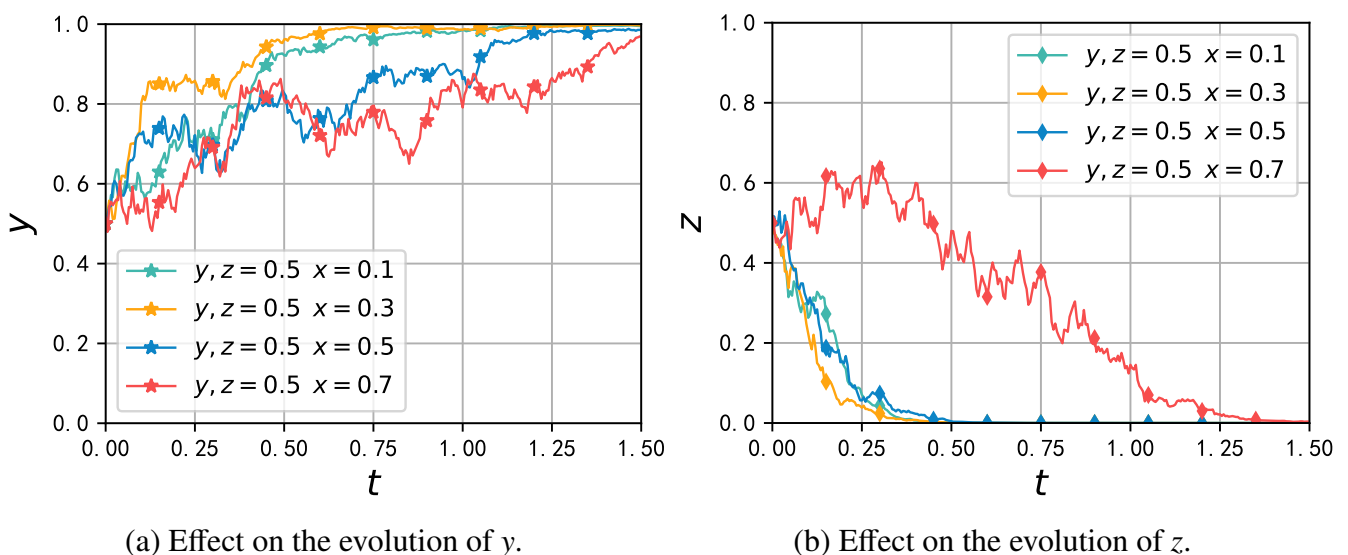


Figure 10. The effect of a change in the initial probability x on the evolution of the system.

An analysis of the simulation results reveals that regardless of how high the manufacturer's initial enthusiasm is (as x increases from 0.1 to 0.7), the evolutionary curve z , representing the software developer, inevitably collapses to 0 (as shown in Figure 10(b)). This collapse simultaneously drags the

manufacturer (curve x) on the supply side into failure, eventually plunging to 0 as well. Conversely, as depicted in Figure 10(a), the curve y , representing the platform's commitment to governance, ascends and stabilizes at 1.0.

This demonstrates that in an environment of structural disequilibrium (characterized by prohibitively high costs associated with underlying compliance and data anonymization, C_{risk}), relying exclusively on the initial enthusiasm of the data supply side cannot reverse the system's trajectory toward a negative locked-in state. Although a high initial participation rate from manufacturers provides the digital platform with sufficient data samples, the supply side will ultimately deplete its confidence and withdraw from the ecosystem due to the protracted erosion of its core interests.

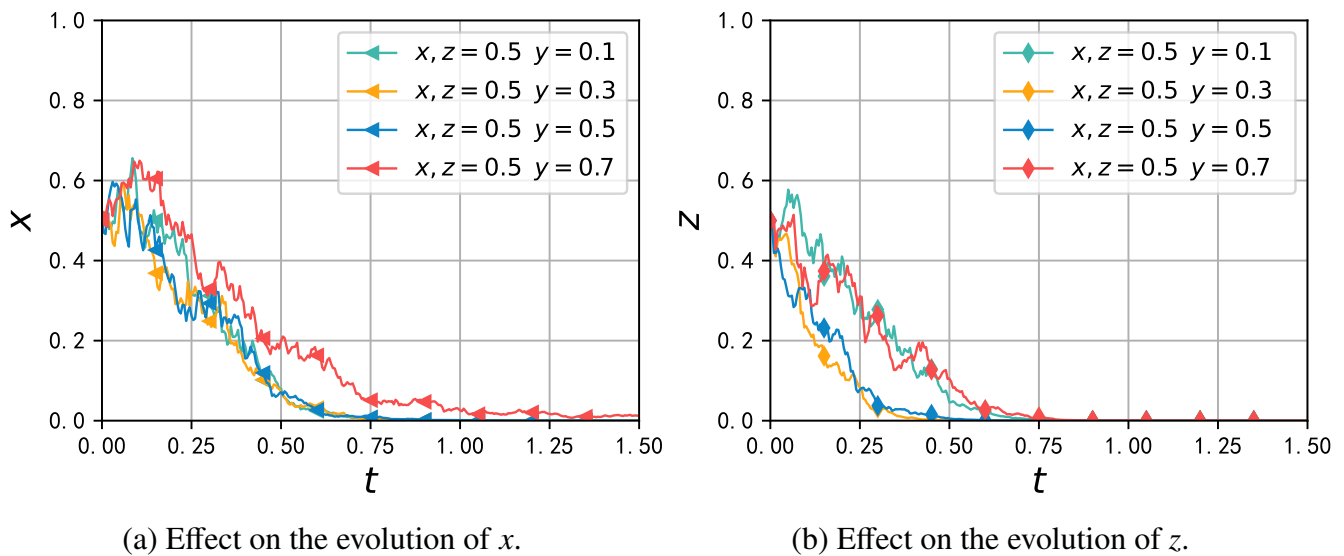


Figure 11. The effect of a change in the initial probability y on the evolution of the system.

5.2.2. Impact of the platform's initial strategy (y)

Figure 11(a) and Figure 11(b) illustrate the evolutionary impact of varying the digital platform's initial willingness to enforce strict governance ($y \in \{0.1, 0.3, 0.5, 0.7\}$) while maintaining the initial probabilities of the core manufacturer and the software developer at a neutral 0.5 ($x = 0.5, z = 0.5$).

