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ERA, 34(5): 3334–3349.
DOI: 10.3934/era.2026150
Received: 26 November 2025
Revised: 26 February 2026
Accepted: 05 March 2026
Published: 17 April 2026

Research article

Hierarchical stability conditions for aperiodic sampled-data control systems with fixed delay

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Abstract: This paper deals with the stability issue of aperiodic sampled-data control systems with fixed delay. First, to obtain the less conservative stability criterion, the hierarchical looped-functional is assembled. In the functional, the integrals in the integral inequalities are directly utilized in the construction. Hence, the integrals of the system equation are prevented to avoid the products of the free matrices and system matrices. Also, the integrals over the segmented interval $[t, t-\gamma]$ are introduced. Then, by employing the generalized free-matrix-based integral inequalities, the integrals of the same quadratic product term over different time intervals are estimated to acquire the linear functions of these intervals. Next, the hierarchical stability conditions are presented in terms of the linear matrix inequalities based on the Schur complement. Finally, some numeric examples are used to illustrate the superiority of the proposed method.

Keywords: aperiodic sampled-data control systems; hierarchical looped functional; stability; generalized free-matrix-based integral inequalities

1. Introduction

With the development of modern technology, sampled-data control systems (SDCSs) received much attention because of their convenience and effectiveness [1–4]. Benefiting from the extensive attention, many of research achievements have been obtained and presented. In control theory, the most fundamental requirement is stability. Thus, a lot of stability analysis approaches were proposed for SDCSs [5–13]. Among them, the input delay method [8] and the looped-functional (LF) method [11, 14] have become the main tools to derive stability criteria [15–18]. As we all know, the

obtained stability conditions based on these two approaches are sufficient ones, which have some conservativeness naturally. And, the sampling period that can be calculated by these stability criteria is the main evaluation indicator. The bigger the sampling period, the less conservative the stability conditions. Consequently, acquiring a larger sampling period and a stability criterion with less conservativeness has become the research focus of SDCSs.

Over the past twenty years, the combination of the Lyapunov stability principle and the linear matrix inequality (LMI) technology has been developed for the stability analysis of SDCSs. By treating the control signals as the delayed ones, the input delay approach was proposed in [19]. As a result, SDCSs were treated as delayed systems, and the stability criteria of SDCSs were the same as the ones for delayed systems [19–21]. Then, the improved input delay approach and the time-dependent functional were provided in [8]. Thereafter, the discontinuous Lyapunov functional (DLF) [22] and the augmented Lyapunov functional [23] were constructed to improve stability analysis methods. Until [11], the proposal of LF attracted much attention and derived excellent results. Additionally, the LF framework was extended by the two-sided LF presented in [17]. By transforming the first-order free-matrix-based integral inequality (FII), the integrals of the system state began to appear in the construction of the LF [24]. Nevertheless, there are no hierarchical types of LF and LMI that have been put forward for SDCSs. And, the signal transmission time was not considered in the above-mentioned articles.

In practical SDCSs, time delay is a ubiquitous and inevitable phenomenon in signal transmission [25]. And, for the vast majority of control systems, time delay is also an unstable factor that can deteriorate the performance and even destroy the stability [26–28]. Thus, research on the stability of SDCSs with time delay has become a concern for many scholars [22, 23, 29, 30]. In [22], a new DLF was used to study the invariant delay's effect on SDCSs, and some stability criteria were provided. Based on the Wirtinger's inequality, a novel DLF was established to enlarge the sampling period [29]. Afterwards, the augmented DLFs were employed to further reduce the stability conditions' conservatism [23, 30]. In [4], by adopting the two-sided LF framework, it has been proved that some fixed delays can increase the constant sampling interval and accelerate the convergence time for some SDCSs. However, the sampling interval was assumed to be constant, which is not in accordance with a real situation. Accordingly, the aperiodic sampling is the focus of this paper by considering the time delay.

Before the application of the Wirtinger-based integral inequality, the system state integrals were never in the derivation of the stability criteria for SDCSs. In [31], the single integral was first employed. But, the integral was not in the construction of the LF. To enhance the relationship between the integral and the other states, a zero equality and a free matrix are introduced. For the zero equality, it is derived from the integral of the system equation. Subsequently, the two-sided LF and double integrals were used to obtain the stability criterion in [17]. However, the LF did not include the integrals, and the zero equalities were introduced along with some free matrices. In the above two articles, the introduction of the system equation integrals results in the coupling of system matrices and free matrices. To avoid the coupling matrices and save computational resources, the integral involved in the transformed FII was used to construct the LF [24]. Thereafter, the new LF, including the integrals involved in the second-order FIIs, is provided in [32]. But, for SDCSs with a fixed delay, there are no hierarchical LFs that have been proposed to obtain the hierarchical LMIs.

Based on prior discussion and the above observation, the stability problem of SDCSs with a fixed delay is researched in this paper. The main contributions are concluded as follows:

- For the first time, the hierarchical-type LF is constructed, which consists of the integrals in the generalized FIIs. And thus, the hierarchical LMIs are assembled.
- By dividing the integral interval into some subintervals, the novel integrals involved in one of the subintervals are introduced. And, these integrals will be employed to construct the hierarchical LF, which can help reduce conservatism.
- For some SDCSs, some fixed delays have been proven to enlarge the periodic sampling period upper boundary and quicken the convergence time.

Notations: $\mathbb{R}^n$ represents the n-dimensional Euclidean space. $\mathbb{R}^{m\times n}$ denotes the union of $m \times n$ real matrices. Superscripts T and -1 stand for the transpose and the inverse of a matrix, respectively. $P > 0$ (< 0) illustrates that matrix P is positive definite (negative definite). In a symmetric matrix, '$*$' is the symbol of the symmetric term. I and 0 are the identity matrix and the zero matrix, respectively. diag$\{\cdots\}$ is a block-diagonal matrix, and $\mathrm{He}\{A\} = A + A^T$.

2. Preliminaries

In this section, first of all, the SDCS model is given as

$$\dot{z}(t) = Cz(t) + Dv(t), \tag{2.1}$$

where $z(t) \in \mathbb{R}^n$ is the system state vector; $v(t) \in \mathbb{R}^s$ is the control input, and C and D are system matrices. In SDCSs, the control signals are transmitted only at a sequence of time instants, i.e., $0 = t_0 < t_1 < \cdots < t_k < \cdots < \infty$. Thus, considering the transmission time, the control input has the following form:

$$v(t) = Kz(t_k - \delta),\ t_k \le t < t_{k+1}, \tag{2.2}$$

where $K \in \mathbb{R}^{s\times n}$ are the given gain matrices, and δ represents the communication delay. In the following, $\tau_k = t_{k+1} - t_k$ is denoted as the sampling intervals, and $\tau \ge \tau_k > 0$ is the upper boundary. Let $\gamma_k = \tau_k + \delta \in (\delta, \gamma]$, γ_k be the sum of τ_k and δ, and $\gamma = \tau + \delta$ be the upper boundary. As the initial condition, $z(s) = \phi(s)$, $s \in [-\gamma, 0]$.

The aim of this article is to derive the stability criterion that allows the boundary τ to be as large as possible. Then, to obtain the main results, the following generalized FII is needed.

