



Research article

Risk optimization of hazardous materials transportation through differentiated speed limit strategies in mixed traffic networks

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Abstract: Hazardous materials (hazmat) transportation accidents may cause severe casualties and environmental damage, highlighting the need for effective risk-mitigation strategies. In mixed traffic environments, hazmat vehicles share road resources with regular traffic, and the resulting congestion effects may further affect travel costs, traffic flow distribution, and transportation risk. This study proposes a speed-limit strategy for hazmat vehicles to reduce network transportation risk. A government-carrier bilevel optimization model was developed to characterize the decision-making relationship between regulatory authorities and transportation users. In the upper-level model, the regulatory authority determines the speed limits for hazmat vehicles on each link to minimize population-exposure-based network transportation risk. In the lower-level model, hazmat and regular traffic are incorporated into a multiclass user equilibrium framework, where the effects of hazmat speed limits on their travel costs and the congestion effects of aggregate traffic flow in the mixed traffic environment are incorporated into link travel costs. To address the computational challenges arising from complementarity constraints and nonlinear travel time functions, the bilevel model was reformulated as a mixed-integer linear programming model and solved using binary-variable linearization and special ordered sets of type 2 (SOS2)-based piecewise linearization. Numerical experiments on the Tilburg and Sioux Falls networks showed that reducing speed limits generally decreases the modeled network transportation risk, with maximum reductions of 35.12% and 7.91%, respectively, while the travel delay of hazmat vehicles correspondingly increases, with maximum increases of 19.71% and 11.15%, respectively. The proposed framework provides decision support for transportation authorities in designing network-level speed-limit strategies while accounting for congestion effects in mixed traffic environments and their implications for hazmat transportation risk.

Keywords: hazardous materials transportation; differentiated speed limits; bilevel programming; multiclass user equilibrium; mixed traffic networks

Mathematics Subject Classification: 90B06, 90B20, 90C11, 90C30

1. Introduction

The rapid pace of industrialization and the continuous growth of emerging industries, such as the lithium-ion battery sector, have substantially increased the demand for hazardous materials (hazmat), collectively driving the continuous expansion of hazmat production and circulation. In recent years, road transportation has become the dominant mode in global hazmat logistics systems due to its flexibility, convenience, and extensive network coverage. Statistics indicate that approximately 70.6%, 59.4%, and 54.5% of hazmat are transported by road in China, the United States, and Europe, respectively [1]. Such high dependence has made road networks the primary carriers of hazmat transportation risk. According to the latest monitoring data from the U.S. Bureau of Transportation Statistics, the number of hazmat incidents involving road transportation in the United States reached a historical high of 24,514 in 2024, representing a 26.3% increase compared with 2019 and accounting for 89.7% of incidents across all transportation modes [2]. The severe risk associated with hazmat transportation has been further demonstrated by several major accidents in recent years. In June 2020, a tanker truck explosion caused by speeding occurred on an expressway in Wenling, Zhejiang Province, China, resulting in 20 fatalities, 175 injuries, and substantial economic losses. In January 2025, a hazmat vehicle carrying approximately 2800 gallons of sulfur dioxide overturned in San Jacinto County, Texas, United States, causing toxic gas leakage; the driver was killed, and 29 students and one school bus driver from a nearby school received emergency medical treatment due to gas exposure. In November 2025, a 13-vehicle rear-end collision involving a tanker truck occurred on a highway in South Korea, resulting in two fatalities, four injuries, three vehicle fires, and several hours of severe traffic disruption. These severe accidents demonstrate the necessity of implementing effective intervention strategies and further highlight hazmat transportation as an important research topic in operations research and transportation systems engineering [3].

To address increasingly severe transportation risks, developing effective road risk-mitigation strategies has become an urgent task for regulatory authorities. However, under market-oriented transportation systems, governments are expected to respect the operational autonomy and cost considerations of carriers, and avoid directly intervening in their routing decisions [4]. These constraints have shifted regulatory efforts toward indirect traffic-management strategies. By modifying roadway operating conditions through measures such as speed management, lane reservation, and signal control, regulators can indirectly influence route choices and reshape the spatial distribution of network traffic flows, thereby guiding hazmat vehicles toward lower-risk corridors and reducing overall system risk.

Among these traffic-management measures, differentiated speed-limit strategies have attracted increasing attention due to their flexibility and relatively low implementation cost. Existing studies have demonstrated the effectiveness of speed management in addressing traffic congestion [5–7] and reducing traffic noise emissions [8–10]. However, the application of speed management to hazmat transportation risk mitigation remains insufficiently explored. Existing regulations for hazmat vehicles generally adopt uniform speed limits rather than link-specific differentiated speed limits at the network level. For example, China typically imposes a maximum speed limit of 80 km/h for hazmat vehicles on expressways, whereas the United States generally applies the same speed regulations to hazmat and regular vehicles. Moreover, existing studies on speed-limit strategies for hazmat transportation have primarily focused on the behavioral responses of hazmat vehicles, while the resulting traffic redistribution under mixed traffic conditions has received limited attention. For example, Wang et

al. [11] investigated speed-limit strategies for hazmat transportation from the perspective of hazmat vehicles. Since hazmat vehicles commonly share transportation infrastructure with regular vehicles [12], incorporating multiclass traffic interactions into speed-management strategies is essential for effective risk mitigation.

Motivated by the above observations, this study investigates the impacts of network-level differentiated speed-limit strategies on hazmat transportation risk and develops an optimization model with an effective solution methodology. The main contributions of this study are summarized as follows:

1. A network-level differentiated speed-limit strategy for hazmat vehicles is proposed to minimize transportation risk by optimizing link-specific speed limits.
2. A government-carrier bilevel optimization model is developed to characterize the interactions between regulatory authorities and traffic users. By explicitly incorporating regular traffic into the lower-level model, the proposed framework captures the travel cost impacts caused by shared roadway resources among different vehicle classes under mixed traffic equilibrium conditions and evaluates the impacts of differentiated speed-limit strategies on both hazmat transportation risk and network operations.

The proposed framework advances existing hazmat transportation risk-management research by integrating differentiated speed-limit strategies into a mixed traffic and network equilibrium framework. The remainder of this paper is organized as follows. Section 2 reviews the relevant literature. Section 3 presents the mathematical model. Section 4 introduces the model reformulation and solution methodology. Section 5 reports the numerical experiments and discusses the results. Finally, Section 6 concludes the paper and outlines future research directions.

2. Literature review

2.1. Risk-mitigation strategies for hazardous materials transportation

Risk-mitigation strategies for hazmat transportation at the road network level primarily rely on traffic-management measures. According to their intervention mechanisms, existing studies can generally be classified into two categories. The first category directly influences carriers' route choices through regulatory interventions, whereas the second indirectly guides route choices by modifying roadway operating conditions.

Strategies based on regulatory interventions have long been the dominant research direction in hazmat transportation. Representative approaches include network design and toll pricing. Kara and Verter [13] first formulated the hazmat transportation network design problem using a bilevel programming framework, in which the upper-level authority minimized the overall network risk by restricting link accessibility, while the lower-level carriers selected routes to minimize transportation costs. Building upon this framework, subsequent studies incorporated various practical considerations, including risk equity constraints [14], competitive interactions [15], emergency response planning [16], and time-dependent transportation networks [17], thereby extending the applicability of hazmat transportation network design models. Nevertheless, network design typically reduces transportation risk by restricting the use of selected links, which may reduce carriers' routing flexibility and lead to inefficient utilization of roadway infrastructure, thereby limiting the practical applicability of network design approaches [18].

In addition to network design, toll pricing has also emerged as an important research direction for hazmat transportation risk management. Marcotte et al. [19] first proposed a bilevel optimization model with differentiated toll pricing for hazmat transportation and demonstrated its effectiveness in mitigating transportation risk. Subsequent studies further incorporated risk equity considerations [20]. To account for traffic congestion, Wang et al. [21] proposed a dual toll pricing strategy for both hazmat vehicles and regular traffic. Recognizing that linear travel time functions cannot adequately capture congestion effects, Esfandeh et al. [22] improved the pricing framework by introducing nonlinear travel time functions. More recently, Chiou [12] explicitly modeled the selfish routing behaviors of hazmat vehicles and regular traffic, demonstrating that dual toll pricing can effectively reduce transportation risk while maintaining overall network efficiency.