The simulation results indicate that regardless of the intensity of the platform's initial governance-related enthusiasm, it is completely insufficient to reverse the systemic exodus of both the supply and demand sides from the ecosystem. Following brief initial fluctuations, the evolutionary trajectories x (Figure 11(a)) and z (Figure 11(b)) precipitously collapse to 0, whereas the platform's strategy y persistently ascends and locks in at the high-level boundary of 1.0.

This phenomenon points to a systemic vulnerability: Although the platform's governance actions reflect a determination to uphold ecological rules, it cannot unilaterally sustain the stability of the tripartite data-sharing network. Faced with high privacy-exposure risks for manufacturers (C_{risk}) and high legal recourse costs for software developers (C_p), the absence of a compensation and risk-hedging mechanism drives both end-users toward a risk-averse nonparticipation strategy.

5.2.3. Impact of the developer's initial strategy (z)

Figure 12(a) and Figure 12(b) illustrate the evolutionary trajectories when the developer's initial willingness to engage in normative use is incrementally increased ($z \in \{0.1, 0.3, 0.5, 0.7\}$), while the initial probabilities of the core manufacturer and the digital platform are held constant ($x = 0.5, y = 0.5$).

As demonstrated by the simulation results, even with a high initial participation willingness from the downstream demand side, the supply-side manufacturer (curve x , shown in Figure 12(a)) still rapidly collapses to 0, compelling the platform's strategy (curve y , shown in Figure 12(b)) to ascend to 1.0. Caught in this negative chain reaction, the developer's strategy (z) ultimately cannot escape the fate of plunging to 0.

A further dynamic concerns: Although third-party software developers constitute the core demand side of the data market, they confront high legal recourse costs (C_p) within the current game environment. When these enforcement costs far exceed the direct cost of data procurement or potential compensation, the developers' tolerance for inferior data quality diminishes significantly. Lacking an effective institutional mechanism to safeguard their fundamental interests, the commercial closed-loop of data transactions fractures, rendering the platform's stringent governance mechanisms practically meaningless.

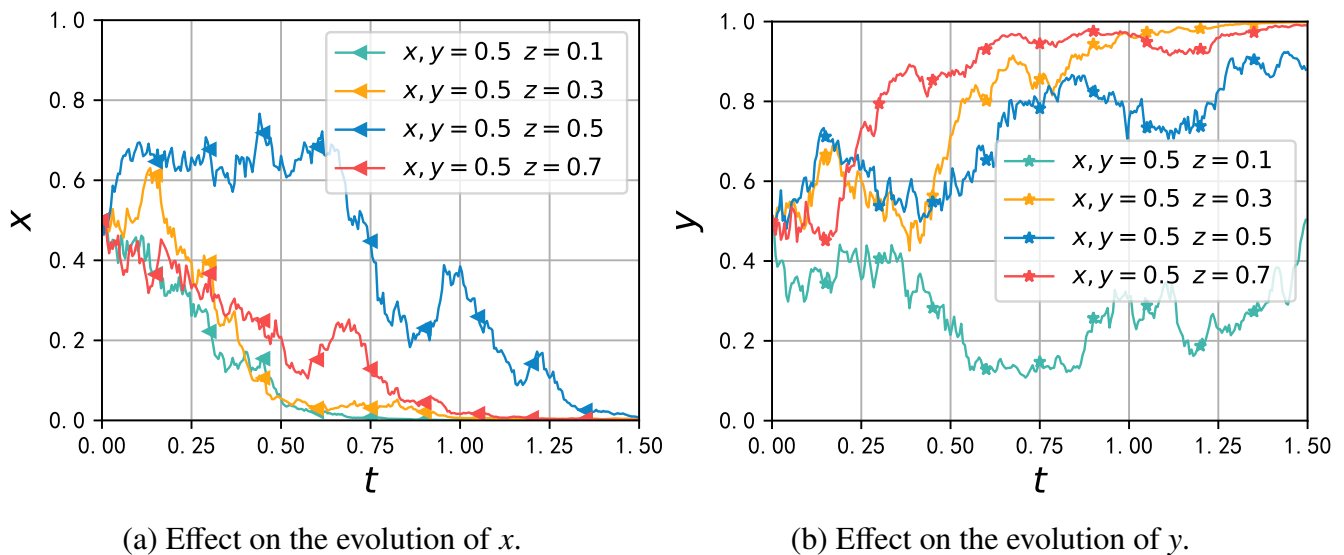


Figure 12. The effect of a change in the initial probability z on the evolution of the system.

5.3. Impact of key parameters on the system's evolution

5.3.1. Impact of the manufacturer's reputation and synergistic benefits (R_r)

Figure 13(a) and Figure 13(b) illustrate the impact of the core manufacturer's reputation and synergistic benefits (R_r) on the system's evolutionary trajectory under deterministic and stochastic perturbation environments, respectively.

As depicted in the deterministic phase-space diagram (Figure 13(a)), when R_r is maintained at a low level ($R_r \in \{7, 18\}$), the evolutionary curve x , representing the manufacturer's choice of high-quality sharing, eventually declines and converges to 0. Corroborating this, the stochastic phase-space diagram (Figure 13(b)) reveals that under low-gain conditions, the trajectory still precipitously drops to 0 despite experiencing intense initial fluctuations.

Conversely, when R_r is substantially amplified ($R_r \in \{22, 48\}$), the evolutionary trajectory of x undergoes a fundamental reversal, ascending rapidly and converging to 1.0. In this scenario, the system exhibits considerable robustness, reliably reaching the system-optimal cooperative equilibrium $E_8(1, 1, 1)$ even under continuous stochastic disturbances.

This comparative simulation indicates that reputation and synergistic gains are the primary endogenous drivers for data providers to adopt proactive strategies. When synergistic returns are low, the manufacturer's total expected payoffs fail to amortize the substantial data processing costs and the acute risks of privacy leakage (C_{risk}). Therefore, a rational manufacturer will inevitably withdraw from the collaboration. However, when the returns increase significantly, the excess profits effectively internalize the potential privacy risks. In a highly volatile environment characterized by random noise, these elevated expected returns significantly bolster the manufacturer's risk tolerance, thereby ensuring a sustainable supply of high-quality data and facilitating the achievement of a cooperative consensus across the entire tripartite system.