Lemma 1 ([33–35]). *For an integrable and differentiable function $z : [l, u] \to \mathbb{R}^n$, an integer $N \ge 0$ and matrices $Z > 0$ and M_N, the following inequality holds for $i = 1, 2$:*

$$-\int_l^u \dot{z}^T(\eta)Z\dot{z}(\eta)d\eta \le (u-l)\xi_{iN}^T M_N \mathcal{Z}_N^{-1} M_N^T \xi_{iN} + \mathrm{He}\{\xi_{iN}^T M_N \Xi_N \Gamma_{iN}\zeta_{iN}\}, \tag{2.3}$$

where

$$\Gamma_{1N}=\begin{bmatrix} I & -I & 0 & 0 & \cdots & 0 \\ 0 & -I & I & 0 & \cdots & 0 \\ 0 & -I & 0 & 2I & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & -I & 0 & 0 & \cdots & NI \end{bmatrix}, \Gamma_{2N}=\begin{bmatrix} I & -I & 0 & 0 & \cdots & 0 \\ I & 0 & -I & 0 & \cdots & 0 \\ I & 0 & 0 & -2I & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ I & 0 & 0 & 0 & \cdots & -NI \end{bmatrix},$$

$$\zeta_{1N}=\mathrm{col}\{z(u), z(l), \rho_1(l,u), \cdots, \rho_N(l,u)\},\ \zeta_{2N}=\mathrm{col}\{z(u), z(l), \bar{\rho}_1(l,u), \cdots, \bar{\rho}_N(l,u)\},$$

$$\rho_a(l,u)=\int_l^u \frac{(u-\eta)^{a-1}}{(u-l)^a} z(\eta)d\eta, a=1,2,\cdots,N,\ \ \bar{\rho}_b(l,u)=\int_l^u \frac{(\eta-l)^{b-1}}{(u-l)^b} z(\eta)d\eta, b=1,2,\cdots,N,$$

$$\Xi_N=\begin{bmatrix} I & 0 & \cdots & 0 \\ I & (-1)^1\binom{1}{1}\binom{2}{1}I & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ I & (-1)^1\binom{N}{1}\binom{N+1}{1}I & \cdots & (-1)^N\binom{N}{N}\binom{2N}{N}I \end{bmatrix}, \mathcal{Z}_N=\mathrm{diag}\{Z, 3Z, \cdots, (2N+1)Z\},$$

and ξ_{iN} is an appropriate vector including ζ_{iN}.

Remark 1. *For SDCS* (2.1) *with input* (2.2)*, as far as the authors know, there are no hierarchical LF constructed by the integrals in the above generalized FII. Accordingly, this is the first attempt to construct a hierarchical LF and establish hierarchical stability conditions via the above FII. Under the framework of LF, first, the states and integrals in the above FIIs are going to be used to construct the LF. Then, in the derivative of the functional, the integrals of the same quadratic product item will be estimated by the above FIIs.*

Remark 2. *For the same integer N, the above generalized FII can derive the lowest conservative results compared with other integral inequalities. And, the larger N, the less conservative. If other integral inequalities are employed, the integral interval will be shown in the denominator. Under these circumstances, it is hard to determine the negative conditions for the integral-interval-dependent functional derivative. Based on the above generalized FII, the integral interval will be presented as the linear variable. Then, the convex analysis method can be used to obtain the negative conditions for the functional derivative.*

3. Main results

In the following, the stability criterion of system (2.1) with (2.2) will be proposed based on the hierarchical LF and generalized FIIs.

Theorem 1. *For positive integer N, some constants $\tau > 0$ and $\delta > 0$, and the given gain matrix K, if there exist positive matrices $P_N \in \mathbb{R}^{(N+2)n\times(N+2)n} > 0$, $S \in \mathbb{R}^{4n\times 4n} > 0$, $Y_1 \in \mathbb{R}^{n\times n} > 0$, $Y_3 \in \mathbb{R}^{n\times n} > 0$, $R_2 \in \mathbb{R}^{n\times n} > 0$, and $R_4 \in \mathbb{R}^{n\times n} > 0$, symmetric matrices $Y_2 \in \mathbb{R}^{n\times n}$, $Z_N \in \mathbb{R}^{(5N+4)n\times(5N+4)n}$, $R_1 \in \mathbb{R}^{n\times n}$, and $R_3 \in \mathbb{R}^{n\times n}$, any matrices $X_N \in \mathbb{R}^{(3N+2)n\times(2N+2)n}$, $Q_{1N} \in \mathbb{R}^{(N+2)n\times(4N+6)n}$, $Q_{2N} \in \mathbb{R}^{2n\times(4N+6)n}$, and*

$M_{iN} \in \mathbb{R}^{(6N+7)n\times(N+2)n}, i = 1, \cdots, 6$, *the following inequalities hold with* $R_1 + Y_3 > 0$, $R_3 + Y_2 > 0$ *and* $Y_2 + Y_3 > 0$, *then, system* (2.1) *is stable.*

$$\begin{bmatrix} \Psi_1 + \tau_k\Psi_2 & \sqrt{\tau_k}\Upsilon_1 & \sqrt{\delta}\Upsilon_2 \\ * & -\Xi_1 & 0 \\ * & * & -\Xi_2 \end{bmatrix} < 0, \tag{3.1}$$

$$\begin{bmatrix} \Psi_1 + \tau_k\Psi_3 & \sqrt{\tau_k}\Upsilon_3 & \sqrt{\delta}\Upsilon_2 \\ * & -\Xi_3 & 0 \\ * & * & -\Xi_2 \end{bmatrix} < 0, \tag{3.2}$$

