Electronic Research Archive

https://www.aimspress.com/journal/era

ERA, 34(12): 9438–9461.
DOI: 10.3934/era.2026396
Received: 21 August 2026
Revised: 28 September 2026
Accepted: 13 October 2026
Published: 12 November 2026

Research article

Practical event-triggered intermittent stabilization of stochastic differential equations driven by G-Brownian motion via nonlinear feedback

Bingjun Wang[1,2,*] **and Mingxia Yuan**[3]

[1] School of Science, Jinling Institute of Technology, Nanjing 211169, China
[2] School of Mathematics, Southeast University, Nanjing 211189, China
[3] Nanjing Vocational Institute of Transport Technology, Nanjing 211188, China

* **Correspondence:** Email: wbj586@jit.edu.cn.

Abstract: This paper studies practical event-triggered intermittent stabilization for stochastic differential equations driven by G-Brownian motion. A nonlinear feedback controller is applied only on prescribed active windows and is implemented through the latest transmitted state. The triggering rule contains a constant practical threshold, which leads to an ultimate bound in the mean-square estimate and provides a positive margin for excluding Zeno behavior. Under explicit Lyapunov conditions, practical exponential stability in G-mean square is established with an explicit decay rate and residual bound. When the constant threshold is removed, the residual term disappears and, provided that the closed-loop solution is globally defined quasi-surely, a capacity Borel–Cantelli argument yields quasi-sure exponential stability. For a positive constant threshold, quasi-sure Zeno-freeness is proved by a finite-horizon increment estimate and a capacity argument. A nonlinear example illustrates the theoretical conditions.

Keywords: G-Brownian motion; event-triggered control; intermittent control; practical stability; quasi-sure stability; Zeno behavior

1. Introduction

Many stochastic models are affected not only by random perturbations but also by uncertainty in the underlying probability law. The G-expectation framework was introduced by Peng [1–3] to model such uncertainty, especially volatility ambiguity. In this framework, the expectation is sublinear and pathwise statements are formulated quasi-surely by means of capacities. Denis et al. [4] developed the associated capacity theory and function spaces for G-Brownian motion paths. Gao [5] established the Burkholder-Davis-Gundy inequality for G-stochastic integrals. The existence and uniqueness of solutions to stochastic differential equations driven by G-Brownian motion (G-SDEs) with

non-Lipschitz coefficients were later studied by Bai and Lin [6], while Lyapunov-type conditions for solvability and stability of G-SDEs were developed by Li et al. [7]. These developments have made G-SDEs a standard tool for stochastic systems under volatility uncertainty.

Stability and stabilization are central problems in stochastic control. In the classical linear expectation framework, tools such as discrete-time stochastic feedback (see [8] and the references therein) have been extensively studied. In the G-expectation framework, Hu et al. [9] established p-moment stability of G-SDEs by means of Lyapunov functions and the G-Itô formula. Subsequent works extended these stability results to discrete-time feedback control [10–14] and delay systems [15]. These results provide a solid foundation for stability analysis within the G-expectation framework, but they typically assume either continuous-time control or periodic sampling, and do not address resource-efficient control scheduling.

Intermittent control and event-triggered control are two effective mechanisms for reducing control effort. Event-triggered control updates the control input only when a state-dependent error threshold is exceeded, so that the transmission rate adapts to the current system behavior. Intermittent control, by contrast, restricts actuation to designated active windows separated by prescheduled rest windows, thereby reducing the update frequency in a time-driven manner. Both mechanisms have been extensively studied for classical stochastic differential equations in the linear expectation framework (see [8, 16, 17]). Within the G-expectation framework, Yang et al. [18] investigated stabilization of G-SDEs via aperiodically intermittent control, Li et al. [19] extended the intermittent adaptive control technique to multi-weights stochastic complex networks driven by G-Brownian motion, and Ma et al. [20] developed a dynamic event-triggered control design for stochastic nonlinear systems driven by G-Brownian motion.