In recent years, increasing attention has been devoted to traffic-management strategies that, unlike regulatory interventions that directly affect carriers' routing decisions or transportation cost, indirectly influence route choices and reshape the spatial distribution of network traffic flows by modifying roadway operating conditions. Representative strategies include lane reservation [23], signal control [24], and speed-limit strategies [11]. Similar to regulatory approaches, these studies are commonly formulated as bilevel optimization problems but place greater emphasis on roadway resource allocation, traffic operations, and the interactions among different traffic participants. Lane reservation reduces transportation risk by physically separating hazmat vehicles from regular traffic. However, it requires dedicated lanes for hazmat vehicles, which may lead to underutilization of roadway capacity when hazmat transportation demand is relatively low. Signal control strategies are mainly applicable to localized traffic environments such as urban intersections. Although they improve traffic operations through optimized signal timing, their influence is generally limited to local areas and is therefore difficult to extend to network-wide risk management. In contrast, differentiated speed-limit strategies require no additional infrastructure and can be implemented using existing traffic-monitoring and speed-enforcement facilities. By modifying the travel times of hazmat vehicles, differentiated speed limits influence their route-choice behavior while offering high implementation flexibility, broad applicability, and relatively low implementation cost.

Despite the practical advantages of differentiated speed-limit strategies, relatively few studies have investigated their application to hazmat transportation risk management under mixed traffic conditions. Among the existing studies, Wang et al. [11] developed a bilevel optimization framework to evaluate link-specific speed-limit strategies for hazmat transportation. However, their model focuses only on hazmat vehicles, with speed limits affecting their route choices through changes in link travel times, without considering congestion interactions with regular traffic or the resulting network-wide traffic redistribution. Consequently, the network-level impacts of differentiated speed-limit strategies under mixed traffic equilibrium conditions remain to be further investigated.

2.2. *Speed-limit strategies and traffic assignment*

Speed limits are widely used as an effective traffic-management measure. Beyond regulating vehicle operating speeds, link-specific speed limits can modify roadway impedance and thereby reshape the spatial distribution of network traffic flows. While early studies primarily focused on the impacts of speed limits on traffic safety, traffic operations, and environmental performance at the individual road level [25–30], analyses of their traffic redistribution effects were largely confined to localized road segments [31, 32]. More recently, increasing attention has been paid to network-level speed management. Yang et al. [5] developed one of the earliest analytical frameworks to investigate the network-wide

traffic redistribution effects of link-specific speed limits under the user equilibrium principle. Building upon this work, subsequent studies extended speed-limit optimization by considering multiple factors, including travel time, accident risk, and vehicle emissions [31], the trade-off between transportation efficiency and equity [7], and drivers' compliance with speed limits [32]. Lin et al. [6] investigated speed-limit settings in a multiclass traffic environment, providing insights into network-level speed-limit optimization under heterogeneous traffic conditions. However, their study primarily focused on improving traffic operational performance rather than mitigating the risks associated with hazmat transportation. More broadly, existing studies on network-level speed-limit strategies have mainly addressed general traffic efficiency, road safety, and environmental performance. The application of speed-limit strategies to hazmat transportation risk management remains relatively underexplored, particularly in mixed traffic environments where hazmat and regular vehicles share roadway resources and interact through congestion effects.

Analyzing the effects of network-level speed-limit strategies requires a traffic assignment model capable of accurately capturing the spatial redistribution of network traffic flows. Traffic assignment models describe how travel demand is distributed over transportation networks and are generally formulated according to Wardrop's equilibrium principles [33]. Among the two classical principles, the user equilibrium assumption provides a more realistic representation of decentralized route choice behavior in practical transportation systems and has therefore become the predominant framework for network-level speed-limit optimization [34]. Accordingly, the user equilibrium principle provides the theoretical foundation for analyzing the network-wide traffic redistribution induced by differentiated speed-limit strategies.

3. Model formulation

3.1. Problem statement

This study considers the design of network-level differentiated speed-limit strategies for hazmat transportation from the perspective of a regulatory authority. Since the regulatory authority and transportation users pursue different objectives, the problem naturally exhibits a hierarchical leader–follower decision-making structure. Therefore, a bilevel programming framework is adopted. Given that hazmat vehicles commonly share transportation infrastructure with regular traffic, a mixed traffic environment comprising hazmat vehicles and regular traffic is explicitly incorporated into the analytical framework. Considering the congestion effects arising from the aggregate traffic flow jointly formed by hazmat vehicles and regular traffic enables a more realistic representation of network traffic conditions, thereby facilitating a more comprehensive assessment of both network transportation risk and traffic efficiency.

To define the modeling scope, the following assumptions are adopted. First, the proposed lower-level model is formulated as a static traffic-assignment model, with the demands of hazmat and regular traffic fixed during the analysis period. Second, the congestion effects in the mixed traffic environment are characterized by changes in link travel-time congestion costs resulting from variations in the aggregate traffic flow jointly formed by hazmat vehicles and regular vehicles. Since this study focuses on hazmat transportation risk optimization at the macroscopic level, microscopic interactions at the individual-vehicle level and their associated uncertainties are not further considered.

Table 1. Notation for the proposed model.

Sets	
N	Set of nodes
A	Set of links
W	Set of origin–destination (OD) pairs
R_w	Set of feasible paths for OD pair w
K	Set of user classes (H : hazmat vehicles; M : regular traffic)
Ω	Feasible set of link and path flows
Variables and Functions	
$f_{w,r}^k$	Path flow of user class k on path r for OD pair w
$c_{w,r}^k$	Path travel cost of user class k on path r for OD pair w
q_a^k	Link flow of user class k on link a
d_w^k	Travel demand of user class k for OD pair w
d_w	Total travel demand for OD pair w
c_w^{k*}	Minimum travel cost of user class k for OD pair w
$t_a^k(\cdot)$	Link travel time function of user class k on link a
u_a^k	Speed limit for hazmat vehicles on link a (decision variable)
$Q_a^k(\cdot)$	Critical link flow corresponding to speed limit u_a^k
TR_a^k	Transportation risk of hazmat vehicle class k ($k \in H$) on link a
$x_{w,r}^k$	Binary variable for linearizing the UE conditions
$g_a(\cdot)$	BPR function without the speed-related term
$b_{a,i}$	SOS2 weight coefficient for breakpoint i on link a
$y_{a,i}$	Flow value at breakpoint i on link a
$\hat{g}_a$	Interpolated value of $g_a(\cdot)$ on link a
$\hat{c}_a^k$	Link travel cost of user class k on link a
Model Parameters	
$\delta_{a,r}$	Link–path incidence indicator (= 1 if link a belongs to path r , and 0 otherwise)
ϕ^k	Passenger-car-equivalent (PCE) factor of user class k
u_{a0}^k	Initial free-flow speed of user class k on link a
l_a	Length of link a
CA_a	Capacity of link a
POP_a	Population density along link a
p_a^k	Average accident probability per unit distance for $k \in H$
r^k	Accident impact range coefficient for $k \in H$
λ_a^k	Lower-bound coefficient of the speed limit on link a for $k \in H$
I	Total number of piecewise linear segments
I_{left}	Number of piecewise linear segments on the left side of the nominal capacity, which determines the piecewise linearization step size
I_{right}	Number of piecewise linear segments beyond the nominal capacity

In the upper-level model, the regulatory authority determines the speed limits for hazmat vehicles on each link to minimize network-wide transportation risk. Transportation risk is quantified using a

population exposure model, and the speed limits are treated as continuous decision variables within a predefined feasible strategy space. Given the upper-level speed-limit decisions, the lower-level model formulates the route-choice behaviors of hazmat vehicles and regular traffic as a multiclass user equilibrium problem. The resulting equilibrium traffic flows are then fed back to the upper-level model for network transportation risk evaluation and speed-limit strategy optimization. The notation of the proposed model is summarized in Table 1 and the overall model framework is illustrated in Figure 1.