where

$$\begin{aligned}
\Psi_1 &= \mathrm{He}\{\Theta_1^T P_N\Theta_2 + \Theta_5^T S\Theta_6 + \Theta_7^T X_N\Theta_8 + \Theta_{10}^T X_N\Theta_{11} + \Theta_{18}^T Q_{1N}(\Theta_{20} - \Theta_{17}) + \Theta_{23}^T Q_{2N}(\Theta_{25} + \Theta_{22}) \\
&\quad + M_{1N}E_1 + M_{2N}E_2 + M_{3N}E_3 + M_{4N}E_4 + M_{5N}E_5 + M_{6N}E_6\} + \Theta_3^T S\Theta_3 - \Theta_4^T S\Theta_4 \\
&\quad + \Gamma^T(\delta Y_1 + \tau Y_2 + \gamma Y_3)\Gamma, \\
\Psi_2 &= \mathrm{He}\{\Theta_7^T X_N\Theta_9 + \Theta_{14}^T Z_N\Theta_{13} + \Theta_{16}^T Q_{1N}\Theta_{17} + \Theta_{18}^T Q_{1N}\Theta_{19}\} + \Theta_{13}^T Z_N\Theta_{13} + \Gamma^T R_1\Gamma + r_7^T R_3 r_7, \\
\Psi_3 &= \mathrm{He}\{\Theta_{12}^T X_N\Theta_{11} + \Theta_{15}^T Z_N\Theta_{13} + \Theta_{21}^T Q_{2N}\Theta_{22} + \Theta_{23}^T Q_{2N}\Theta_{24}\} - \Theta_{13}^T Z_N\Theta_{13} + \Gamma^T R_2\Gamma + r_7^T R_4 r_7, \\
\Upsilon_1 &= [M_{4N}, M_{6N}], \Upsilon_2 = [M_{1N}, M_{2N}], \Upsilon_3 = [M_{3N}, M_{5N}], \Xi_1 = \mathrm{diag}\{\mathcal{R}_{2N}, \mathcal{R}_{4N}\}, \Xi_2 = \mathrm{diag}\{\mathcal{Y}_{1N}, \mathcal{Y}_{3N}\}, \\
\Xi_3 &= \mathrm{diag}\{\mathcal{R}_{1N} + \mathcal{Y}_{3N}, \mathcal{R}_{3N} + \mathcal{Y}_{2N}\}, \mathcal{R}_{aN} = \mathrm{diag}\{R_a, 3R_a, \cdots, (2N+1)R_a\}, a = 1, 2, 3, 4, \\
\mathcal{Y}_{bN} &= \mathrm{diag}\{Y_b, 3Y_b, \cdots, (2N+1)Y_b\}, b = 1, 2, 3, \Theta_1 = [\Gamma^T, r_7^T, r_1^T - r_2^T, \delta\dot{r}_9^T, \cdots, \delta\dot{r}_{N+7}^T]^T, \\
\delta\dot{r}_{c+7} &= r_1 - (c-1)r_{c+6}, c = 2, 3, \cdots, N, \Theta_2 = [r_1^T, r_2^T, \delta r_8^T, \cdots, \delta r_{N+7}^T]^T, \Theta_3 = [r_1^T, r_2^T, r_1^T, \Gamma^T]^T, \\
\Theta_4 &= [r_1^T, r_2^T, r_2^T, r_7^T]^T, \Theta_5 = [\Gamma^T, r_7^T, 0, 0]^T, \Theta_6 = [\delta r_1^T, \delta r_2^T, \delta r_8^T, r_1^T - r_2^T]^T, \\
\Theta_7 &= [\Gamma^T, r_7^T, r_1^T, r_1^T - r_{N+9}^T, \cdots, r_1^T - (N-1)r_{2N+7}^T, r_2^T, r_2^T - r_{4N+9}^T, \cdots, r_2^T - (N-1)r_{5N+7}^T, r_1^T - r_2^T, \delta\dot{r}_9^T, \\
&\quad \cdots, \delta\dot{r}_{N+7}^T]^T, \Theta_8 = [r_1^T - r_5^T, r_2^T - r_6^T, 0_N, 0_N]^T, \Theta_9 = [0, 0, r_{2N+8}^T, \cdots, r_{3N+7}^T, r_{5N+8}^T, \cdots, r_{6N+7}^T]^T, \\
\Theta_{10} &= [r_1^T - r_3^T, r_2^T - r_4^T, 0_N, 0_N, \delta(r_8^T - r_{3N+8}^T), \cdots, \delta(r_{N+7}^T - r_{4N+7}^T)]^T, \\
\Theta_{11} &= [\Gamma^T, r_7^T, -r_1^T, r_{2N+9}^T - r_1^T, \cdots, (N-1)r_{3N+7}^T - r_1^T, -r_2^T, r_{5N+9}^T - r_2^T, \cdots, (N-1)r_{6N+7}^T - r_2^T]^T, \\
\Theta_{12} &= [0, 0, r_{N+8}^T, \cdots, r_{2N+7}^T, r_{4N+8}^T, \cdots, r_{5N+7}^T]^T, \\
\Theta_{13} &= [r_3^T, r_5^T, r_4^T, r_6^T, \delta r_{3N+8}^T, \cdots, \delta r_{4N+7}^T, r_{N+8}^T, \cdots, r_{2N+7}^T, r_{2N+8}^T, \cdots, r_{3N+7}^T, r_{4N+8}^T, \cdots, r_{5N+7}^T, r_{5N+8}^T, \\
&\quad \cdots, r_{6N+7}^T]^T, \Theta_{16} = [\Gamma^T, r_7^T, r_1^T - r_2^T, \delta\dot{r}_9^T, \cdots, \delta\dot{r}_{N+7}^T]^T, \\
\Theta_{14} &= [0_4, 0_N, r_1^T - r_{N+8}^T, \cdots, r_1^T - Nr_{2N+7}^T, 0_N, r_2^T - r_{4N+8}^T, \cdots, r_2^T - Nr_{5N+7}^T, 0_N]^T, \\
\Theta_{15} &= [0_4, 0_N, 0_N, r_{2N+8}^T - r_1^T, \cdots, Nr_{3N+7}^T - r_1^T, 0_N, r_{5N+8}^T - r_2^T, \cdots, Nr_{6N+7}^T - r_2^T]^T, \\
\Theta_{17} &= [\Theta_2^T, r_3^T, r_5^T, r_4^T, r_6^T, \delta r_{3N+8}^T, \cdots, \delta r_{4N+7}^T, r_{2N+8}^T, \cdots, r_{3N+7}^T, r_{5N+8}^T, \cdots, r_{6N+7}^T]^T, \\
\Theta_{18} &= [r_1^T - r_3^T, r_2^T - r_4^T, \delta(r_8^T - r_{3N+8}^T), \cdots, \delta(r_{N+7}^T - r_{4N+7}^T)]^T, \Theta_{19} = [\Theta_1^T, 0_4, 0_N, 0_N, 0_N]^T, \\
\Theta_{20} &= [0_{2+N}, 0_4, 0_N, r_{2N+8}^T - r_1^T, \cdots, Nr_{3N+7}^T - r_1^T, r_{5N+8}^T - r_2^T, \cdots, Nr_{6N+7}^T - r_2^T]^T, \\
\Theta_{21} &= [\Gamma^T, r_7^T]^T, \Gamma = Cr_1 + DKr_4, \Theta_{23} = [r_1^T - r_5^T, r_2^T - r_6^T]^T, \Theta_{24} = [\Theta_1^T, 0_4, 0_N, 0_N, 0_N]^T, \\
\Theta_{22} &= [\Theta_2^T, r_3^T, r_5^T, r_4^T, r_6^T, \delta r_{3N+8}^T, \cdots, \delta r_{4N+7}^T, r_{N+8}^T, \cdots, r_{2N+7}^T, r_{4N+8}^T, \cdots, r_{5N+7}^T]^T, \\
\Theta_{25} &= [0_{2+N}, 0_4, 0_N, r_1^T - r_{N+8}^T, \cdots, r_1^T - Nr_{2N+7}^T, r_2^T - r_{4N+8}^T, \cdots, r_2^T - Nr_{5N+7}^T]^T, \\
E_1 &= \Xi_N\Gamma_{2N}\mathrm{col}\{r_1, r_2, r_8, \cdots, r_{7+N}\}, E_2 = \Xi_N\Gamma_{2N}\mathrm{col}\{r_3, r_4, r_{8+3N}, \cdots, r_{7+4N}\},
\end{aligned}$$

$E_3 = \Xi_N\Gamma_{2N}\text{col}\{r_1, r_3, r_{8+N}, \cdots, r_{7+2N}\}, E_4 = \Xi_N\Gamma_{1N}\text{col}\{r_5, r_1, r_{8+2N}, \cdots, r_{7+3N}\},$
$E_5 = \Xi_N\Gamma_{2N}\text{col}\{r_2, r_4, r_{8+4N}, \cdots, r_{7+5N}\}, E_6 = \Xi_N\Gamma_{1N}\text{col}\{r_6, r_2, r_{8+5N}, \cdots, r_{7+6N}\},$
$r_j = \left[0_{n\times(j-1)n}, I_n, 0_{n\times(6N+7-j)n}\right], j = 1, \cdots, 6N+7.$