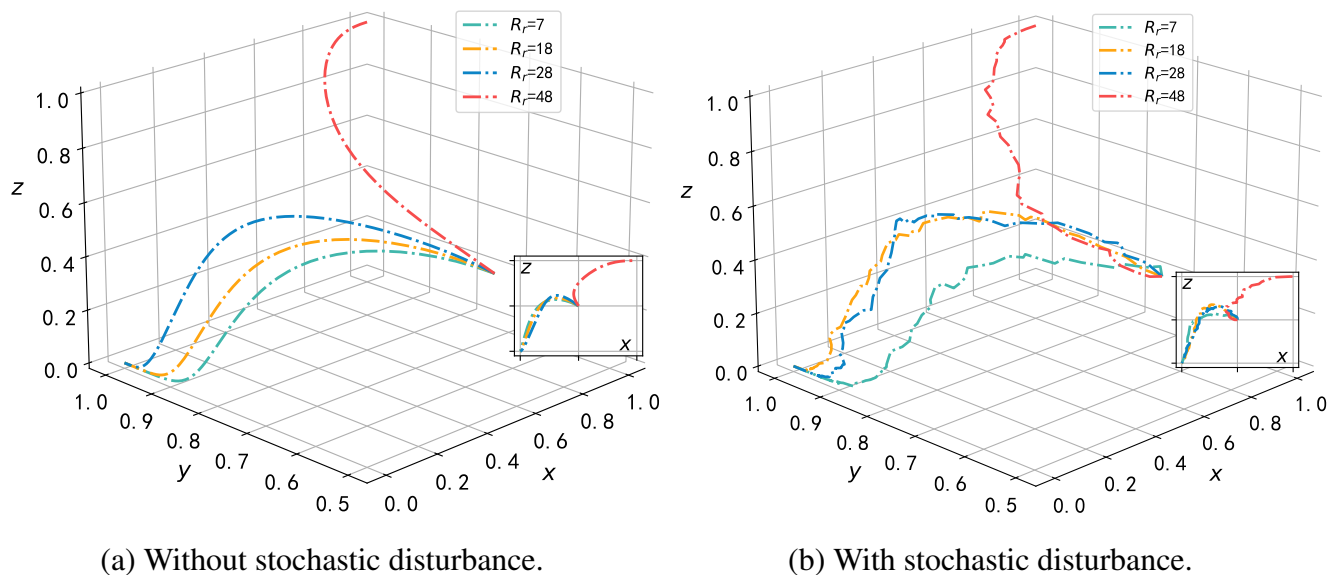


Figure 13. The effect of changes in reputation and synergistic benefits R_r on evolutionary pathways.

5.3.2. Impact of the manufacturer's risk exposure cost (C_{risk})

Figure 14(a) and Figure 14(b) analyze the impact of C_{risk} , representing the privacy and data leakage risk exposure cost faced by the core manufacturer, on the system's evolutionary trajectory under deterministic and stochastic environments, respectively.

As illustrated in the deterministic phase-space diagram (Figure 14(a)), the system's evolution exhibits acute sensitivity to risk costs. When the risk exposure cost is maintained at a low level ($C_{risk} = 16$), the evolutionary curve x , representing the manufacturer's strategy of high-quality sharing, eventually overcomes the initial resistance and converges to 1.0, culminating in systemic cooperation. Conversely, when C_{risk} escalates to 36 or beyond ($C_{risk} \in \{36, 52, 72\}$), the trajectory of curve x undergoes a fundamental reversal, precipitously converging toward 0. Furthermore, the convergence rate accelerates in direct proportion to the magnitude of the risk cost.

The stochastic phase-space diagram (Figure 14(b)) elucidates this dynamic even more intuitively: Under excessively high risk conditions (e.g., $C_{risk} = 72$), the curve x plummets directly and irreversibly to the 0 state without undergoing any initial oscillatory probing.

This implies that acute risk exposure directly deteriorates the expected net returns of the high-quality sharing strategy. While synergistic benefits can adequately offset risk-induced losses when the cost remains within a controllable threshold (low C_{risk}), data providers manifest a strong risk aversion in complex environments permeated by Gaussian white noise. Once the potential risk cost surpasses a critical threshold, even marginal random fluctuations in the external environment can trigger panic and strict risk-avoidance behaviors among manufacturers. This precipitates a rapid disruption in the upstream data supply, which consecutively degrades the expected returns for both the digital platform and third-party developers, ultimately culminating in a complete systemic breakdown of the tripartite partnership.

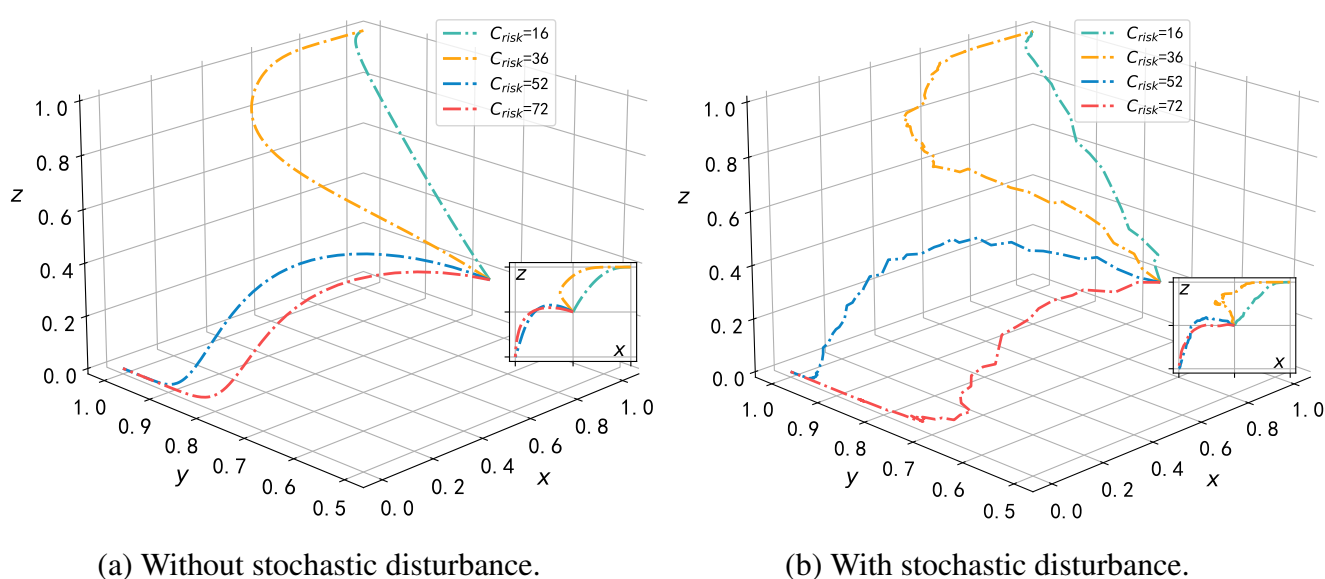


Figure 14. The effect of changes in risk exposure cost C_{risk} on evolutionary pathways.