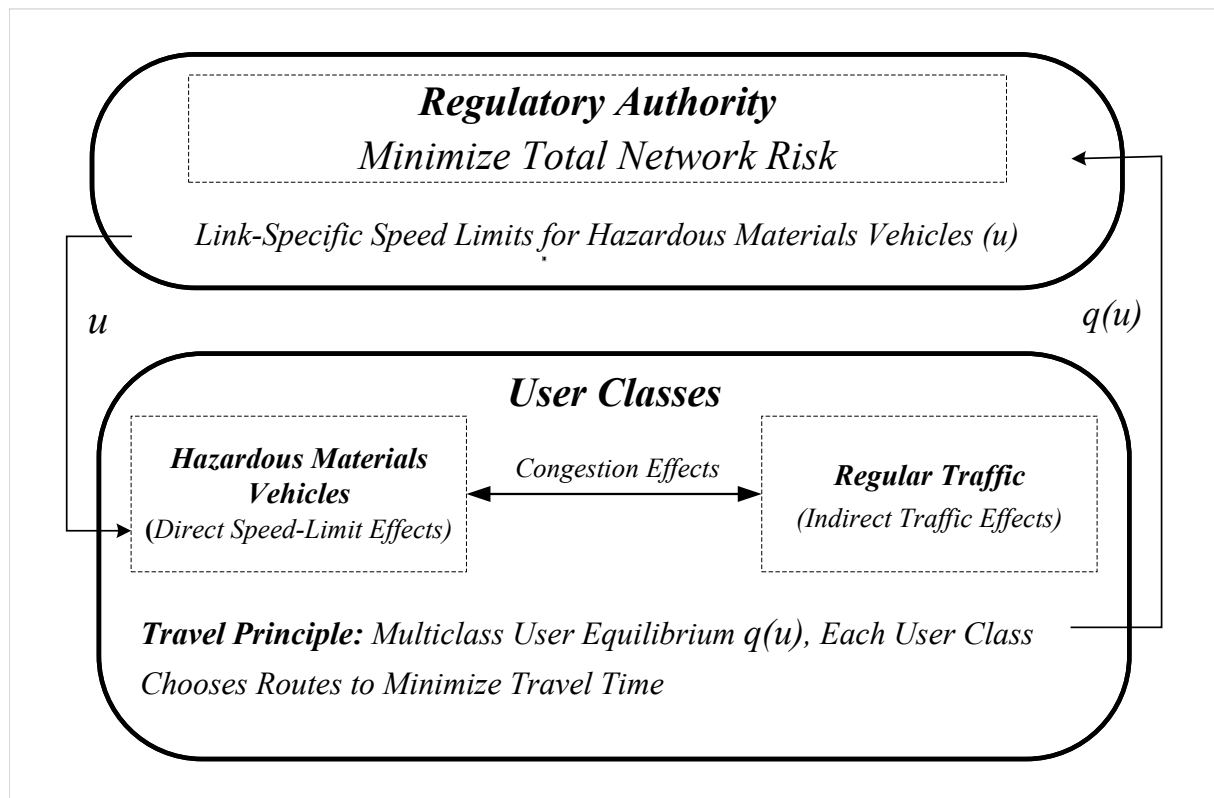


Figure 1. Bilevel analytical framework.

3.2. Lower-level network equilibrium model

The lower-level model determines the equilibrium traffic flow distribution under a given speed-limit strategy, providing the traffic flow pattern required for upper-level optimization. Route-choice behavior is governed by link travel times, with those of hazmat vehicles affected directly by the imposed speed limits.

3.2.1. Link travel time function

The classical Bureau of Public Roads (BPR) function is widely used to characterize link travel times because it effectively captures congestion effects. Accordingly, it is adopted to represent link travel cost. Specifically, the BPR function consists of a free-flow travel-time term and a congestion term. The former is determined by the free-flow speed and therefore reflects the effect of speed-limit changes, while the latter characterizes the congestion effects associated with the interaction between regular and hazmat vehicles in the mixed traffic environment.

Let K denote the set of all user classes, where H and M represent the sets of hazmat vehicles and regular vehicles, respectively, satisfying $K = H \cup M$ and $H \cap M = \emptyset$. In the BPR function, u_{a0}^k denotes the free-flow speed of user class k on link a , α and β are the BPR parameters, CA_a is the link capacity, and l_a is the length of link a .

To account for differences in passenger-car-equivalent factors across user classes, the total equivalent flow on link a is defined as the weighted sum of the link flows of all user classes:

$$q_a = \sum_{k \in K} q_a^k \phi^k \quad \forall a \in A \quad (1)$$

Regular vehicles: The link travel time of regular vehicles is represented by the conventional BPR function:

$$t_a^k(q_a) = \frac{l_a}{u_{a0}^k} \left[1 + \alpha \left(\frac{q_a}{CA_a} \right)^\beta \right] \quad \forall a \in A, k \in M \quad (2)$$

Hazmat vehicles: The proposed link-specific speed-limit strategy influences route choice by modifying travel costs. Following Yang et al. [5], a horizontal segment is incorporated into the travel time function for hazmat vehicles to reflect the additional travel time associated with speed restrictions. Thus, the link travel time function for hazmat vehicles is formulated as

$$t_a^k(q_a, u_a^k) = \begin{cases} \frac{l_a}{u_a^k}, & q_a \leq Q_a^k(u_a^k) \\ \frac{l_a}{u_{a0}^k} \left[1 + \alpha \left(\frac{q_a}{CA_a} \right)^\beta \right], & q_a > Q_a^k(u_a^k) \end{cases} \quad \forall a \in A, k \in H \quad (3)$$

In Eq. (3), u_a^k denotes the speed limit imposed on hazmat vehicles on link a , which is determined by the upper-level model, and $Q_a^k(u_a^k)$ denotes the corresponding critical link flow determined by enforcing the continuity of the piecewise travel time function:

$$Q_a^k(u_a^k) = CA_a \left[\frac{1}{\alpha} \left(\frac{u_{a0}^k}{u_a^k} - 1 \right) \right]^{\frac{1}{\beta}} \quad \forall a \in A, k \in H \quad (4)$$

It should be noted that u_{a0}^k denotes the initial free-flow speed of user class k in the absence of the differentiated speed-limit strategy considered in this study. It is used to characterize the baseline traffic operating conditions rather than serving as a speed-limit decision variable. For regular vehicles ($k \in M$), u_{a0}^k is determined according to the existing speed limits and roadway characteristics of each link, which may vary substantially across links. In contrast, for hazmat vehicles ($k \in H$), existing baseline speed-limit regulations generally specify uniform speed limits rather than differentiated speed limits across links.

3.2.2. Mixed traffic assignment model

Given the link travel time functions defined above, the lower-level model is formulated as a multiclass user equilibrium traffic assignment problem. Consider a directed transportation network $G = (N, A)$, where N and A denote the sets of nodes and links, respectively. Let W denote the set of origin–destination (OD) pairs, and let R_w denote the set of feasible paths connecting each OD pair w .

For each OD pair w and user class k , let d_w^k denote the travel demand and $f_{w,r}^k$ denote the corresponding path flow on path r . The traffic assignment model is defined by the following flow and demand relationships:

$$q_a^k = \sum_{w \in W} \sum_{r \in R_w} \delta_{a,r} f_{w,r}^k \quad \forall a \in A, k \in K \quad (5)$$

$$d_w^k = \sum_{r \in R_w} f_{w,r}^k \quad \forall w \in W, k \in K \quad (6)$$

$$d_w = \sum_{k \in K} d_w^k \quad \forall w \in W \quad (7)$$

where Eq. (5) defines the relationship between link flows and path flows through the link–path incidence parameter $\delta_{a,r}$, with $\delta_{a,r} = 1$ if link a belongs to path r and $\delta_{a,r} = 0$ otherwise. Eqs. (6) and (7) ensure demand conservation and aggregation for each user class and OD pair.

The travel cost of path r for user class k between OD pair w is obtained by summing the travel times of all links along the path:

$$c_{w,r}^k = \sum_{a \in A} t_a^k(\cdot) \delta_{r,a} \quad \forall k \in K, w \in W, r \in R_w \quad (8)$$

where $t_a^k(\cdot)$ denotes the corresponding link travel time function defined by Eqs. (2) and (3).