Proof: First, the following Lyapunov functional $V(t)$ and LF $\mathfrak{V}(t)$ are given:

$$W(t) = V(t) + \mathfrak{V}(t), \tag{3.3}$$

where $V(t) = \sum_{i=1}^{2} V_i(t)$, $\mathfrak{V}(t) = \sum_{i=1}^{4} \mathfrak{V}_i(t)$,

$$V_1(t) = v_1^T(t)P_N v_1(t) + \int_{t-\delta}^{t} v_2^T(t,\eta)S\, v_2(t,\eta)d\eta,$$
$$V_2(t) = \int_{-\delta}^{0}\int_{t+\theta}^{t} \dot{z}^T(\eta)Y_1\dot{z}(\eta)d\eta d\theta + \int_{-\gamma}^{-\delta}\int_{t+\theta}^{t} \dot{z}^T(\eta)Y_2\dot{z}(\eta)d\eta d\theta + \int_{-\gamma}^{0}\int_{t+\theta}^{t} \dot{z}^T(\eta)Y_3\dot{z}(\eta)d\eta d\theta,$$
$$\mathfrak{V}_1(t) = 2v_3^T(t)X_N v_4(t), \mathfrak{V}_2(t) = (t_{k+1}-t)(t-t_k)v_5^T(t)Z_N v_5(t),$$
$$\mathfrak{V}_3(t) = 2(t_{k+1}-t)v_6^T(t)Q_{1N}v_7(t) + 2(t-t_k)v_8^T(t)Q_{2N}v_9(t),$$
$$\mathfrak{V}_4(t) = (t_{k+1}-t)\int_{t_k}^{t} \dot{z}^T(\eta)R_1\dot{z}(\eta)d\eta - (t-t_k)\int_{t}^{t_{k+1}} \dot{z}^T(\eta)R_2\dot{z}(\eta)d\eta + (t_{k+1}-t)\int_{t_k-\delta}^{t-\delta} \dot{z}^T(\eta)R_3\dot{z}(\eta)d\eta$$
$$- (t-t_k)\int_{t-\delta}^{t_{k+1}-\delta} \dot{z}^T(\eta)R_4\dot{z}(\eta)d\eta,$$
$$v_1(t) = [z^T(t), z^T(t-\delta), \delta\bar{w}_N^T(t-\delta,t)]^T, v_2(t,\eta) = [z^T(t), z^T(t-\delta), z^T(\eta), \dot{z}^T(\eta)]^T,$$
$$v_3(t) = [v_{10}^T(t), (t-t_k)\bar{w}_N^T(t_k,t), (t-t_k)\bar{w}_N^T(t_k-\delta,t-\delta), \delta\bar{w}_N^T(t-\delta,t) - \delta\bar{w}_N^T(t_k-\delta,t_k)]^T,$$
$$v_4(t) = [v_8^T(t), (t_{k+1}-t)w_N^T(t,t_{k+1}), (t_{k+1}-t)\times w_N^T(t-\delta,t_{k+1}-\delta)]^T,$$
$$v_5(t) = [v_{11}^T, \delta\bar{w}_N^T(t_k-\delta,t_k), \bar{w}_N^T(t_k,t), w_N^T(t,t_{k+1}), \bar{w}_N^T(t_k-\delta,t-\delta), w_N^T(t-\delta,t_{k+1}-\delta)]^T,$$
$$v_6(t) = [v_{10}^T(t), \delta\bar{w}_N^T(t-\delta,t) - \delta\bar{w}_N^T(t_k-\delta,t_k)]^T, v_8(t) = [z^T(t) - z^T(t_{k+1}), z^T(t-\delta) - z^T(t_{k+1}-\delta)]^T,$$
$$v_7(t) = [v_1^T(t), v_{11}^T, \delta\bar{w}_N^T(t_k-\delta,t_k), w_N^T(t,t_{k+1}), w_N^T(t-\delta,t_{k+1}-\delta)]^T,$$
$$v_9(t) = [v_1^T(t), v_{11}^T, \delta\bar{w}_N^T(t_k-\delta,t_k), \bar{w}_N^T(t_k,t), \bar{w}_N^T(t_k-\delta,t-\delta)]^T,$$
$$v_{10}(t) = [z^T(t) - z^T(t_k), z^T(t-\delta) - z^T(t_k-\delta)]^T, v_{11} = [z^T(t_k), z^T(t_{k+1}), z^T(t_k-\delta), z^T(t_{k+1}-\delta)]^T,$$
$$w_N(l,u) = [\rho_1^T(l,u), \cdots, \rho_N^T(l,u)]^T, \bar{w}_N(l,u) = [\bar{\rho}_1^T(l,u), \cdots, \bar{\rho}_N^T(l,u)]^T.$$

Then, differentiating $W(t)$ yields

$$\dot{V}_1(t) = 2\dot{v}_1^T(t)P_N v_1(t) + v_2^T(t,t)S\, v_2(t,t) - v_2^T(t,t-\delta)S\, v_2(t,t-\delta) + 2\int_{t-\delta}^{t} v_2^T(t,\eta)S\frac{\partial}{\partial t}v_2(t,\eta)d\eta$$
$$= \xi_N^T(t)[\text{He}\{\Theta_1^T P_N\Theta_2 + \Theta_5^T S\Theta_6\} + \Theta_3^T S\Theta_3 - \Theta_4^T S\Theta_4]\xi_N(t),$$
$$\dot{V}_2(t) = \delta\dot{z}^T(t)Y_1\dot{z}(t) + \tau\dot{z}^T(t)Y_2\dot{z}(t) + \gamma\dot{z}^T(t)Y_3\dot{z}(t) + \psi_1(t) + \psi_2(t) + \psi_3(t)$$
$$= \xi_N^T(t)\Gamma^T(\delta Y_1 + \tau Y_2 + \gamma Y_3)\Gamma\xi_N(t) + \psi_1(t) + \psi_2(t) + \psi_3(t),$$
$$\dot{\mathfrak{V}}_1(t) = 2\dot{v}_3^T(t)X_N v_4(t) + 2v_3^T(t)X_N\dot{v}_4(t)$$
$$= \xi_N^T(t)\text{He}\{\Theta_7^T X_N\Theta_8 + \Theta_{10}^T X_N\Theta_{11} + (t_{k+1}-t)\Theta_7^T X_N\Theta_9 + (t-t_k)\Theta_{12}^T X_N\Theta_{11}\}\xi_N(t),$$