Despite these advances, to the best of our knowledge, no existing work integrates state-dependent event-triggering into an intermittent control architecture for G-SDEs. Such an integration is of practical value: the intermittent schedule provides a predictable rest pattern for energy harvesting or maintenance, while event-triggering within the active windows suppresses unnecessary transmissions whenever the state deviation remains small. Motivated by this gap, this paper develops a practical event-triggered intermittent control strategy for G-SDEs, which synthesizes the advantages of time-based and state-based mechanisms. Compared to [19], our approach enhances adaptivity by embedding a state-dependent event-triggering law within each active window, so that transmissions are issued only when truly warranted by system state deviation—not just by elapsed time. Compared to [20], our framework introduces explicit rest intervals in the hybrid control schedule, further reducing the average update rate and offering practitioners more flexibility in balancing stability and control energy. Moreover, the triggering rule combines a constant practical threshold, ensuring Zeno-freeness and robust steady-state behavior, with a decaying term that improves the transient response. This combination broadens the applicability of both intermittent and event-triggered control.

The main difficulty of this paper lies in establishing Zeno-freeness for the hybrid triggering mechanism under the G-expectation framework, where the quadratic variation is uncertain and standard comparison arguments for inter-event times are not directly applicable. We overcome this by deriving a high-moment increment estimate for the stopped solution and applying a capacity Markov inequality. A related design issue is the trade-off between the active-window duty ratio and the triggering threshold: a larger threshold reduces transmissions but weakens the effective decay rate

during active windows. We address this by introducing a Lyapunov parameter μ_ε that depends explicitly on the threshold ρ, and by deriving a balance condition $\lambda_* = \theta\mu_\varepsilon - (1-\theta)\alpha_r > 0$. The main contributions are fourfold. First, we propose a hybrid event-triggered intermittent control strategy for G-SDEs that unifies time-based and state-based scheduling. Unlike the purely intermittent schemes [18, 19], which update the control only at preassigned instants, our scheme adapts transmissions to the state deviation within each active window. Second, we establish practical G-mean-square exponential stability with explicit decay rates and residual bounds. In contrast to the purely event-triggered design [20], our estimate quantifies how the duty ratio and the triggering threshold jointly determine the ultimate bound. Third, we prove quasi-sure Zeno-freeness under a strictly positive threshold and establish global well-posedness of the closed-loop system. Fourth, we provide a nonlinear example and numerical simulations that illustrate the theoretical conditions and demonstrate the reduction in control transmissions.

The remainder of this paper is organized as follows. Section 2 recalls the key concepts and technical preliminaries on G-expectation throughout the paper. Section 3 presents the controlled model. Section 4 establishes the quasi-sure Zeno-free property of the event sequence and proves the global well-posedness of the closed-loop solution. Section 5 derives practical exponential stability in G-mean square. Section 6 establishes quasi-sure exponential stability. Section 7 provides an illustrative nonlinear example to demonstrate the main results.

2. Preliminaries

We recall the notation and preliminary results used throughout the paper. For more details on G-expectation, G-Brownian motion, and related function spaces, one can see [4, 21, 22].

Let $\mathbb{R}^n$ be the n-dimensional Euclidean space with norm $|\cdot|$, and let A^T denote the transpose of a matrix A. For real numbers a, b, set $a \vee b = \max\{a, b\}$ and $a \wedge b = \min\{a, b\}$. Denote by $\mathbb{R}_+ = [0, \infty)$ the set of nonnegative real numbers and by $\mathbb{N} = \{0, 1, 2, \ldots\}$ the set of nonnegative integers. We denote by $C_{b,\mathrm{Lip}}(\mathbb{R}^n)$ the space of bounded Lipschitz continuous functions on $\mathbb{R}^n$. If $V(t, x) \in C^{1,2}(\mathbb{R}_+ \times \mathbb{R}^n; \mathbb{R}_+)$, then V is nonnegative and is continuously differentiable in t and twice continuously differentiable in x.

Let $\Omega = C_0([0, \infty); \mathbb{R})$ be the space of real-valued continuous paths $\omega = (\omega_t)_{t\geq 0}$ with $\omega_0 = 0$, equipped with the metric of uniform convergence on compact subsets of $[0, \infty)$. For each $T > 0$, let $\Omega_T = C_0([0, T]; \mathbb{R})$ denote the restriction of Ω to $[0, T]$, equipped with the Borel σ-field $\mathcal{B}(\Omega_T)$. The canonical process is $B_t(\omega) = \omega_t$, and we use the canonical filtration $\mathcal{F}_t := \sigma(B_s : 0 \leq s \leq t),\ t \geq 0$.