Let c_w^{k*} denote the minimum travel cost of user class k for OD pair w . According to Wardrop's first principle, for each user class and each OD pair, all used routes have equal and minimal travel cost. Accordingly, the path flows and travel costs satisfy the following user equilibrium conditions:

$$(c_{w,r}^k - c_w^{k*}) f_{w,r}^k = 0 \quad \forall w \in W, r \in R_w, k \in K \quad (9)$$

$$c_{w,r}^k - c_w^{k*} \geq 0 \quad \forall w \in W, r \in R_w, k \in K \quad (10)$$

$$f_{w,r}^k \geq 0 \quad \forall w \in W, r \in R_w, k \in K \quad (11)$$

Eqs. (9)–(11) are the complementarity conditions of the user equilibrium model, ensuring that positive flow is assigned only to minimum-cost paths while satisfying the non-negativity of path flows.

Collectively, Eqs. (1)–(11) formulate the lower-level multiclass user equilibrium traffic assignment model.

3.3. Risk assessment model

Transportation risk in hazmat transportation is commonly evaluated using a probability–consequence framework. Accident consequences are typically measured in terms of potential casualties or property losses [35]. For hazmat transportation, the magnitude of accident consequences is primarily determined by the affected area, which depends on the physical and chemical properties of the hazardous material. Gopalan et al. [36] characterized the accident impact area using an impact radius associated with each type of hazardous material. In this study, the impact radius associated with each hazmat vehicle

user class k is denoted by r^k . Harwood et al. [37] reported that the accident probability of hazmat transportation generally ranges from 10^{-9} to 10^{-6} accidents per vehicle-kilometer. Therefore, the probability–consequence framework is adopted in this study, where accident probability is represented by the average accident probability per unit distance and accident consequence is quantified using population exposure.

The link-level transportation risk associated with hazmat user class k is calculated as

$$TR_a^k = p_a^k l_a POP_a \pi(r^k)^2 \quad \forall a \in A, k \in H \quad (12)$$

where p_a^k is the average accident rate per unit distance for hazmat vehicles of user class k , l_a is the length of link a , POP_a is the average population density along link a , and r^k is the impact radius associated with hazmat vehicle user class k .

Two remarks regarding the proposed risk model are worth noting. First, the objective of this study is to investigate the impacts of differentiated speed-limit strategies on hazmat transportation risk through traffic flow redistribution. Therefore, the proposed model emphasizes the indirect effect of speed management on route choice rather than the direct relationship between vehicle speed and accident probability. Second, although vehicle speed is widely recognized as an important factor affecting traffic safety, its relationship with accident probability remains inconclusive in the literature [26]. Some studies have reported a positive correlation between speed and accident probability [38–40], whereas others have found either no statistically significant relationship or even a negative correlation [41, 42]. In addition, accident risk has been shown to depend on the deviation between an individual vehicle's speed and the average speed of the surrounding traffic stream. Given the lack of consensus regarding the relationship between speed and accident probability, the direct effect of vehicle speed on accident probability is therefore not explicitly considered in the proposed risk model.

3.4. Bilevel optimization model for link-specific speed limits

Based on the analytical framework presented in Section 3.1, the optimization of link-specific speed limits for hazmat transportation is formulated as a bilevel programming model. The upper-level model determines the link-specific speed limits for hazmat vehicles, whereas the lower-level model determines the corresponding multiclass user equilibrium traffic flow under the given speed-limit strategy.

In the upper-level model, the regulatory authority determines the link-specific speed-limit decision variables u_a^k for hazmat vehicles with the objective of minimizing the total transportation risk over the network. The variables u_a^k are continuous decision variables, and their feasible values are bounded by the prescribed speed-limit range defined relative to the corresponding free-flow speeds. The lower-level model is represented by the multiclass user equilibrium model developed in Section 3.2, which determines the equilibrium traffic flow distribution corresponding to a given speed-limit strategy. The resulting bilevel optimization model is formulated as follows.

Upper-level model:

$$\min_{\mathbf{u}} \quad \sum_{a \in A} \sum_{k \in H} p_a^k l_a POP_a \pi(r^k)^2 q_a^k \quad (13)$$

$$\lambda_a^k \leq \frac{u_a^k}{u_{a0}^k} \leq 1 \quad \forall a \in A, k \in H \quad (14)$$

Lower-level model:

$$\text{Eqs. (1)–(11)} \quad (15)$$

Equation (13) is the upper-level objective function, which minimizes the total transportation risk over the network by summing the risk contributions of hazmat vehicle flows across all links and hazmat-vehicle user classes. Constraint (14) defines the feasible strategy space for the link-specific speed limits. The parameter λ_a^k is a user-defined lower-bound coefficient for the speed limit, satisfying $0 < \lambda_a^k \leq 1$, and represents the ratio of the minimum allowable speed limit to the corresponding free-flow speed.

4. Model reformulation

Directly solving the proposed bilevel optimization model is computationally challenging. The primary difficulties arise from the complementarity conditions in the lower-level user equilibrium model and the nonlinear travel time functions. Specifically, the lower-level user equilibrium conditions are formulated as the complementarity constraints given by Eqs. (9)–(11), resulting in a mathematical program with complementarity constraints (MPCC). MPCCs violate the Mangasarian–Fromovitz constraint qualification, thereby rendering the Karush–Kuhn–Tucker optimality conditions inapplicable and preventing the direct application of many standard nonlinear programming algorithms. In addition, the nonlinear travel time functions defined by Eqs. (2) and (3) further increase the computational complexity of the model. To overcome these difficulties, the original bilevel optimization model is reformulated as a mixed-integer linear programming (MILP) model. First, the complementarity conditions in the lower-level multiclass user equilibrium model are linearized using binary auxiliary variables. Second, the travel time functions are reformulated and approximated using SOS2-based piecewise linearization. The resulting MILP model can then be solved using commercial optimization solvers.

4.1. Linearization of complementarity conditions

To reformulate the complementarity condition in Eq. (9), a binary variable $x_{w,r}^k$ is introduced to indicate whether path r carries positive flow [43,44]. The binary variable is defined as

$$x_{w,r}^k = \begin{cases} 1, & \text{if } f_{w,r}^k > 0 \\ 0, & \text{if } f_{w,r}^k = 0 \end{cases} \quad \forall k \in K, w \in W, r \in R_w \quad (16)$$

Thus, the complementarity condition can be rewritten as

$$x_{w,r}^k (c_{w,r}^k - c_w^{k*}) = 0 \quad \forall k \in K, w \in W, r \in R_w \quad (17)$$

Applying a variable-bound Big- M reformulation based on disjunctive programming, Eqs. (16)–(17) are transformed into the following set of linear constraints. Let $\bar{c}$ and $\bar{f}$ denote valid upper bounds on path cost and path flow:

$$c_{w,r}^k - c_w^{k*} \leq \bar{c} (1 - x_{w,r}^k) \quad \forall k \in K, w \in W, r \in R_w \quad (18)$$

$$f_{w,r}^k \leq \bar{f} x_{w,r}^k \quad \forall k \in K, w \in W, r \in R_w \quad (19)$$

$$x_{w,r}^k \in \{0, 1\} \quad \forall k \in K, w \in W, r \in R_w \quad (20)$$

In the numerical implementation, the upper bound on path flow, $\bar{f}$, can be set as the maximum travel demand among all user classes and OD pairs:

$$\bar{f} = \max_{w \in W, k \in K} d_w^k \quad (21)$$

The upper bound on the path cost, $\bar{c}$, can be determined based on the maximum possible travel time of each link within its allowable flow range and should be chosen sufficiently large to cover the costs of all feasible paths.

4.2. Reformulation and piecewise linearization of travel time functions

The improved BPR function defined in Eq. (3) captures the speed-limit effect for hazardous materials vehicles by introducing a constant travel-time segment determined by the imposed speed limit. However, directly piecewise linearizing Eq. (3) is challenging because the transition point $Q_a^k(u_a^k)$ connecting the constant travel-time segment and the congestion curve varies with the upper-level speed-limit decision variable u_a^k . In addition, $Q_a^k(u_a^k)$ is determined by the nonlinear relationship defined in Eq. (4). Therefore, directly linearizing this decision-dependent piecewise structure requires additional treatment and substantially increases the complexity of the MILP formulation.