$$\begin{aligned}
\dot{\mathfrak{V}}_2(t) &= [(t_{k+1}-t)-(t-t_k)]v_5^T(t)Z_Nv_5(t) + 2(t_{k+1}-t)(t-t_k)\dot{v}_5^T(t)Z_Nv_5(t)\\
&= \xi_N^T(t)\{[(t_{k+1}-t)-(t-t_k)]\Theta_{13}^TZ_N\Theta_{13} + 2[(t_{k+1}-t)\Theta_{14}^T + (t-t_k)\Theta_{15}^T]Z_N\Theta_{13}\}\xi_N(t),\\
\dot{\mathfrak{V}}_3(t) &= 2(t_{k+1}-t)[\dot{v}_6^T(t)Q_{1N}v_7(t) + v_6^T(t)Q_{1N}\dot{v}_7(t)] - 2v_6^T(t)Q_{1N}v_7(t) + 2(t-t_k)[\dot{v}_8^T(t)Q_{2N}v_9(t)\\
&\quad + v_8^T(t)Q_{2N}\dot{v}_9(t)] + 2v_8^T(t)Q_{2N}v_9(t)\\
&= \xi_N^T(t)\mathrm{He}\{(t_{k+1}-t)[\Theta_{16}^TQ_{1N}\Theta_{17} + \Theta_{18}^TQ_{1N}\Theta_{19}] + \Theta_{18}^TQ_{1N}(\Theta_{20}-\Theta_{17}) + (t-t_k)[\Theta_{21}^TQ_{2N}\Theta_{22}\\
&\quad + \Theta_{23}^TQ_{2N}\Theta_{24}] + \Theta_{23}^TQ_{2N}(\Theta_{25}+\Theta_{22})\}\xi_N(t),\\
\dot{\mathfrak{V}}_4(t) &= (t_{k+1}-t)(\dot{z}^T(t)R_1\dot{z}(t) + \dot{z}^T(t-\delta)R_3\dot{z}(t-\delta)) + (t-t_k)(\dot{z}^T(t)R_2\dot{z}(t) + \dot{z}^T(t-\delta)R_4\dot{z}(t-\delta))\\
&\quad + \psi_4(t) + \psi_5(t) + \psi_6(t) + \psi_7(t)\\
&= \xi_N^T(t)[(t_{k+1}-t)(\Gamma^TR_1\Gamma + r_7^TR_3r_7) + (t-t_k)\times(\Gamma^TR_2\Gamma + r_7^TR_4r_7)]\xi_N(t) + \psi_4(t) + \psi_5(t)\\
&\quad + \psi_6(t) + \psi_7(t),
\end{aligned}$$

where

$$\dot{v}_1(t) = [\dot{z}^T(t), \dot{z}^T(t-\delta), \delta\dot{\bar{w}}_N^T(t-\delta,t)]^T, \delta\dot{\bar{w}}_N(t-\delta,t) = [z^T(t)-z^T(t-\delta), \delta\dot{\bar{\rho}}_2^T(t-\delta,t),\cdots,\delta\dot{\bar{\rho}}_N^T(t-\delta,t)]^T,$$
$$\delta\dot{\bar{\rho}}_a(t-\delta,t) = z(t)-(a-1)\bar{\rho}_{a-1}(t-\delta,t), a = 2,3,\cdots,N, v_2(t,t) = [z^T(t), z^T(t-\delta), z^T(t), \dot{z}^T(t)]^T,$$
$$v_2(t,t-\delta) = [z^T(t), z^T(t-\delta), z^T(t-\delta), \dot{z}^T(t-\delta)]^T, \frac{\partial}{\partial t}v_2(t,\eta) = [\dot{z}^T(t), \dot{z}^T(t-\delta), 0, 0]^T,$$
$$\int_{t-\delta}^{t} v_2(t,\eta)d\eta = [\delta z^T(t), \delta z^T(t-\delta), \delta\bar{\rho}_1^T(t-\delta,t), z^T(t)-z^T(t-\delta)]^T,$$
$$\dot{v}_3(t) = [\dot{v}_{10}^T(t), \bar{w}_N^T(t_k,t) + (t-t_k)\dot{\bar{w}}_N^T(t_k,t), \bar{w}_N^T(t_k-\delta,t-\delta) + (t-t_k)\dot{\bar{w}}_N^T(t_k-\delta,t-\delta), \delta\dot{\bar{w}}_N^T(t-\delta,t)]^T,$$
$$\dot{v}_{10}(t) = [\dot{z}^T(t), \dot{z}^T(t-\delta)]^T, \dot{\bar{w}}_N(t_k,t) = [\dot{\bar{\rho}}_1^T(t_k,t),\cdots,\dot{\bar{\rho}}_N^T(t_k,t)]^T,$$
$$\dot{\bar{\rho}}_b(t_k,t) = \frac{1}{t-t_k}(z(t)-b\bar{\rho}_b(t_k,t)), b = 1,2,\cdots,N,$$
$$\dot{\bar{w}}_N(t_k-\delta,t-\delta) = [\dot{\bar{\rho}}_1^T(t_k-\delta,t-\delta),\cdots,\dot{\bar{\rho}}_N^T(t_k-\delta,t-\delta)]^T,$$
$$\dot{\bar{\rho}}_c(t_k-\delta,t-\delta) = \frac{1}{t-t_k}(z(t-\delta)-c\bar{\rho}_c(t_k-\delta,t-\delta)), c = 1,2,\cdots,N,$$
$$\dot{v}_4(t) = [\dot{v}_8^T(t), (t_{k+1}-t)\dot{w}_N^T(t,t_{k+1}) - w_N^T(t,t_{k+1}), (t_{k+1}-t)\dot{w}_N^T(t-\delta,t_{k+1}-\delta) - w_N^T(t-\delta,t_{k+1}-\delta)]^T,$$
$$\dot{v}_8(t) = [\dot{z}^T(t), \dot{z}^T(t-\delta)]^T, \dot{w}_N(t,t_{k+1}) = [\dot{\rho}_1^T(t,t_{k+1}),\cdots,\dot{\rho}_N^T(t,t_{k+1})]^T,$$
$$\dot{\rho}_d(t,t_{k+1}) = \frac{1}{t_{k+1}-t}(d\rho_d(t,t_{k+1})-z(t)), d = 1,2,\cdots,N,$$
$$\dot{w}_N(t-\delta,t_{k+1}-\delta) = [\dot{\rho}_1^T(t-\delta,t_{k+1}-\delta),\cdots,\dot{\rho}_N^T(t-\delta,t_{k+1}-\delta)]^T,$$
$$\dot{\rho}_e(t-\delta,t_{k+1}-\delta) = \frac{1}{t_{k+1}-t}(e\rho_e(t-\delta,t_{k+1}-\delta)-z(t-\delta)), e = 1,2,\cdots,N,$$
$$\dot{v}_5(t) = [0_4, 0_N, \dot{\bar{w}}_N^T(t_k,t), \dot{w}_N^T(t,t_{k+1}), \dot{\bar{w}}_N^T(t_k-\delta,t-\delta), \dot{w}_N^T(t-\delta,t_{k+1}-\delta)]^T,$$
$$\dot{v}_6(t) = [\dot{v}_{10}^T(t), \delta\dot{\bar{w}}_N^T(t-\delta,t)]^T, \dot{v}_7(t) = [\dot{v}_1^T(t), 0_4, 0_N, \dot{w}_N^T(t,t_{k+1}), \dot{w}_N^T(t-\delta,t_{k+1}-\delta)]^T,$$
$$\dot{v}_9(t) = [\dot{v}_1^T(t), 0_4, 0_N, \dot{\bar{w}}_N^T(t_k,t), \dot{\bar{w}}_N^T(t_k-\delta,t-\delta)]^T, \psi_1(t) = -\int_{t-\delta}^{t}\dot{z}^T(\eta)Y_1\dot{z}(\eta)d\eta,$$
$$\psi_2(t) = -\int_{t-\gamma}^{t-\delta}\dot{z}^T(\eta)Y_2\dot{z}(\eta)d\eta, \psi_3(t) = -\int_{t-\gamma}^{t}\dot{z}^T(\eta)Y_3\dot{z}(\eta)d\eta, \psi_4(t) = -\int_{t_k}^{t}\dot{z}^T(\eta)R_1\dot{z}(\eta)d\eta,$$