Let

$$\mathrm{Lip}(\Omega_T) = \{\varphi(B_{t_1}, \ldots, B_{t_m}) : m \geq 1,\ 0 \leq t_1 \leq \cdots \leq t_m \leq T,\ \varphi \in C_{b,\mathrm{Lip}}(\mathbb{R}^m)\}.$$

According to [22], there exists a sublinear functional $\hat{\mathbb{E}} : \mathrm{Lip}(\Omega_T) \to \mathbb{R}$, called the G-expectation, such that (i) $\hat{\mathbb{E}}[X] \geq \hat{\mathbb{E}}[Y]$ if $X \geq Y$; (ii) $\hat{\mathbb{E}}[c] = c$ for every constant $c \in \mathbb{R}$; (iii) $\hat{\mathbb{E}}[X + Y] \leq \hat{\mathbb{E}}[X] + \hat{\mathbb{E}}[Y]$; (iv) $\hat{\mathbb{E}}[\lambda X] = \lambda\hat{\mathbb{E}}[X]$ for every $\lambda \geq 0$.

The triplet $(\Omega_T, \mathrm{Lip}(\Omega_T), \hat{\mathbb{E}})$ is called a sublinear expectation space, and elements of $\mathrm{Lip}(\Omega_T)$ are called random variables. Thus, $\hat{\mathbb{E}}$ is generally not additive, in contrast to the classical linear expectation.

The associated function $G : \mathbb{R} \to \mathbb{R}$ is defined by

$$G(a) = \frac{1}{2}(a^+\overline{\sigma}^2 - a^-\underline{\sigma}^2), \qquad a \in \mathbb{R},$$

where $a^+ = a \vee 0$, $a^- = (-a) \vee 0$, and $[\underline{\sigma}^2, \overline{\sigma}^2]$ is the volatility uncertainty interval. The G-expectation $\hat{\mathbb{E}}$ is generated by the nonlinear heat equation (see [22]):

$$\begin{cases} \partial_t u - G(\partial^2_{xx} u) = 0, & (t, x) \in (0, \infty) \times \mathbb{R}, \\ u(0, x) = \varphi(x), & \varphi \in C_{b,\mathrm{Lip}}(\mathbb{R}), \end{cases}$$

whose unique viscosity solution admits the representation

$$u(t, x) = \hat{\mathbb{E}}[\varphi(x + \sqrt{t}B_1)].$$

Here B_1 is the value at $t = 1$ of the canonical process $B = (B_t)_{t \geq 0}$ under $\hat{\mathbb{E}}$, and B_1 is G-normally distributed with upper and lower variances

$$\overline{\sigma}^2 = \hat{\mathbb{E}}[|B_1|^2], \qquad \underline{\sigma}^2 = -\hat{\mathbb{E}}[-|B_1|^2].$$

The canonical process B_t is called a G-Brownian motion, whose increments are stationary, independent, and G-normally distributed. Unlike the classical Brownian motion, the quadratic variation process $\langle B \rangle_t$ of a G-Brownian motion is itself uncertain and takes values in $[\underline{\sigma}^2 t, \overline{\sigma}^2 t]$.

For $p \geq 1$, $L^p_G(\Omega_T)$ is the completion of $\mathrm{Lip}(\Omega_T)$ under the norm

$$\|X\|_{p,G} = (\hat{\mathbb{E}}[|X|^p])^{1/p}.$$

Let $M^{p,0}_G(0, T)$ be the space of simple adapted processes

$$\eta_t = \sum_{k=0}^{N-1} \xi_k \mathbf{1}_{[r_k, r_{k+1})}(t), \qquad 0 = r_0 < r_1 < \cdots < r_N = T,$$

where $\xi_k \in L^p_G(\Omega_{r_k})$. The space $M^p_G(0, T)$ is the completion of $M^{p,0}_G(0, T)$ with respect to

$$\|\eta\|_{M^p_G(0,T)} = (\hat{\mathbb{E}}[\int_0^T |\eta_t|^p dt])^{1/p}.$$

If $\eta \in M^2_G(0, T)$, the stochastic integral $\int_0^t \eta_s \, dB_s$ is well defined. If $\eta \in M^1_G(0, T)$, then $\int_0^t \eta_s \, ds$ and $\int_0^t \eta_s \, d\langle B \rangle_s$ are well defined.