To reduce the complexity of the above linearization process, motivated by the modeling idea in Ref. [6], which represents speed-limit effects through adjustments of free-flow speed, this study represents the speed-limit effect by adjusting the free-flow speed rather than explicitly modeling the speed-dependent transition point $Q_a^k(u_a^k)$. Specifically, the initial free-flow speed u_{a0}^k in the standard BPR function is replaced by the upper-level decision variable u_a^k , resulting in the following speed-adjusted standard BPR function:

$$t_a^k(q_a, u_a^k) = \frac{l_a}{u_a^k} \left[1 + \alpha \left(\frac{q_a}{CA_a} \right)^\beta \right] \quad \forall a \in A, k \in H \quad (22)$$

It should be noted that the speed-adjusted standard BPR function is not strictly equivalent to the improved BPR function defined in Eq. (3) over the entire flow range. Figure 2 compares the two function formulations and their corresponding piecewise linear approximations. Although the original function shapes are different, their piecewise linear approximations exhibit similar increasing trends and structural characteristics, suggesting that both formulations can capture the main effect of speed limits on travel time. Specifically, reducing u_a^k represents imposing a lower speed limit on hazardous materials vehicles, which increases their free-flow travel time and, consequently, their link travel time.

It should also be emphasized that, compared with Eq. (3), Eq. (22) generally overestimates the travel time of hazmat vehicles over the overall flow range, particularly on links with medium to high traffic flows. Accordingly, Eq. (22) provides a more stringent representation of the increase in travel time caused by speed limits. From a traffic-engineering perspective, this overestimation represents a more pessimistic estimation of travel time under speed-limit and congestion effects. From the perspective of risk minimization, such a treatment does not lead to an underestimation of hazmat transportation risk, but may further compromise transportation efficiency.

Therefore, in the MILP model, the speed-adjusted standard BPR function is adopted as an alternative representation of Eq. (3), preserving the essential impact of speed-limit decisions on

hazmat vehicle travel time while avoiding the additional complexity introduced by the decision-dependent piecewise structure.

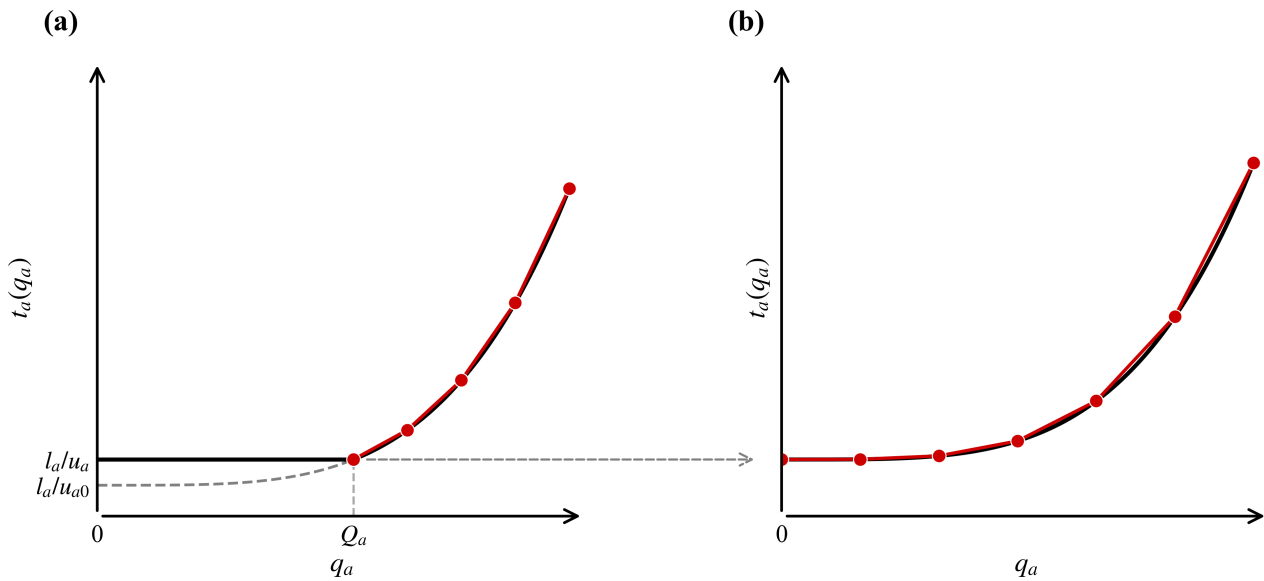


Figure 2. Comparison of travel time functions and their piecewise linear approximations: (a) improved BPR function for hazmat vehicles; (b) speed-adjusted standard BPR function.

Equation (22) has the same BPR functional form as Eq. (2), with the only difference being that the initial free-flow speed u_{a0}^k in Eq. (2) is replaced by the upper-level speed-limit decision variable u_a^k . Therefore, the speed-related term can be separated from the common congestion term, allowing a unified piecewise linearization scheme to be applied to the congestion function. The resulting congestion function is defined as

$$g_a(q_a) = l_a \left[1 + \alpha \left(\frac{q_a}{CA_a} \right)^\beta \right] \quad \forall a \in A \quad (23)$$

The flow linearization range of each link is divided into I intervals, where I_{left} and I_{right} denote the numbers of intervals to the left and right of the nominal capacity CA_a , respectively, with $I = I_{\text{left}} + I_{\text{right}}$. The corresponding breakpoints are defined as

$$y_{a,i} = \frac{CA_a}{I_{\text{left}}}(i-1) \quad \forall a \in A, i = 1, \dots, I+1 \quad (24)$$

Special ordered sets of type 2 (SOS2) are then employed to construct the piecewise linear approximation. Let $b_{a,i}$ denote the interpolation weights associated with breakpoint $y_{a,i}$, where $0 \leq b_{a,i} \leq 1$. The resulting formulation is given by

$$\sum_{i=1}^{I+1} b_{a,i} = 1 \quad \forall a \in A \quad (25)$$

$$\sum_{i=1}^{I+1} b_{a,i} y_{a,i} = q_a \quad \forall a \in A \quad (26)$$

$$\hat{g}_a = \sum_{i=1}^{I+1} b_{a,i} g_a(y_{a,i}) \quad \forall a \in A \quad (27)$$

$$(b_{a,1}, b_{a,2}, \dots, b_{a,I+1}) \in \text{SOS2} \quad \forall a \in A \quad (28)$$

Eq. (25) requires the interpolation weights to form a convex combination. Eq. (26) links the original flow variable with its piecewise representation, while Eq. (27) constructs the piecewise linear approximation of the congestion function. Finally, the SOS2 constraint in Eq. (28) restricts at most two adjacent interpolation weights to be nonzero, thereby ensuring a valid piecewise linear interpolation.

To facilitate the subsequent MILP reformulation, an auxiliary variable $\hat{c}_a^k$ is introduced to represent the link travel cost of user class k on link a in the lower-level user equilibrium model. For hazmat vehicles, according to Eq. (22), the link travel cost can be expressed as $\hat{c}_a^k = \hat{g}_a / u_a^k$, where $\hat{g}_a$ denotes the linearized value of the congestion function $g_a(q_a)$ in Eq. (23). Based on the allowable range of the upper-level speed-limit decision variable given in Eq. (14), $\lambda_a^k u_{a0}^k \leq u_a^k \leq u_{a0}^k$, the feasible range of $\hat{c}_a^k$ can be bounded as follows:

$$\frac{\hat{g}_a}{u_{a0}^k} \leq \hat{c}_a^k \leq \frac{\hat{g}_a}{\lambda_a^k u_{a0}^k} \quad \forall a \in A, k \in H \quad (29)$$

This bound incorporates the effect of the upper-level speed-limit decision into the link travel cost of hazmat vehicles in the lower-level user equilibrium model. Specifically, a lower speed limit leads to a higher feasible upper bound for $\hat{c}_a^k$, thereby affecting hazmat vehicle route choices and the resulting equilibrium traffic flow distribution. Thus, although the upper-level speed-limit decision variable u_a^k does not explicitly appear in the subsequent user equilibrium constraints, its effect on the lower-level traffic assignment is captured through the auxiliary variable $\hat{c}_a^k$.

After solving the MILP model, the optimal speed-limit decisions u_a^{k*} and equilibrium traffic flows q_a^* are directly obtained. Subsequently, the obtained u_a^{k*} and q_a^* are substituted into the original travel time functions to recalculate the actual link travel times, which are then used for subsequent performance evaluation.