$$\psi_5(t) = -\int_t^{t_{k+1}} \dot{z}^T(\eta)R_2\dot{z}(\eta)d\eta, \psi_6(t) = -\int_{t_k-\delta}^{t-\delta} \dot{z}^T(\eta)R_3\dot{z}(\eta)d\eta, \psi_7(t) = -\int_{t-\delta}^{t_{k+1}-\delta} \dot{z}^T(\eta)R_4\dot{z}(\eta)d\eta,$$

$$\begin{aligned}\xi_N(t) = [&z^T(t), z^T(t-\delta), z^T(t_k), z^T(t_k-\delta), z^T(t_{k+1}), z^T(t_{k+1}-\delta), \dot{z}^T(t-\delta), \bar{w}_N^T(t-\delta,t), \bar{w}_N^T(t_k,t),\\ &w_N^T(t,t_{k+1}), \bar{w}_N^T(t_k-\delta,t_k), \bar{w}_N^T(t_k-\delta,t-\delta), w_N^T(t-\delta,t_{k+1}-\delta)]^T.\end{aligned}$$

By dividing $\psi_2(t)$ and $\psi_3(t)$ into different parts, the following equalities hold:

$$\begin{aligned}\psi_2(t) &= -\int_{t_k-\delta}^{t-\delta} \dot{z}^T(\eta)Y_2\dot{z}(\eta)d\eta - \int_{t-\gamma}^{t_k-\delta} \dot{z}^T(\eta)Y_2\dot{z}(\eta)d\eta,\\ \psi_3(t) &= -\int_{t_k}^{t} \dot{z}^T(\eta)Y_3\dot{z}(\eta)d\eta - \int_{t_k-\delta}^{t_k} \dot{z}^T(\eta)Y_3\dot{z}(\eta)d\eta - \int_{t-\gamma}^{t_k-\delta} \dot{z}^T(\eta)Y_3\dot{z}(\eta)d\eta.\end{aligned}$$

Thus,

$$\sum_{i=2}^{3}\psi_i(t) = -\int_{t_k-\delta}^{t-\delta} \dot{z}^T(\eta)Y_2\dot{z}(\eta)d\eta - \int_{t_k}^{t} \dot{z}^T(\eta)Y_3\dot{z}(\eta)d\eta - \int_{t_k-\delta}^{t_k} \dot{z}^T(\eta)Y_3\dot{z}(\eta)d\eta - \int_{t-\gamma}^{t_k-\delta} \dot{z}^T(\eta)(Y_2+Y_3)\dot{z}(\eta)d\eta.$$

Based on $Y_2 + Y_3 > 0$, the following inequality holds:

$$\sum_{i=2}^{3}\psi_i(t) \le -\int_{t_k-\delta}^{t-\delta} \dot{z}^T(\eta)Y_2\dot{z}(\eta)d\eta - \int_{t_k}^{t} \dot{z}^T(\eta)Y_3\dot{z}(\eta)d\eta - \int_{t_k-\delta}^{t_k} \dot{z}^T(\eta)Y_3\dot{z}(\eta)d\eta.$$

Define

$$\bar{\psi}_3(t) = -\int_{t_k-\delta}^{t_k} \dot{z}^T(\eta)Y_3\dot{z}(\eta)d\eta, \bar{\psi}_4(t) = -\int_{t_k}^{t} \dot{z}^T(\eta)(R_1+Y_3)\dot{z}(\eta)d\eta, \bar{\psi}_6(t) = -\int_{t_k-\delta}^{t-\delta} \dot{z}^T(\eta)(R_3+Y_2)\dot{z}(\eta)d\eta.$$

Next, employing the generalized FII to $\psi_1(t)$, $\bar{\psi}_3(t)$, $\bar{\psi}_4(t)$, $\psi_5(t)$, $\bar{\psi}_6(t)$, and $\psi_7(t)$ yields

$$\begin{aligned}\psi_1(t) &\le \delta\xi_N^T(t)M_{1N}\mathcal{Y}_{1N}^{-1}M_{1N}^T\xi_N(t) + \mathrm{He}\{\xi_N^T(t)M_{1N}\Xi_N\Gamma_{2N}\zeta_{2N}^1(t)\},\\ \bar{\psi}_3(t) &\le \delta\xi_N^T(t)M_{2N}\mathcal{Y}_{3N}^{-1}M_{2N}^T\xi_N(t) + \mathrm{He}\{\xi_N^T(t)M_{2N}\Xi_N\Gamma_{2N}\zeta_{2N}^2(t)\},\\ \bar{\psi}_4(t) &\le (t-t_k)\xi_N^T(t)M_{3N}(\mathcal{R}_{1N}+\mathcal{Y}_{3N})^{-1}M_{3N}^T\xi_N(t) + \mathrm{He}\{\xi_N^T(t)M_{3N}\Xi_N\Gamma_{2N}\zeta_{2N}^3(t)\},\\ \psi_5(t) &\le (t_{k+1}-t)\xi_N^T(t)M_{4N}\mathcal{R}_{2N}^{-1}M_{4N}^T\xi_N(t) + \mathrm{He}\{\xi_N^T(t)M_{4N}\Xi_N\Gamma_{1N}\zeta_{1N}^1(t)\},\\ \bar{\psi}_6(t) &\le (t-t_k)\xi_N^T(t)M_{5N}(\mathcal{R}_{3N}+\mathcal{Y}_{2N})^{-1}M_{5N}^T\xi_N(t) + \mathrm{He}\{\xi_N^T(t)M_{5N}\Xi_N\Gamma_{2N}\zeta_{2N}^4(t)\},\\ \psi_7(t) &\le (t_{k+1}-t)\xi_N^T(t)M_{6N}\mathcal{R}_{4N}^{-1}M_{6N}^T\xi_N(t) + \mathrm{He}\{\xi_N^T(t)M_{6N}\Xi_N\Gamma_{1N}\zeta_{1N}^2(t)\},\end{aligned}$$

where

$$\begin{aligned}&\zeta_{1N}^1(t) = [z^T(t_{k+1}), z^T(t), w_N^T(t,t_{k+1})]^T, \zeta_{1N}^2(t) = [z^T(t_{k+1}-\delta), z^T(t-\delta), w_N^T(t-\delta,t_{k+1}-\delta)]^T,\\ &\zeta_{2N}^1(t) = [z^T(t), z^T(t-\delta), \bar{w}_N^T(t-\delta,t)]^T, \zeta_{2N}^2(t) = [z^T(t_k), z^T(t_k-\delta), \bar{w}_N^T(t_k-\delta,t_k)]^T,\\ &\zeta_{2N}^3(t) = [z^T(t), z^T(t_k), \bar{w}_N^T(t_k,t)]^T, \zeta_{2N}^4(t) = [z^T(t-\delta), z^T(t_k-\delta), \bar{w}_N^T(t_k-\delta,t-\delta)]^T.\end{aligned}$$