According to [4], there exists a weakly compact family $\mathcal{P}$ of probability measures such that

$$\hat{\mathbb{E}}[X] = \sup_{P \in \mathcal{P}} E_P[X], \qquad X \in L^1_G(\Omega_T).$$

The corresponding capacity is

$$\mathbb{C}(A) = \sup_{P \in \mathcal{P}} P(A), \qquad A \in \mathcal{B}(\Omega_T).$$

If $\mathbb{C}(A) = 0$, then A is called a polar set. A property holds quasi-surely, abbreviated q.s., if it holds outside a polar set.

We shall use the following capacity Markov inequality.

Lemma 2.1 ([22]). *Let $X \in L_G^p(\Omega_T)$, $p \geq 1$. Then, for every $a > 0$,*

$$\mathbb{C}(|X| > a) \leq \frac{\hat{\mathbb{E}}[|X|^p]}{a^p}.$$

Lemma 2.2 ([5]). *For $p \geq 2$ and $\eta \in M_G^p(0,T)$, there exists a positive constant C'_p, depending only on p, such that*

$$\hat{\mathbb{E}}[\sup_{0\leq u\leq T} |\int_0^u \eta_r \, dB_r|^p] \leq C'_p T^{\frac{p}{2}-1} \int_0^T \hat{\mathbb{E}}[|\eta_r|^p]dr.$$

Lemma 2.3 ([5]). *For $p \geq 1$ and $\eta \in M_G^p(0,T)$, there exists a positive constant C''_p, depending only on p, such that*

$$\hat{\mathbb{E}}[\sup_{0\leq u\leq T} |\int_0^u \eta_r \, d\langle B\rangle_r|^p] \leq C''_p T^{p-1} \int_0^T \hat{\mathbb{E}}[|\eta_r|^p]dr.$$

Remark 2.4. Although we consider one-dimensional G-Brownian motion in this paper, the multidimensional case can be treated similarly by using componentwise G-stochastic integrals and the corresponding quadratic covariation processes.

3. Controlled model

Consider the uncontrolled G-SDE

$$dx(t) = f(t,x(t))dt + h(t,x(t))d\langle B\rangle_t + g(t,x(t))dB_t, \qquad x(0) = x_0 \in \mathbb{R}^n, \tag{3.1}$$

where $f(\cdot,x), h(\cdot,x) \in M_G^1(0,T;\mathbb{R}^n)$ and $g(\cdot,x) \in M_G^2(0,T;\mathbb{R}^n)$ for each $x \in \mathbb{R}^n$. Here $M_G^p([0,T];\mathbb{R}^n)$ denotes the space of $\mathbb{R}^n$-valued adapted processes whose components belong to $M_G^p(0,T)$. Assume that $f(t,0) = h(t,0) = g(t,0) = 0$, so the system admits the trivial solution. The trivial solution may be unstable under suitable Lyapunov-type conditions (see [7]). The aim of this paper is to design a feedback controller u in the drift term and an intermittent event-triggered implementation so that the resulting controlled system becomes stable. The controlled G-SDE is

$$dx(t) = [f(t,x(t)) + u(t)]dt + h(t,x(t))d\langle B\rangle_t + g(t,x(t))dB_t, \qquad x(0) = x_0 \in \mathbb{R}^n. \tag{3.2}$$

Before specifying the controller, we introduce the stability notions used in this paper.

Definition 3.1. The trivial solution of the controlled G-SDE is said to be practically exponentially stable in G-mean square if there exist constants $M \geq 1$, $\lambda > 0$, and $K > 0$ such that

$$\hat{\mathbb{E}}[|x(t)|^2] \leq Me^{-\lambda t}|x_0|^2 + K, \qquad t \geq 0.$$

If $K = 0$, the solution is said to be exponentially stable in G-mean square.

Definition 3.2. The trivial solution is called quasi-surely exponentially stable if there exists $\lambda > 0$ such that

$$\limsup_{t\to\infty} \frac{1}{t} \log|x(t)| \leq -\lambda, \qquad \text{q.s.},$$

where we use the convention $\log 0 = -\infty$.