5.3.3. Impact of the software developer's legal recourse costs (C_p)

Figure 15(a) and Figure 15(b) illustrate the impact of the third-party software developer's legal recourse costs (C_p) on the demand-side strategic decision-making process under deterministic and stochastic environments, respectively.

As depicted in the deterministic phase-space diagram (Figure 15(a)), the evolutionary trajectory of the system undergoes a qualitative bifurcation as the value of C_p escalates. When C_p remains at a relatively low level ($C_p = 18$), the evolutionary curve z , representing the software developer's strategy of normative use, successfully converges to 1.0. However, as C_p increases to 38, 55, and 85, the curve z exhibits an accelerated downward trend, ultimately plunging to 0. Corroborated by the stochastic phase-space diagram (Figure 15(b)), it is evident that elevated C_p values reduce the system's resilience to random environmental disturbances, forcing the demand-side trajectory to collapse to 0 within a significantly compressed timeframe.

This phenomenon reveals that legal recourse costs directly dictate the data consumer's tolerance threshold for inferior data. When the cost of seeking redress exceeds the potential innovation returns

(R_i) , the developer's expected net profit turns negative. To avert the accumulation of high sunk costs associated with the procurement of low-quality data, rational developers will either abandon normative practices or exit the ecosystem entirely. This strategic non-compliance and severe attrition on the demand side disrupt the data market's structural integrity. Once the core consumer base is depleted, the digital platform's stringent data governance efforts are stripped of their commercial viability, ultimately rendering the entire data-sharing ecosystem dysfunctional.

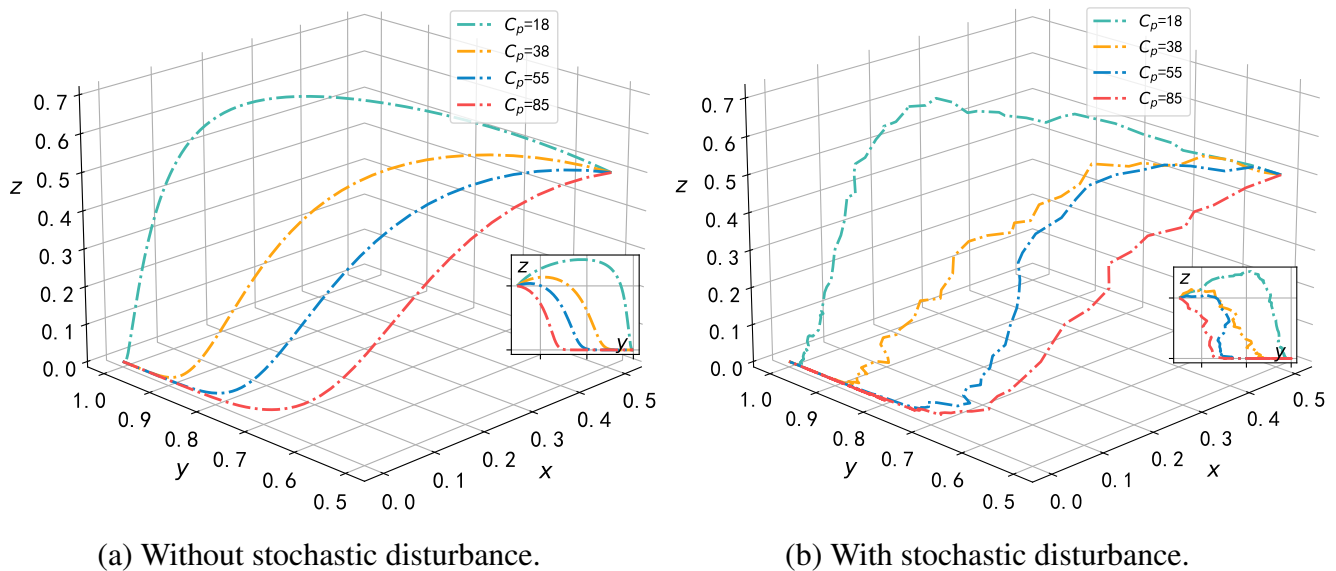


Figure 15. The effect of changes in legal recourse costs C_p on evolutionary pathways.

5.3.4. Impact of the digital platform's data governance capability (a)

Figure 16(a) and Figure 16(b) illustrate the critical role of the digital transformation platform's technical capability (a) in identifying inferior data on the system's evolutionary trajectory under deterministic and stochastic environments, respectively.

As depicted in the deterministic phase-space diagram (Figure 16(a)), when the governance capability remains at a low baseline (e.g., $a = 0.18$), the system fails to achieve tripartite cooperation, with all evolutionary trajectories ultimately collapsing to 0. However, when the governance capability a is elevated to 0.38 or beyond ($a \in \{0.38, 0.55, 0.78\}$), the evolutionary curve y , representing the platform's strategy of strict governance, rapidly and preferentially converges to 1.0. Driven by this strong governance environment, the trajectories x and z , representing the supply and demand sides, respectively, subsequently converge toward the cooperative boundary of 1.0.

The stochastic phase-space diagram (Figure 16(b)) further corroborates this by demonstrating that higher values of a strengthen the restoring force toward the cooperative state during the initial evolutionary stages. This strengthens the system's resilience against exogenous random disturbances and steers the system robustly toward the system-optimal cooperative equilibrium $E_8(1, 1, 1)$.