Therefore, one has

$$\dot{W}(t) \le \xi_N^T(t)[\frac{t_{k+1}-t}{\tau_k}\Pi_1(\tau_k) + \frac{t-t_k}{\tau_k}\Pi_2(\tau_k)]\xi_N(t),$$

where

$$\Pi_1(\tau_k) = \Psi_1 + \tau_k\Psi_2 + \delta\Psi_4 + \tau_k\Psi_5, \Pi_2(\tau_k) = \Psi_1 + \tau_k\Psi_3 + \delta\Psi_4 + \tau_k\Psi_6,$$
$$\Psi_4 = M_{1N}\mathcal{Y}_{1N}^{-1}M_{1N}^T + M_{2N}\mathcal{Y}_{3N}^{-1}M_{2N}^T, \Psi_5 = M_{4N}\mathcal{R}_{2N}^{-1}M_{4N}^T + M_{6N}\mathcal{R}_{4N}^{-1}M_{6N}^T,$$
$$\Psi_6 = M_{3N}(\mathcal{R}_{1N} + \mathcal{Y}_{3N})^{-1}M_{3N}^T + M_{5N}(\mathcal{R}_{3N} + \mathcal{Y}_{2N})^{-1}M_{5N}^T.$$

Obviously, $\Pi_1(\tau_k)$ and $\Pi_2(\tau_k)$ are the linear functions of τ_k. Accordingly, using the convex analysis method for $\tau_k \in (0, \tau]$, we can determine the negative of $W(t)$. That is to say, the negativity of $\Pi_1(\tau_k)$ and $\Pi_2(\tau_k)$ at the start point (set as 10^{-5}) and the end point (τ) will give $\Pi_1(\tau_k) < 0$ and $\Pi_2(\tau_k) < 0$. By using the Schur complement, $\Pi_1(\tau_k) < 0$ and $\Pi_2(\tau_k) < 0$ are equal to LMIs (3.1) and (3.2), respectively. Hence, $\dot{W}(t) < 0$ and the SDCS (2.1) is asymptotically stable. That completes the proof.

Remark 3. *With regard to SDCSs, it has been proved that the second order FII can obtain less conservative results [17]. Meanwhile, the integrals of the system equation, as well as the couplings of additional free matrices and system matrices, are inescapable. These free matrices introduce substantial computational complexity to the stability conditions. In this paper, the integrals in the generalized FIIs are adopted to construct the LF, and thus, the integral of the system equation and free matrices are avoided.*

Remark 4. *In the above Theorem 1, the hierarchical LF $\mathfrak{V}_i(t)$ is created to decrease the conservatism. Obviously, $\mathfrak{V}_i(t_k) = \mathfrak{V}_i(t_{k+1}) = 0$ satisfies the LF conditions in [11]. Because the matrices involved in $\mathfrak{V}_i(t)$ do not have to be positive, the derived criterion thus has lower conservatism.*

Remark 5. *In the proof of Theorem 1, we used the generalized FII in Lemma 1 to deal with the integral quadratic terms. Thus, the linear functions of the integral interval can be derived, which helps to determine the negativity of $\dot{W}(t)$. And, compared with other integral inequality methods, Lemma 1 is better and can obtain less conservative results. However, the utilization of the generalized FII will introduce a lot of computational complexity.*

4. Numerical examples

This part provides two examples with the same parameters as the mentioned references to demonstrate the advantages of the proposed analysis method. We use MATLAB software, YALMIP tool, and SDPT3 solver to obtain the sampling upper boundaries. In the following, for the aperiodic sampling, the lower boundary is set as 10^{-5} in accordance with the given references. And, the calculating algorithm is presented in Table 1.

Table 1. Aperiodic sampling interval upper boundary calculating algorithm.

Algorithm: The calculation of Theorem 1.
Step 1: Given step size $\Delta\tau$ and δ, let $\tau = d_0 = 0$.
Step 2: Calculate the LMIs in Theorem 1.
Step 3: If it is feasible, let $d_0 = \tau$, $\tau = \Delta\tau + \tau$ and proceed to **Step 2**. Otherwise, proceed to **Step 4**.
Step 4: Output d_0, which is the calculated upper bound τ.

Example 1: First, let us introduce the following example with system parameters:

$$C = \begin{bmatrix} 0 & 1 \\ 0 & -0.1 \end{bmatrix}, D = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix}, K = \begin{bmatrix} -3.75 & -11.5 \end{bmatrix}. \tag{4.1}$$

By calculating the LMIs in Theorem 1 and the ones in [4, 30, 36], the maximal upper boundaries of the aperiodic sampling interval corresponding to various fixed communication delays are listed in Table 2. From Table 2, we know that 1.5378 obtained from Theorem 1 with $N = 1$ is bigger than 1.4688 in [30], 1.5325 in [4], and 1.5363 in [36] when $\delta = 0.1$. When $N = 1$, 1.0831 for $\delta = 0.4$ is larger than 1.0459 in [30], 1.0800 in [4], and 1.0809 in [36], and 0.8290 for $\delta = 0.6$ is better than 0.7877 in [30], 0.8278 in [4], and 0.8289 in [36]. For $N = 2, 3, 4$, the calculated sampling upper boundaries are superior to those derived in [4, 30, 36] when $\delta = 0.1, 0.2, 0.4, 0.6$. Also from Table 2, one can tell that the sampling upper boundaries τ increase when the integer N increases, which illustrates that the increase of N decreases the conservativeness of Theorem 1.

In Table 3, the maximum upper boundaries of the periodic sampling interval for different constant delays are displayed. Based on Table 3, the value 2.1171 computed by Theorem 1 with $N = 1$ is bigger than 2.0580 in [4] and 2.1133 in [36] when $\delta = 0.2$. For $N = 2$, 1.9348 obtained from Theorem 1 with $\delta = 0.1$ is larger than 1.9223 in [4] and 1.9337 in [36], 2.2633 calculated by Theorem 1 with $\delta = 0.4$ is greater than 1.7843 in [4] and 2.0274 in [36], and 1.1328 derived for $\delta = 0.6$ is better than 1.0266 in [4] and 1.0764 in [36]. For $\delta = 0.1, 0.2, 0.4, 0.6$, the periodic sampling upper boundaries computed from Theorem 1 are more competitive than the ones in [4, 36] when $N = 3, 4$. And according to Table 3, we can see that the constant sampling interval τ increases as the integer N increases. In addition, there is a difference between the results derived from [4, 36] and Theorem 1. In [4, 36], the biggest values of the sampling period upper bound were derived when $\delta = 0.2$. By computing Theorem 1 with $N = 2, 3, 4$, the largest sampling period upper bound is obtained for $\delta = 0.4$. From the above comparison, one can conclude that the proposed analysis method is superior, and Theorem 1 is better than others in [4, 30, 36].

Table 2. Aperiodic sampling interval upper boundaries τ for various fixed communication delays δ.

Methods\δ	0.1	0.2	0.4	0.6
[30]	1.4688	1.3122	1.0459	0.7877
[4]	1.5325	1.3618	1.0800	0.8278
[36]	1.5363	1.3665	1.0809	0.8289
Theorem 1 (N = 1)	1.5378	1.3627	1.0831	0.8290
Theorem 1 (N = 2)	1.5444	1.3810	1.0926	0.8459
Theorem 1 (N = 3)	1.5458	1.3841	1.0940	0.8506
Theorem 1 (N = 4)	1.5460	1.3842	1.0940	0.8584

Table 3. Periodic sampling interval upper boundaries τ for different constant delays δ.