Let

$$0 = t_0 < s_0 < t_1 < s_1 < t_2 < s_2 < \cdots$$

be an intermittent schedule. The intervals $[t_k, s_k)$ are the active control windows, whereas $[s_k, t_{k+1})$ are the rest windows. Within each active window, the event times are defined recursively until the terminal time s_k is reached, with $\tau_{k,0} = t_k$ and $\tau_{k,j} < \tau_{k,j+1} \le s_k$ whenever $\tau_{k,j} < s_k$. For $t \in [\tau_{k,j}, \tau_{k,j+1}) \cap [t_k, s_k)$, define

$$\eta(t) = \tau_{k,j}, \qquad e(t) = x(\eta(t)) - x(t).$$

The controller is

$$u(t) = \begin{cases} F(x(\eta(t))), & t \in [t_k, s_k), \\ 0, & t \in [s_k, t_{k+1}), \end{cases} \tag{3.3}$$

where $F : \mathbb{R}^n \to \mathbb{R}^n$ is a nonlinear feedback map satisfying $F(0) = 0$.

The triggering law on each active window is

$$\tau_{k,j+1} = \inf\{t \in (\tau_{k,j}, s_k) : |x(t) - x(\tau_{k,j})|^2 > \rho|x(t)|^2 + \delta_0 + \delta_1 e^{-\beta t}\} \wedge s_k, \tag{3.4}$$

where $\rho > 0$, $\delta_0 \ge 0$, $\delta_1 \ge 0$, and $\beta > 0$ are prescribed constants. Consequently, for each $j \ge 0$ and every $t \in [\tau_{k,j}, \tau_{k,j+1})$,

$$|e(t)|^2 \le \rho|x(t)|^2 + \delta_0 + \delta_1 e^{-\beta t}. \tag{3.5}$$

Remark 3.3. The exponentially decaying term $\delta_1 e^{-\beta t}$ in (3.4) serves two purposes. First, it relaxes the threshold at the initial stage, when $e^{-\beta t}$ is close to one, thereby reducing unnecessary transmissions during the transient phase. As time evolves, the term decays and the effective threshold approaches δ_0, so the triggering law becomes increasingly sensitive to state deviations near the steady state. Second, it improves the transient estimate without affecting the ultimate bound: the term δ_1 enters the stability estimate only through the exponentially decaying factor $e^{-\gamma t}$ in (5.8), whereas the residual bound $D\delta_0$ is determined solely by the constant threshold. When $\delta_1 = 0$, the triggering law reduces to the pure constant-threshold form, and the estimate (5.8) reduces accordingly.

The proposed hybrid event-triggered intermittent control strategy can be illustrated as follows.

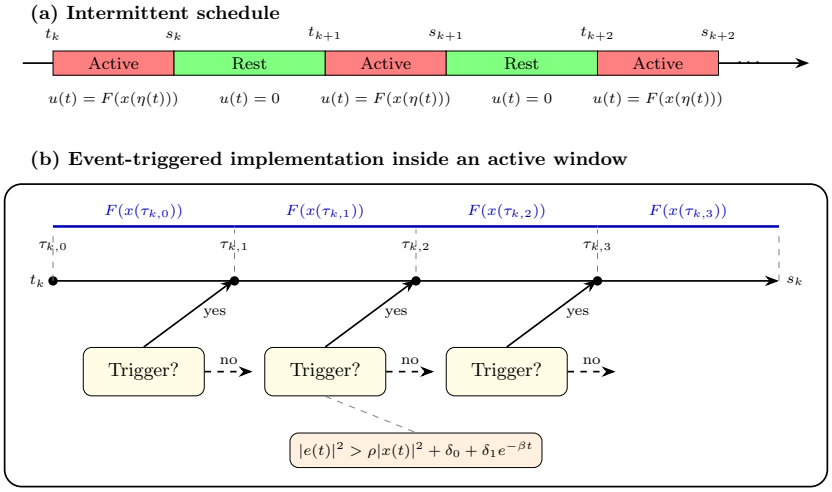


Figure 1. Hybrid event-triggered intermittent control mechanism. (a) Alternating active and rest windows: the active window $[t_k, s_k)$ is followed by the rest window $[s_k, t_{k+1})$. (b) Zoom into an active window: at each triggering instant the condition is evaluated; if it is satisfied, the control input is updated; otherwise the previous value is held. The active window is terminated at s_k.