This indicates that parameter a characterizes the digital platform's regulatory efficiency and technical efficacy. A heightened governance capability amplifies the probability of detecting compliance violations. This directly escalates the expected penalty for manufacturers, effectively deterring opportunistic behaviors and perfunctory sharing. Concurrently, it reduces the probability of third-party software

developers procuring substandard data, thereby safeguarding the core interests of the demand side. Consequently, a formidable platform governance capability acts as a critical technical safeguard: It strengthens the system's resilience against exogenous random disturbances, institutionalizes multi-party trust, and decisively propels the tripartite ecosystem toward long-term, sustainable cooperation.

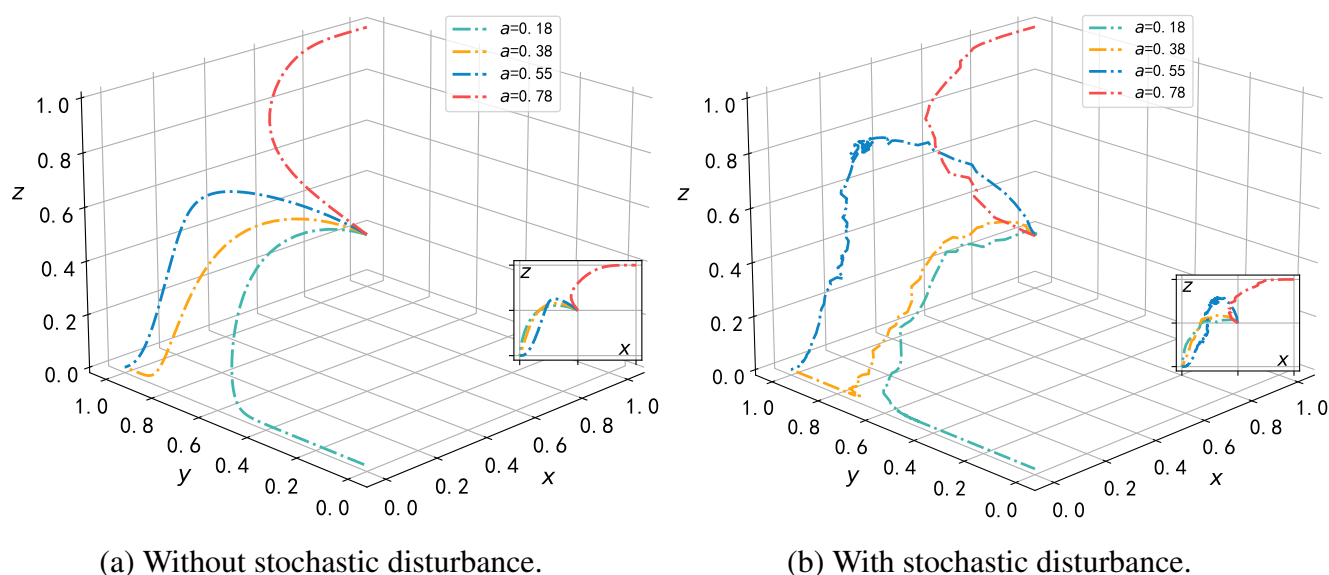


Figure 16. The effect of changes in platform governance capability a on evolutionary pathways.

5.3.5. Impact of the platform's accountability and enforcement intensity (i)

Figure 17(a) and Figure 17(b) examine the impact of the digital platform's accountability and enforcement intensity (i) against non-compliant manufacturers under deterministic and stochastic environments, respectively.

As depicted in the deterministic phase-space diagram (Figure 17(a)), when the enforcement intensity is weak (e.g., $i = 0.15$), the evolutionary curve x eventually converges to 0, signifying a breakdown in tripartite cooperation. However, when i is amplified to 0.35 or beyond ($i \in \{0.35, 0.65, 0.85\}$), the trajectory of x undergoes a fundamental reversal, rapidly converging to 1.0. Furthermore, the rate of convergence accelerates significantly in tandem with the increase in i .

A comparative analysis of the stochastic phase-space diagram (Figure 17(b)) reveals that a low value of i induces severe irregular fluctuations during the initial stages of the system's evolution. Conversely, a high-intensity enforcement regime strengthens the restoring force toward the cooperative state, enabling the evolutionary trajectory to reach the cooperative equilibrium in a smoother and more stable manner.

This dynamic indicates that the intensity of accountability dictates the actual efficacy of the punitive mechanism. A lenient enforcement regime fails to deter, and may even inadvertently incentivize, opportunistic behaviors among data providers, causing their strategic selections to oscillate repeatedly during the early phases of the game. In contrast, high-intensity accountability directly and substantially escalates the economic penalties associated with perfunctory sharing. This stringent punitive mechanism discourages opportunistic tendencies, thereby ensuring a stable and continuous output of high-quality data. Ultimately, it significantly fortifies the system's resilience against exogenous random shocks. We

emphasize that the governance parameters (a, i) affect the deterministic drift term of the replicator's dynamics rather than the diffusion intensity σ ; accordingly, their stabilizing effect operates indirectly by reinforcing the convergence toward the cooperative state, not by directly reducing σ .