For regular vehicles, no differentiated speed-limit strategy is imposed, and their free-flow speed remains at the initial value u_{a0}^k . Therefore, the corresponding link travel cost is given by

$$\hat{c}_a^k = \frac{\hat{g}_a}{u_{a0}^k} \quad \forall a \in A, k \in M \quad (30)$$

4.3. MILP formulation

Based on the above reformulations, the original bilevel model is reformulated as the following MILP model.

$$\min \sum_{a \in A} \sum_{k \in H} p_a^k l_a \text{POP}_a \pi(r^k)^2 q_a^k \quad (31)$$

s.t.

$$q_a = \sum_{k \in K} q_a^k \phi^k \quad \forall a \in A \quad (32)$$

$$q_a^k = \sum_{w \in W} \sum_{r \in R_w} f_{w,r}^k \delta_{r,a} \quad \forall a \in A, k \in K \quad (33)$$

$$d_w^k = \sum_{r \in R_w} f_{w,r}^k \quad \forall k \in K, w \in W \quad (34)$$

$$d_w = \sum_{k \in K} d_w^k \quad \forall w \in W \quad (35)$$

$$c_{w,r}^k - c_w^{k*} \leq \bar{c} (1 - x_{w,r}^k) \quad \forall k \in K, w \in W, r \in R_w \quad (36)$$

$$c_{w,r}^k - c_w^{k*} \geq 0 \quad \forall k \in K, w \in W, r \in R_w \quad (37)$$

$$f_{w,r}^k \leq \bar{f} x_{w,r}^k \quad \forall k \in K, w \in W, r \in R_w \quad (38)$$

$$f_{w,r}^k \geq 0 \quad \forall k \in K, w \in W, r \in R_w \quad (39)$$

$$x_{w,r}^k \in \{0, 1\} \quad \forall k \in K, w \in W, r \in R_w \quad (40)$$

$$y_{a,i} = \frac{CA_a}{I_{\text{left}}} (i - 1) \quad \forall a \in A, i = 1, \dots, I + 1 \quad (41)$$

$$\sum_{i=1}^{I+1} b_{a,i} = 1 \quad \forall a \in A \quad (42)$$

$$\sum_{i=1}^{I+1} b_{a,i} y_{a,i} = q_a \quad \forall a \in A \quad (43)$$

$$\hat{g}_a = \sum_{i=1}^{I+1} b_{a,i} g_a(y_{a,i}) \quad \forall a \in A \quad (44)$$

$$(b_{a,1}, b_{a,2}, \dots, b_{a,I+1}) \in \text{SOS2} \quad \forall a \in A \quad (45)$$

$$\frac{\hat{g}_a}{u_{a0}^k} \leq \hat{c}_a^k \leq \frac{\hat{g}_a}{\lambda_a^k u_{a0}^k} \quad \forall a \in A, k \in H \quad (46)$$

$$\hat{c}_a^k = \frac{\hat{g}_a}{u_{a0}^k} \quad \forall a \in A, k \in M \quad (47)$$

$$c_{w,r}^k = \sum_{a \in A} \hat{c}_a^k \delta_{r,a} \quad \forall k \in K, w \in W, r \in R_w \quad (48)$$

5. Numerical experiments

Numerical experiments are conducted on two networks with different topologies: the Tilburg network in the Netherlands, with 18 nodes and 44 directed links, and the Sioux Falls network in the United States, with 24 nodes and 76 directed links. The corresponding network topologies are shown in Figure 3.

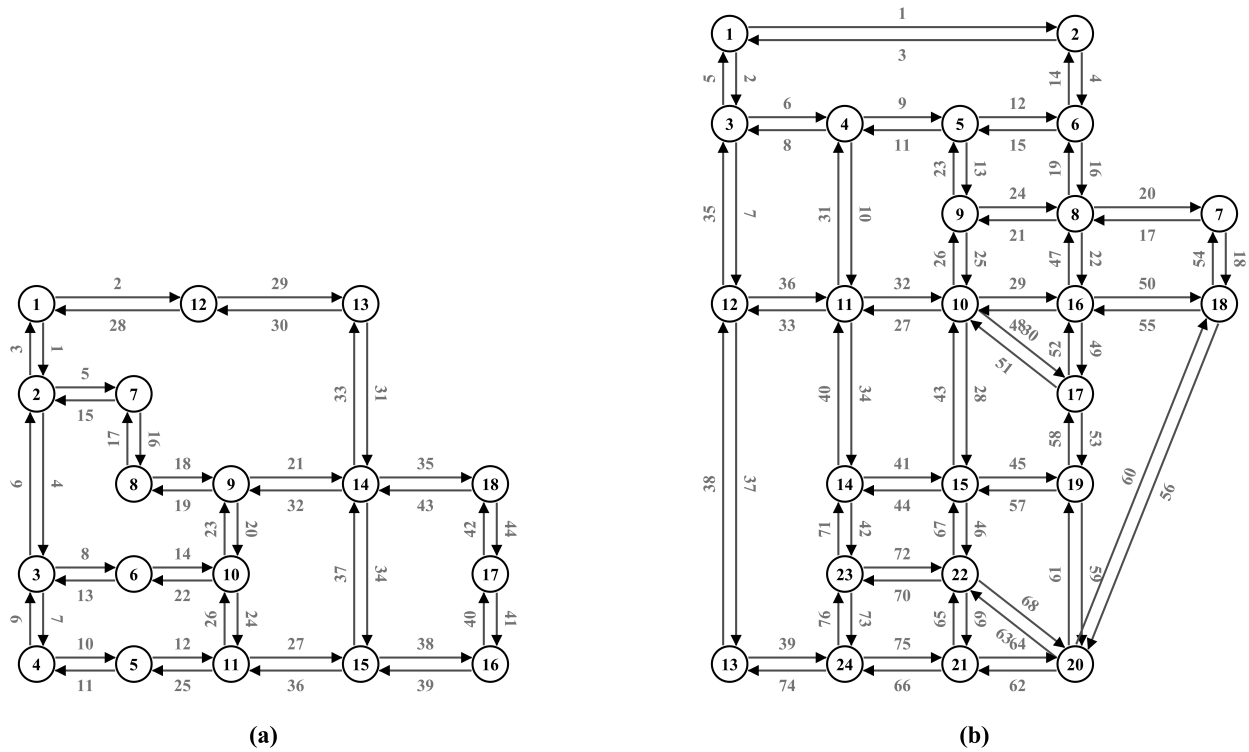


Figure 3. Transportation networks used in the numerical experiments: (a) Tilburg network; (b) Sioux Falls network.

Table 2. Key parameters used in the numerical experiments.

Parameter	Value
BPR α/β	0.15/4
PCE (regular/hazmat)	1/3
Impact radius	0.8 km
Accident rate	10^{-7} vehicle-km $^{-1}$
$I_{\text{left}}/I_{\text{right}}$	4/8
Genetic algorithm population/generations	50/100
Diagonalization tolerance	10^{-4}

The network parameters and regular traffic demand for the Tilburg and Sioux Falls networks are obtained from the Rijkswaterstaat data repository [45] and the Transportation Networks for Research repository [46], respectively. The two networks contain 12 and 552 regular traffic OD pairs, respectively.

To simplify the analysis, regular traffic is assumed to consist of passenger cars, while hazmat traffic is represented by hazmat trucks. Since public hazmat shipment data are unavailable, 11 hazmat OD pairs for Tilburg and 20 for Sioux Falls are synthetically generated following Esfandeh et al. [13]. Population data are obtained from Statistics Netherlands [47] and the U.S. Census Bureau [48], respectively. The relevant network data and generated hazmat OD demand data are provided in Supplementary Tables S1–S2 and S3, respectively.

The key parameters used in the numerical experiments are summarized in Table 2.

To investigate the effects of the speed-limit strategy under different control intensities, five scenarios are considered for each network, with the speed-limit coefficient λ decreasing from 0.9 to 0.5 in increments of 0.1. Thus, the allowable speed ranges from 90% to 50% of the free-flow speed. The proposed MILP model and the heuristic algorithm used for numerical validation are implemented in MATLAB. The MILP model is solved using Gurobi Optimizer 13.0.0 on a Windows 10 system with an Intel Core i5-9400F CPU at 2.90 GHz and 16 GB of RAM. The pseudocode of the heuristic validation procedure is provided in the Supplementary Materials.