Methods\δ	0.1	0.2	0.4	0.6
[4]	1.9223	2.0850	1.7843	1.0266
[36]	1.9337	2.1133	2.0274	1.0764
Theorem 1 (N = 1)	1.9263	2.1171	1.8845	1.0550
Theorem 1 (N = 2)	1.9348	2.1438	2.2633	1.1328
Theorem 1 (N = 3)	1.9399	2.1618	2.3105	1.2676
Theorem 1 (N = 4)	1.9420	2.1645	2.3310	1.3086

To prove the validity of the results calculated by Theorem 1, 1.3842 for $\delta = 0.2$ and $N = 4$ are chosen to draw the system state and sampled system state trajectories. By choosing $[1.89, -1.95]$ as the initial condition, the state and sampled state curves are respectively displayed in Figures 1 and 2. Based on Figures 1 and 2, the state trajectories are convergent. That is to say, system (4.1) is stable under the condition that $\tau = 1.3842$ and $\delta = 0.2$. And thus, the validity of Theorem 1 is proved.

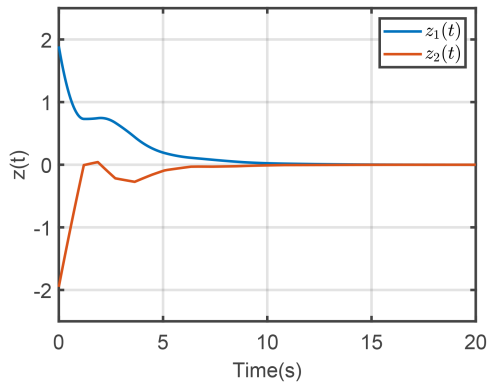

Figure 1. State trajectories of Example 1.

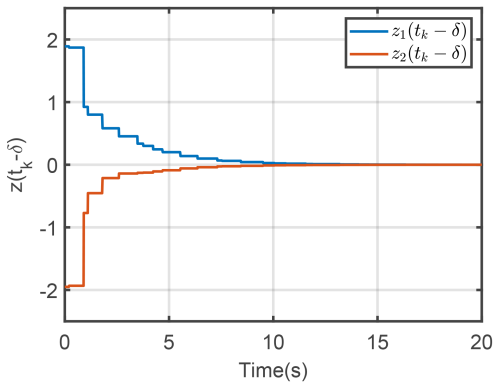

Figure 2. Sampled state trajectories of Example 1.

Example 2: Consider the system with the following parameters:

$$C = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}, DK = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}. \tag{4.2}$$

Table 4. Aperiodic sampling interval upper boundaries τ for different constant delays δ.

Methods\δ	0.2	0.4	0.6
[4]	2.5538	2.2545	2.0660
[36]	2.6484	2.3082	2.1188
Theorem 1 (N = 1)	2.5925	2.2669	2.0846
Theorem 1 (N = 2)	2.6483	2.3306	2.1192
Theorem 1 (N = 3)	2.6632	2.3473	2.1289
Theorem 1 (N = 4)	2.6650	2.3487	2.1311

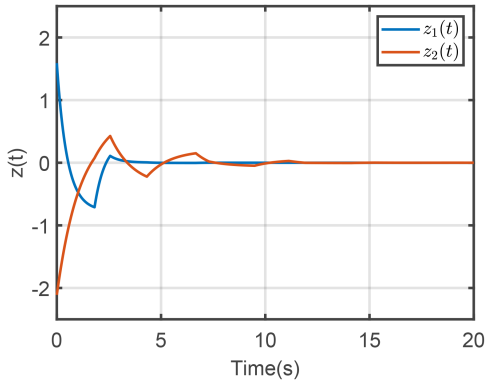

Figure 3. State trajectories of Example 2.

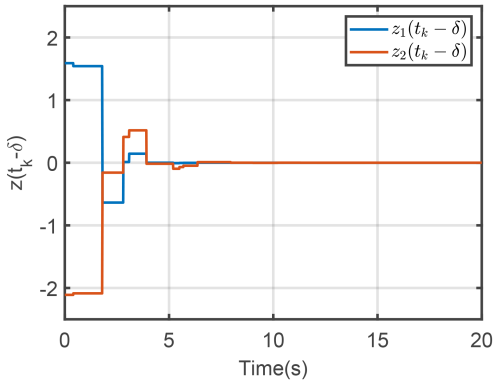

Figure 4. Sampled state trajectories of Example 2.

In Table 4, the calculated maximal aperiodic sampling period upper boundaries corresponding to different fixed delays are listed. From Table 4, we can see that 2.3306 derived from Theorem 1 with $N = 2$ is larger than 2.2545 in [4] and 2.3082 in [36] when $\delta = 0.4$. And, for $\delta = 0.6$, 2.1192 calculated by Theorem 1 with $N = 2$ is also better than 2.0660 in [4] and 2.1188 in [36]. When $\delta = 0.2$, 2.6632 computed by Theorem 1 with $N = 3$ is greater than 2.5538 in [4] and 2.6484 in [36]. For $\delta = 0.2, 0.4, 0.6$, the aperiodic sampling period upper boundaries computed by Theorem 1 are bigger than those in [4, 36] when $N = 3, 4$, and the upper boundaries τ increase with the increase of the positive integer N. Therefore, the proposed Theorem 1 is performs better than others in [4, 36].

In this example, for $\delta = 0.4$ and $N = 4$, 2.3487 is selected to obtain the state and sampled state trajectories. By setting the initial condition as $[1.59, -2.11]$, we obtain Figures 3 and 4, presenting the state and sampled state curves, respectively. From Figures 3 and 4, one can derive the same conclusion as in Example 1.

Remark 6. *In Theorem 1, we introduce too many decision variables. Although the proposed Theorem 1 can provide larger sampling period upper boundaries, the high computational cost will limit the application of the proposed stability conditions with bigger N. Therefore, reducing the computational complexity of the analysis method is a future direction of our work.*

5. Conclusions

This article proposed the hierarchical stability conditions for aperiodic SDCSs with fixed delay. First, to enlarge the upper boundaries of the variable sampling interval under different communication delays, a new type of hierarchical LF is provided. In the assembled LF, the integrals in the generalized FIIs are directly employed. Thus, the system equation integrals are omitted to prevent the matrices from tangling. Then, some subintervals are separated and rearranged to decrease the conservatism. Through the utilization of the generalized FII, the integrals of the same quadratic product item are well addressed. Next, the LMIs with hierarchy are derived by using the Schur complement to the nonlinear part introduced by the generalized FIIs. Eventually, the merits and validity of the provided method are demonstrated through some classic examples. Although the provided analysis approach can obtain less conservative results than the analytical solution, the difficulty of the application mainly comes from the high computational cost for larger N. Consequently, how to decrease the computational complexity is our future research topic.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

Acknowledgments

This research was supported by Hunan Provincial Natural Science Foundation of China [No. 2022JJ60009], and the Scientific Research Fund of Hunan Provincial Department of Education [Nos. 21C1315, 24C1080].

Conflict of interest

The authors declare there are no conflicts of interest.

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