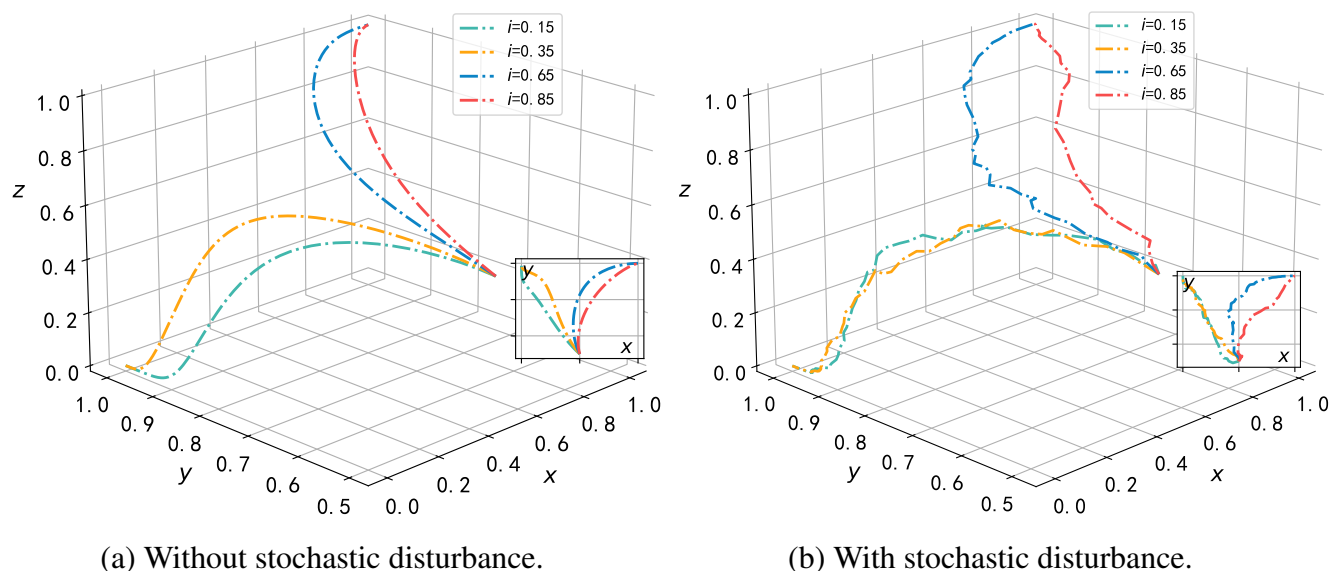


Figure 17. The effect of changes in accountability and enforcement intensity i on evolutionary pathways.

5.3.6. Combined effect of the platform's accountability intensity (i) and breach compensation (D_d)

Figure 18(a) and Figure 18(b) illustrate the synergistic cross-effect between the digital platform's accountability intensity (i) and the breach compensation (D_d) disbursed to third-party software developers under deterministic and stochastic environments, respectively.

Both the deterministic and stochastic phase-space diagrams demonstrate that under a low-accountability regime ($i = 0.15$), continuously escalating the compensation amount ($D_d \in \{16, 42, 62\}$) fails to stabilize the system. The evolutionary trajectories of all three agents continue to oscillate violently, ultimately collapsing to 0. Conversely, under a high-intensity accountability regime ($i = 0.85$), a moderate increase in compensation (D_d) significantly accelerates the convergence of the tripartite trajectories (x, y, z) toward the cooperative equilibrium of 1.0. Concurrently, as seen in Figure 18(b), the amplitude of stochastic fluctuations is reduced.

This dynamic reveals the dual nature of platform penalties as an *ex post* compensation mechanism. When the digital platform lacks the internal capability to effectively enforce accountability, increasing compensation commitments fails to deter non-compliant manufacturers. Instead, it merely exacerbates the platform's financial burden, ultimately compelling it to abandon its governance role entirely. In stark contrast, when the platform possesses a strong accountability capacity, it can efficiently offset its compensation expenditures by levying punitive fines on violators. Under these optimal conditions, elevated compensation guarantees effectively diminish the developers' perceived risk and substantially bolster trust on the demand side. This synergistic mechanism facilitates rapid and resilient systemic cooperation, even amidst complex stochastic disturbances.

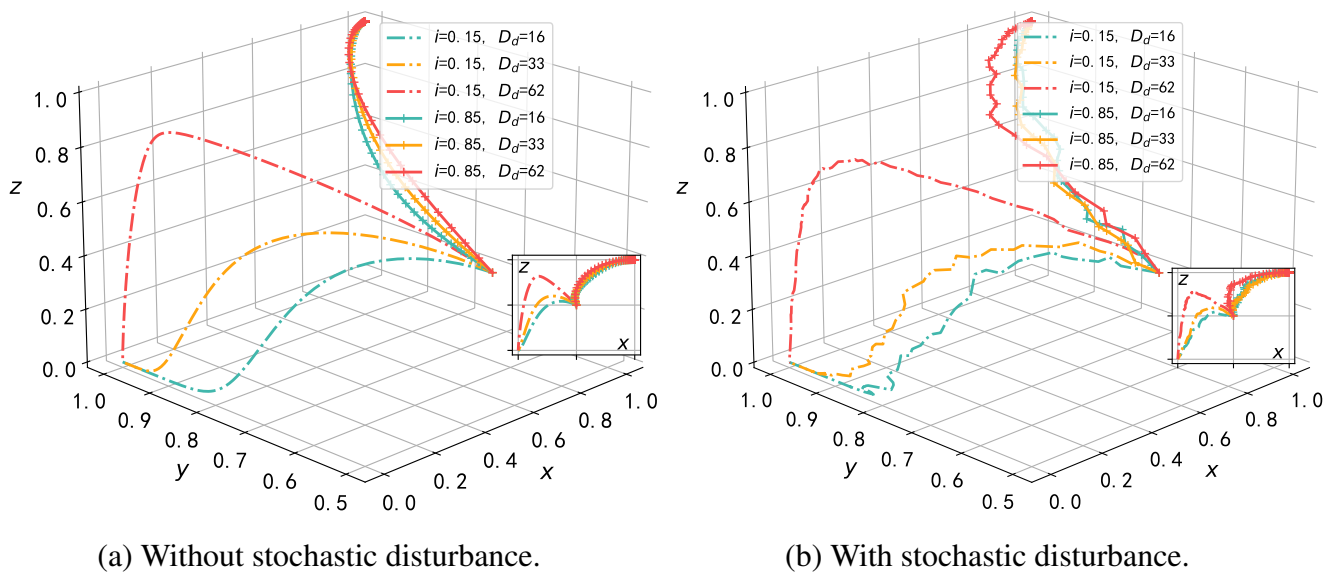


Figure 18. The combined effect of the platform's accountability intensity i and breach compensation D_d on evolutionary pathways.

5.3.7. Combined effect of the platform's accountability intensity (i) and government subsidies (S_g)

Figure 19(a) and Figure 19(b) reveal the synergistic mechanism between the platform's internal accountability intensity (i) and external government subsidies (S_g) under deterministic and stochastic environments, respectively.

As demonstrated by both the deterministic and stochastic phase-space diagrams, under a low-accountability regime ($i = 0.15$), a mere infusion of government subsidies (increasing S_g from 8 to 65) not only fails to foster tripartite cooperation but paradoxically exacerbates the chaos and oscillatory volatility within the system's evolutionary trajectory. It is only when coupled with a high-intensity accountability regime ($i = 0.85$) that an increase in S_g has a positive effect. This optimal combination significantly compresses the system's evolutionary timeframe, enabling the trajectories to converge smoothly and rapidly to the system-optimal cooperative equilibrium $E_8(1, 1, 1)$, while strengthening the system's resilience against stochastic disturbances.