5.1. Numerical validation of the MILP reformulation

The no-speed-limit scenario is used as the reference scenario throughout the numerical experiments. The relative changes in transportation risk and travel delay are calculated as follows:

$$\text{Change of Risk} = \frac{R(\lambda) - R_0}{R_0} \times 100\% \quad (49)$$

$$\text{Change of Delay} = \frac{D(\lambda) - D_0}{D_0} \times 100\% \quad (50)$$

where $R(\lambda)$ and $D(\lambda)$ denote the total transportation risk and travel delay under the speed-limit scenario with coefficient λ , respectively, and R_0 and D_0 are the corresponding values under the no-speed-limit scenario.

To validate the effectiveness of the MILP reformulation, a genetic algorithm (GA) is employed to solve the original bilevel model, with the lower-level user equilibrium problem solved using the diagonalization algorithm proposed by Zhang and Nie [49], as detailed in Supplementary Algorithm 1. The computational results obtained by the two approaches under the same speed-limit settings on the Tilburg network are presented in Table 3.

As shown in Table 3, the two approaches exhibit generally consistent trends across different speed-limit settings. Specifically, as λ decreases, the transportation risk generally decreases, while the travel delay of hazmat vehicles generally increases. It should be noted that the two approaches produce somewhat different numerical results due to the differences between Eqs. (22) and (3), the piecewise linearization error in the MILP model, and the stochastic nature of the GA. Nevertheless, despite the differences in the specific results, the two approaches exhibit generally consistent trends, indicating that the MILP reformulation can capture the main variation patterns of the original bilevel model under the tested network and speed-limit settings. More importantly, the comparison confirms that the main conclusion of this study, namely, that speed-limit strategies can reduce hazardous-materials transportation risk at the expense of increased travel delay for hazmat vehicles, remains unchanged. Therefore, the MILP reformulation is adopted for all subsequent numerical experiments, and the results presented in Sections 5.2 and 5.3 are obtained using the MILP model.

Table 3. Numerical validation of the MILP reformulation on the Tilburg network.

Method	λ	Change of Risk	Change of Delay	
			Hazmat	Regular
GA + Diagonalization	0.9	-2.42%	+2.49%	+0.19%
	0.8	-16.70%	+7.39%	+1.40%
	0.7	-26.17%	+10.52%	+0.99%
	0.6	-32.52%	+14.43%	+1.04%
	0.5	-34.50%	+15.59%	+1.08%
MILP + Gurobi	0.9	-9.74%	+4.11%	+0.93%
	0.8	-26.45%	+9.57%	+2.62%
	0.7	-29.27%	+18.76%	+3.25%
	0.6	-33.96%	+18.69%	+3.13%
	0.5	-35.12%	+19.71%	+3.17%

Note: The average computational times for the GA + Diagonalization and MILP with Gurobi approaches were 516 s and 27 s, respectively. The final MIP gap was 0% for all MILP instances.

5.2. Network risk and transportation efficiency analysis

Following the validation on the Tilburg network, the analysis is further extended to the Sioux Falls network. Table 4 presents the changes in transportation risk and travel delay of hazmat vehicles and regular traffic under five speed-limit scenarios for both networks. Consistent with the MILP results for the Tilburg network shown in Table 3, the speed-limit strategy exhibits the same overall effect in both networks: reducing the speed limit decreases transportation risk while increasing the travel delay of hazmat vehicles. In the Tilburg network, as λ decreases from 0.9 to 0.5, the reduction in transportation risk increases from 2.42% to 35.12%, while the travel delay of hazmat vehicles increases by 2.49%–19.71%. In the Sioux Falls network, the corresponding reduction in transportation risk increases from 0.61% to 7.91%, while the travel delay of hazmat vehicles increases by 3.20%–11.15%.

Although the two networks exhibit the same overall trend, the magnitude of the effects differs considerably. The speed-limit strategy leads to a much greater reduction in transportation risk in the Tilburg network than in the Sioux Falls network, while also resulting in a larger increase in hazmat travel delay. Therefore, the actual operating conditions of the network should be considered when designing speed-limit strategies. In particular, for relatively congested networks such as Sioux Falls, reducing vehicle speeds may provide limited additional benefits in terms of transportation risk reduction.

Additionally, the changes in travel delay of regular traffic are further examined. In the Tilburg network, the travel delay of regular traffic increases by approximately 0.93%–3.25%; in comparison, the changes in the Sioux Falls network range from -0.03% to 0.04%, with slight decreases observed under the $\lambda = 0.8$ and $\lambda = 0.7$ scenarios. While reducing transportation risk and accounting for the resulting increase in hazmat travel delay, selecting speed-limit strategies with smaller impacts on regular traffic can help achieve a balance between transportation risk and traffic operating efficiency.

Table 4. Changes in transportation risk and travel delay under five speed-limit scenarios on the Sioux Falls network.

λ	Change of Risk	Change of Delay	
		Hazmat	Regular
0.9	−0.61%	+3.20%	+0.01%
0.8	−5.31%	+9.14%	−0.01%
0.7	−7.56%	+7.73%	−0.03%
0.6	−7.91%	+11.15%	+0.04%
0.5	−7.91%	+11.15%	+0.04%

Note: The MILP model was solved using Gurobi with a time limit of 10,000 s. The final MIP gap was below 1% for all instances.

The Gini coefficient is calculated for both networks to assess the fairness of the spatial distribution of transportation risk and provide an additional perspective on network-wide risk equity. As shown in Table 5, the coefficients remain close to their baseline values under the five speed-limit scenarios, indicating relatively limited changes in the overall spatial distribution of transportation risk.

Table 5. Network transportation risk equity measured by the Gini coefficient under five speed-limit scenarios.

Network	Baseline	λ				
		0.9	0.8	0.7	0.6	0.5
Tilburg	0.687	0.647	0.674	0.663	0.678	0.684
Sioux Falls	0.655	0.632	0.640	0.656	0.663	0.663

Figure 4 further shows the distribution of link transportation risk. In the Tilburg network, the interquartile range narrows and the mean link risk decreases, while in the Sioux Falls network, the interquartile range also decreases and the maximum link risk is reduced. However, Link 43 in the Tilburg network remains the highest-risk link, and its risk increases relative to the no-speed-limit scenario. These results suggest that, although the overall spatial distribution of transportation risk remains relatively stable, localized changes in critical high-risk links may still occur. The implications of speed-limit strategies for risk equity warrant further investigation in future research, particularly with respect to localized changes in high-risk links.

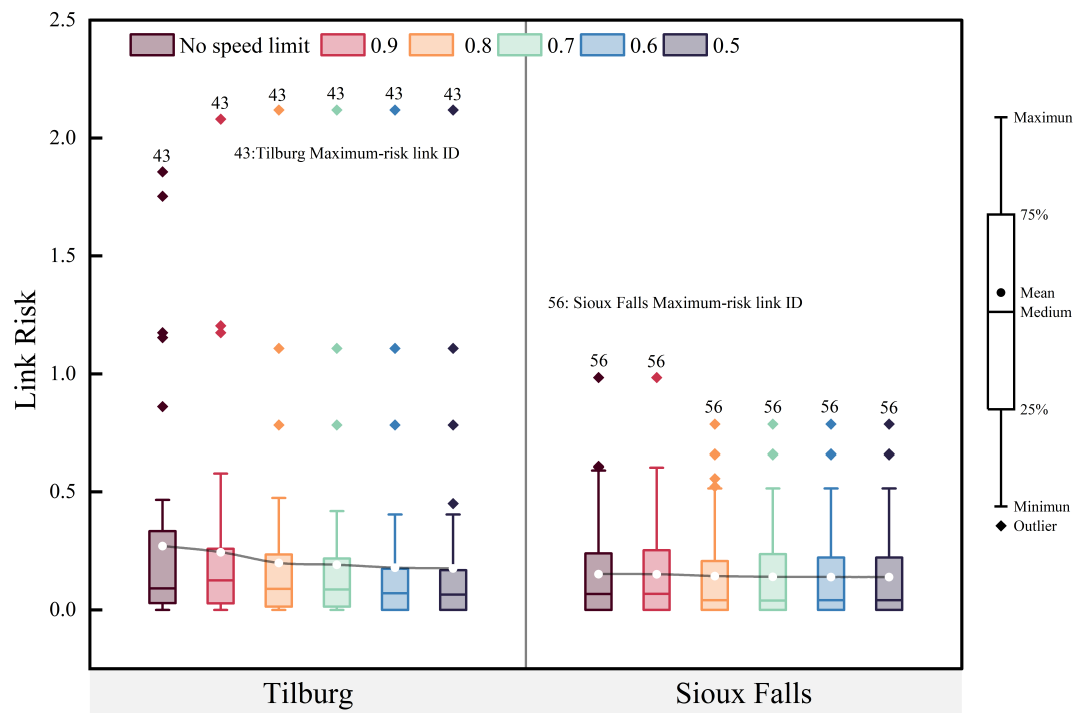


Figure 4. Boxplots of link transportation risks under five speed-limit scenarios for the Tilburg and Sioux Falls networks.