An important economic dynamic is that: government subsidies function as an exogenous incentive tool to catalyze an ecosystem's development. However, under conditions of pronounced information asymmetry, if the digital transformation platform lacks strong internal oversight and accountability capabilities, the injection of external capital may inadvertently incentivize the participants to fabricate data or engage in rent-seeking behaviors to fraudulently misappropriate subsidies. This not only results in the severe misallocation of public funds but also amplifies systemic volatility.

Conversely, when the platform institutes a stringent accountability mechanism, government subsidies can truly operate as an effective economic lever. These exogenous subsidies reduce the platform's internal governance cost pressures (C_{gov}), empowering the platform to sustainably incentivize high-quality data provision through equitable profit-sharing mechanisms. Ultimately, this structural synergy strengthens the system's resilience against external random disturbances and ensures the long-term robustness and stable operation of the data-sharing ecosystem. We clarify that rent-seeking is best understood as one specific manifestation of the subsidy failure mechanism under information asymmetry,

in which subsidies without internal accountability induce misappropriation through falsifying data, so that only subsidies coupled with strict accountability exert a positive effect.

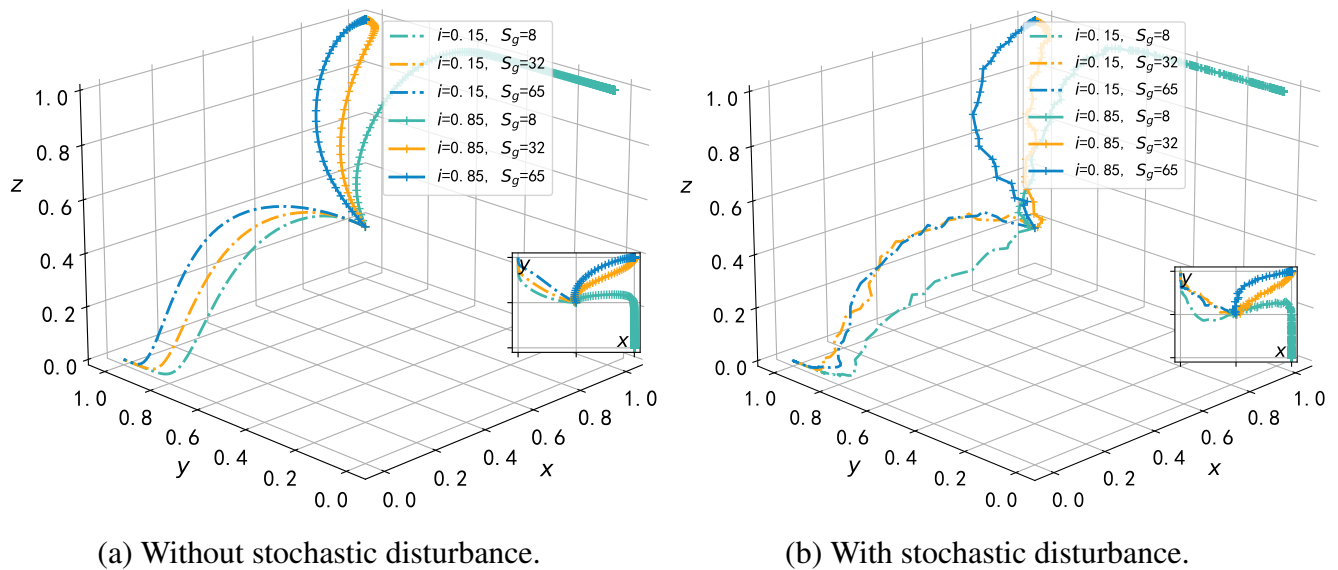


Figure 19. The combined effect of the platform's accountability intensity i and government subsidies S_g on evolutionary pathways.

6. Conclusions, limitations, and future work

6.1. Conclusions

This paper developed a tripartite stochastic evolutionary game model to investigate the strategic interactions among core sports equipment manufacturers, digital transformation platforms, and third-party software developers in the data-sharing ecosystem of the sports equipment manufacturing industry. Drawing on evolutionary game theory and stochastic differential equations, we derived the conditions under which the tripartite system converges to cooperative equilibria and analyzed the impact of Gaussian white noise, which represents technology supply disruption risks.

The theoretical analysis produces two principal findings. First, the deterministic stability analysis identifies four conditionally stable equilibrium points among the eight pure-strategy configurations: $E_1(0, 0, 0)$, $E_3(0, 1, 0)$, $E_6(1, 0, 1)$, and $E_8(1, 1, 1)$. The system-optimal cooperative equilibrium $E_8(1, 1, 1)$ is achievable only when the platform's governance capability and accountability intensity, combined with the manufacturer's reputation and synergistic benefits, exceed the privacy risk exposure costs, and when government subsidies exceed the platform's governance costs. These stability conditions, summarized in Table 5, provide a rigorous basis for identifying the parameter thresholds at which stakeholder strategies change. Second, the stochastic stability analysis reveals that Gaussian white noise tightens the conditions for equilibrium stability: The system requires the drift terms of the replicator's dynamics to overcome not only the deterministic payoff barriers but also the Lyapunov stability margin introduced by random disturbances. The stochastic disturbance intensity parameters ($\sigma_1, \sigma_2, \sigma_3$) directly influence whether the system converges to a stable state or exhibits persistent fluctuations.

The numerical simulations demonstrate several parameter-dependent behaviors. High privacy risk exposure costs (C_{risk}) are the primary deterrent to manufacturers' high-quality data-sharing, and once these costs exceed a threshold, manufacturers withdraw from cooperation regardless of their initial willingness. The platform's data governance capability (a) and accountability intensity (i) exhibit threshold effects: Below a minimum level, they fail to prevent ecosystem collapse; above this level, they effectively strengthen the system's resilience against stochastic disturbances and stabilize the cooperative equilibrium. Government subsidies (S_g) and breach compensation (D_d) are effective only when coupled with strict platform accountability; in its absence, increased financial transfers may encourage rent-seeking rather than cooperation. These findings suggest that policy interventions should prioritize strengthening platforms' governance capacity over simply increasing subsidy levels, and that subsidy mechanisms should be designed with built-in accountability conditions. In operational terms, these results point to three concrete governance instruments: A tiered platform certification regime that links subsidy eligibility to meeting the governance threshold (Sections 5.3.4–5.3.5); audit-conditional disbursement of breach compensation (D_d) and subsidies (S_g), released upon verified compliance (Sections 5.3.6–5.3.7); and a deposit-or-insurance requirement for platforms, calibrated to the accountability intensity i , to pre-fund compensation obligations.