5.3. Spatial redistribution of traffic flows

This section examines the spatial redistribution of traffic flows under the speed-limit strategy for hazmat vehicles. As discussed above, the scenario with $\lambda = 0.7$ represents a moderate level of speed control and effectively reduces transportation risk in both networks. Therefore, this scenario is selected to further analyze the spatial redistribution of traffic flows. Figure 5 illustrates the changes in traffic flow distribution under the $\lambda = 0.7$ scenario, while Figure 6 presents the corresponding link-specific speed-limit settings.

As shown in Figure. 5, the speed-limit strategy changes the route choices of hazmat vehicles, and these route shifts subsequently lead to the spatial redistribution of network traffic flows. In the Tilburg network, hazmat flows are mainly diverted from the original major route (Nodes 2–7–8–9–10–11–5–4) to the peripheral route (Nodes 2–3–4). In the Sioux Falls network, hazmat flows are mainly redistributed from the original route (Nodes 2–6–8–7–18–20) to the alternative route (Nodes 2–1–3–12–13–24–21–20).

Meanwhile, the flow redistribution caused by the route shifts of hazmat vehicles is accompanied by corresponding adjustments in regular traffic flows. Further analysis shows that links with increased hazmat flows generally experience reductions in regular traffic flows, whereas links with decreased hazmat flows tend to experience increases in regular traffic flows. This indicates that the route shifts of hazmat vehicles not only change their own spatial distribution, but also induce the redistribution of regular traffic flows through changes in aggregate traffic flows, resulting in a certain degree of spatial separation between hazmat and regular traffic on some links. Overall, the results indicate that the speed-limit strategy can alter the spatial distribution of mixed traffic flows by inducing route shifts of hazmat vehicles and corresponding adjustments in regular traffic flows.

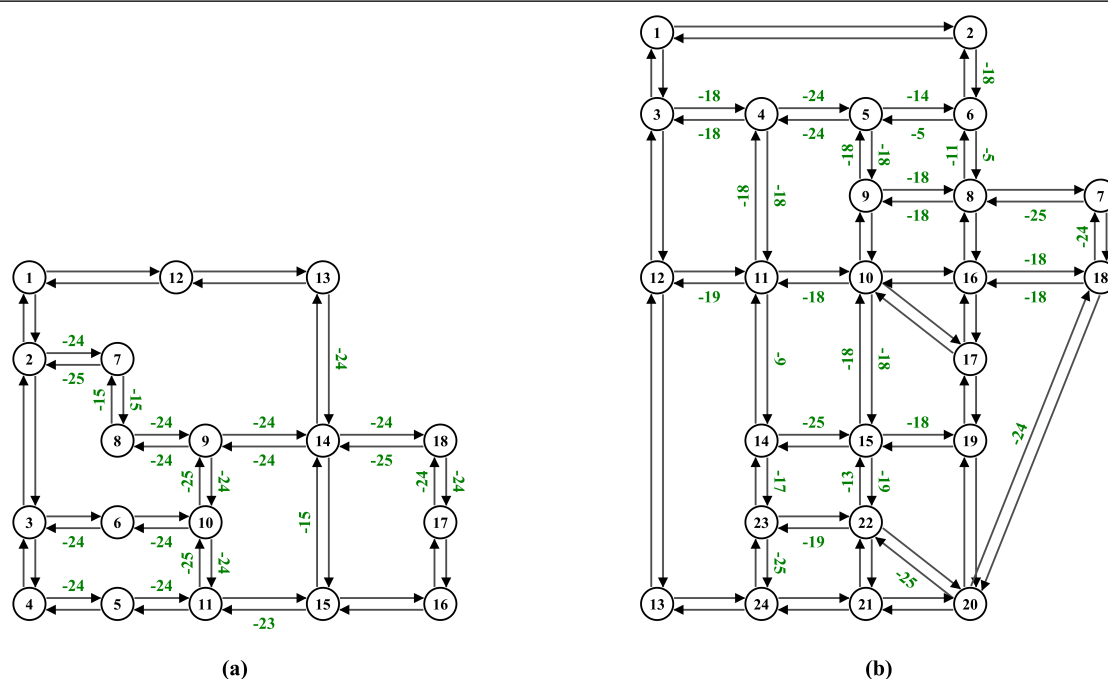


Figure 6. Link-specific speed-limit reductions for hazmat vehicles under $\lambda = 0.7$: (a) Tilburg network; (b) Sioux Falls network. The green labels indicate the changes in speed limits (km/h) relative to the initial free-flow speeds of the corresponding links, where negative values represent speed reductions.

6. Conclusions

This study investigates hazmat transportation risk optimization under mixed traffic conditions and proposes a network-level risk optimization framework based on speed-limit strategies. A government-carrier bilevel optimization model is developed, where the upper-level model determines link-specific speed-limit decisions for hazmat vehicles; the lower-level model considers the effects of hazmat speed limits on their travel costs as well as congestion effects in the mixed traffic environment, and employs a multiclass user equilibrium model to determine the route choices and traffic-flow distributions of hazmat and regular vehicles. The resulting hazmat traffic-flow distribution is fed back to the upper level to reevaluate network transportation risk and optimize the speed-limit decisions.

To address the computational challenges of the original bilevel model, the model is reformulated as a mixed-integer linear programming (MILP) model. Binary auxiliary variables are introduced to handle the complementarity constraints, while the travel time functions are reformulated and approximated using SOS2-based piecewise linearization. Numerical validation shows that the MILP model exhibits generally consistent trends with the original bilevel model solved using a genetic algorithm combined with the diagonalization algorithm, supporting the validity of the proposed model reformulation.

Numerical experiments on the Tilburg and Sioux Falls networks show that speed-limit strategies can generally reduce the modeled hazmat transportation risk by changing the travel costs of hazmat vehicles, which in turn affects their route choices and leads to a redistribution of traffic flows. The magnitude of risk reduction varies across the two networks, indicating that the specific traffic conditions and network characteristics of the target network affect the effectiveness of speed-limit strategies. Further spatial

analysis indicates that the route shifts of hazmat vehicles resulting from changes in travel costs can affect the spatial distribution of traffic flows across the network, with hazmat and regular traffic exhibiting a certain degree of spatial separation on some links.

This study mainly analyzes hazmat transportation risk from a macroscopic network perspective and does not explicitly consider microscopic driving behaviors, such as car-following, lane-changing, and overtaking, induced by speed limits for hazmat vehicles, or the dynamic evolution of traffic conditions. Future research can extend the proposed network optimization framework by incorporating dynamic traffic flow models and microscopic traffic behavior models to further investigate the effects of dynamic traffic conditions and microscopic driving behaviors on hazmat transportation risk and the effectiveness of speed-limit strategies. In addition, real-world accident data can be incorporated to further analyze the relationship between speed-limit strategies and accident probabilities and consequences, thereby improving hazmat transportation risk assessment.

Author contributions

Conceptualization: Guanxiang Zhang and Haidong Huang. Methodology: Haidong Huang. Software: Haidong Huang. Formal analysis: Haidong Huang. Validation: Guanxiang Zhang and Haidong Huang. Investigation: Haidong Huang. Writing—original draft: Haidong Huang. Writing—review and editing: Guanxiang Zhang and Haidong Huang. Supervision: Guanxiang Zhang.

Use of Generative-AI tools declaration

The authors used ChatGPT (OpenAI) to improve the readability and language of the manuscript. All scientific content, methodology, analysis, and conclusions were developed and verified by the authors, who take full responsibility for the content of this article.

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Conflict of interest

The authors declare that they have no competing interests.

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