Concretely, the threshold behavior of a and i (Sections 5.3.4–5.3.5) supports a tiered data-quality certification regime: Platforms exceeding the governance threshold earn certification and the corresponding subsidy eligibility, while those below it face graduated sanctions. The interaction effects in Sections 5.3.6–5.3.7 further suggest that breach compensation (D_d) and government subsidies (S_g) should be disbursed through a performance-contingent mechanism, whereby funds are released only upon verified compliance with audit standards, linking financial transfers to the accountability intensity i that the model identifies as a precondition for cooperative equilibrium. Governance tools such as privacy-enhancing technologies, deposits or insurance, and compensation-sharing rules currently lack directly corresponding parameters in our model's framework and constitute directions for future research (Section 6.3).

In summary, this study provides a formal framework for understanding tripartite coordination in data-sharing ecosystems under technology supply disruption risks. The results indicate that the platform is not merely a passive data intermediary but an active governance agent whose strategic decisions regarding oversight of data quality and accountability enforcement qualitatively shape the equilibrium structure of the ecosystem.

6.2. Limitations

Despite the contributions discussed above, several limitations of the present study should be acknowledged.

Agent homogeneity and deterministic strategy sets: The current model assumes homogeneous agents within each stakeholder group and binary strategy choices. In practice, sports equipment manufacturers differ substantially in size, digital maturity, and data generation capacity, implying heterogeneous sharing thresholds. Similarly, platforms vary in their technological specialization and governance capacity, and software developers differ in their data-processing capabilities and market reach. The binary strategy assumptions, while enabling closed-form analytical solutions, abstract from the continuous or multi-level nature of real-world decisions regarding data quality levels, governance intensity, and usage compliance.

Stochastic modeling simplifications: The model incorporates Gaussian white noise as an additive disturbance to capture exogenous randomness from technology supply disruptions. However, this formulation treats all random disturbances as symmetric and memoryless, whereas real-world supply disruptions may exhibit asymmetric effects, serial correlation, or regime-switching behavior. The current framework does not distinguish among different types of stochastic shocks (e.g., policy-driven versus market-driven) or allow for state-dependent noise intensities.

Empirical calibration: The numerical simulations rely on parameter values calibrated from published literature, expert consultation, and industry reports, rather than on direct estimation from firm-level panel data. While the sensitivity analyses confirm that the qualitative results are robust across parameter ranges, the specific threshold values reported in the simulation results should be interpreted as illustrative rather than as precise empirical predictions. Systematic empirical validation using structured surveys, transaction-level platform data, or comparative case studies would strengthen the evidence base for the model's predictions.

Simulation methodology: This study presents representative trajectories selected from multiple independent runs per parameter configuration. While these trajectories convey general evolutionary patterns, they do not capture the full statistical distribution of possible paths, such as convergence probabilities or the sizes of basins of attraction. Ensemble-averaged trajectories with confidence intervals from large-scale Monte Carlo simulations (e.g., 500–1000 runs per configuration) would provide a more rigorous statistical foundation and are planned for future work.

Scope of the game structure: The model focuses on a single representative platform and does not account for platform competition or multi-homing behavior by manufacturers and developers. In practice, multiple digital transformation platforms may compete for data resources, and manufacturers may participate in multiple platforms simultaneously, introducing additional strategic dimensions not captured in the current framework.

6.3. Future work

Building on the limitations identified above, there are several directions for future research.

First, future work could relax the assumption of homogeneous agents by introducing agent heterogeneity through agent-based modeling with empirically calibrated distributions of cost structures, risk preferences, and digital maturity levels. Multi-level or continuous strategy spaces could be explored to capture the gradations of data quality, governance intensity, and usage compliance observed in practice.

Second, the stochastic modeling framework could be extended by incorporating more complex noise processes. Poisson jump processes could capture discontinuous shocks such as sudden technological breakthroughs or policy reversals, while regime-switching models could represent alternating periods of supply chain stability and disruption. State-dependent noise intensities could reflect the observation that uncertainty may be higher when strategies are far from consensus. Likewise, the common Brownian motion $w(t)$ could be generalized to a vector of partially correlated, stakeholder-specific shock processes that capture the distinct shock sources of manufacturers (component supply), platforms (infrastructure and regulation), and developers (data availability and APIs).

Third, future research should conduct large-scale Monte Carlo simulations (500–1000 independent runs per parameter configuration) to derive ensemble-averaged evolutionary trajectories with confidence intervals. This would enable the quantification of convergence probabilities—for example, the proportion of runs converging to the system-optimal cooperative equilibrium $E_8(1, 1, 1)$ versus collapsing to

$E_1(0, 0, 0)$ under a given parameter set—offering a more complete characterization of the system's stochastic robustness.

Fourth, empirical validation is a necessary next step. Specific approaches include structured surveys of sports equipment manufacturers and platform operators to estimate the key parameters' distributions, transaction-level analysis of data-sharing contracts on existing industrial internet platforms, and comparative case studies of firms at different stages of digital transformation to trace their strategic evolution paths.

Fifth, comparative studies applying the tripartite framework to other manufacturing sectors with similar structural characteristics (e.g., medical devices, consumer electronics, automotive components) would help assess the generalizability of the findings beyond the sports equipment industry.

Finally, a further direction is to formally compare the payoff vectors $U_M(E_k)$, $U_P(E_k)$, and $U_D(E_k)$ across all eight equilibria to verify Pareto optimality in the strict payoff vector sense.

Author contributions

All authors contributed to the study's conception and design. All authors read and approved the final manuscript.

Use of generative AI tools declaration

The authors declare that they did not utilize any artificial intelligence (AI) tools in the creation of this article.

Acknowledgments

This work was supported by the Humanities and Social Science Fund of Ministry of Education of China (Grant No. 25YJAZH032), the Ministry of Education Humanities and Social Science Project (No. 21YJCZH197), the National Social Science Fund of China (Grant No. 23BJY205), and the Shanxi Provincial Research Foundation for Basic Research (No. 202303021221184).

Conflict of interest

The authors declare that they have no competing interests.

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