Assumption 3.4. There exist constants $L > 0$ and $L_F > 0$ such that, for $t \geq 0$ and $x, y \in \mathbb{R}^n$,

$$|f(t,x) - f(t,y)| \vee |h(t,x) - h(t,y)| \vee |g(t,x) - g(t,y)| \leq L|x - y|.$$

Moreover,

$$|F(x) - F(y)| \leq L_F|x - y|.$$

Assumption 3.5. There exist a Lyapunov function $V \in C^{1,2}(\mathbb{R}_+ \times \mathbb{R}^n; \mathbb{R}_+)$ and positive constants $c_1, c_2, \alpha_c, \alpha_r, c_3$ such that

$$c_1|x|^2 \leq V(t,x) \leq c_2|x|^2,$$

and, for $(t,x) \in \mathbb{R}_+ \times \mathbb{R}^n$,

$$\mathcal{L}_c V(t,x) \leq -\alpha_c V(t,x), \tag{3.6}$$
$$\mathcal{L}_r V(t,x) \leq \alpha_r V(t,x), \tag{3.7}$$
$$|V_x(t,x)|^2 \leq c_3 V(t,x), \tag{3.8}$$

where

$$\mathcal{L}_c V(t,x) := V_t(t,x) + V_x(t,x)^T(f(t,x) + F(x)) + G(2V_x(t,x)^T h(t,x) + g(t,x)^T V_{xx}(t,x)g(t,x)),$$
$$\mathcal{L}_r V(t,x) := V_t(t,x) + V_x(t,x)^T f(t,x) + G(2V_x(t,x)^T h(t,x) + g(t,x)^T V_{xx}(t,x)g(t,x)).$$

Assumption 3.6. There exist constants $\theta \in (0,1)$, $T_{\min} > 0$, and $T_{\max} > 0$ such that, for every $k \geq 0$,

$$P_k := t_{k+1} - t_k \in [T_{\min}, T_{\max}], \qquad s_k - t_k \geq \theta P_k.$$

Remark 3.7. Assumption 3.5 imposes uniform exponential decay/growth bounds on the Lyapunov generator. While such conditions are standard and convenient for analysis, they may be conservative for some classes of systems. In principle, the analysis could be extended to allow for average dwell-time type conditions, where only the average decay rate over multiple windows is required to be negative, or to stability criteria based on time-varying bounds. Such relaxation has been explored in classical sampled-data and switched systems, and could be adapted to the G-expectation setting. Investigating these generalizations would be an interesting direction for future work.

Remark 3.8. Under Assumption 3.4, the closed-loop coefficients are globally Lipschitz between consecutive event times. Hence, a unique local adapted solution exists on each inter-event interval (see [22]). The global-in-time construction of the solution is discussed in Section 4.

4. Quasi-sure zeno-freeness

Throughout this section, $\tau_{k,j}$ denotes the j-th event time inside the k-th active window, as defined in Section 3.

For a fixed finite horizon $T > 0$, define the counting process

$$N_t := \#\{(k, j) : t_k \le t,\ \tau_{k,j} < s_k,\ \tau_{k,j} \le t\}, \qquad t \ge 0, \tag{4.1}$$

which counts the number of nonterminal event times up to time t. For $N \ge 1$, define the stopped horizon

$$\sigma_N := T \wedge \inf\{t \ge 0 : N_t \ge N\}, \tag{4.2}$$

with the convention $\inf \emptyset = +\infty$. Thus, σ_N is the first time at which N events have occurred, capped at T.

For each k and j, the event time $\tau_{k,j}$ is a stopping time with respect to the completed filtration $\{\mathcal{F}_t\}$, since it is defined as the first hitting time of a closed set by a continuous adapted process (see [21]). Consequently, N_t is an $\{\mathcal{F}_t\}$-adapted counting process, and σ_N is a stopping time.

On each interval $[\tau_{k,j}, \tau_{k,j+1})$ such that $\tau_{k,j+1} \le \sigma_N$, the control input $u(t)$ is held constant and is measurable with respect to $\mathcal{F}_{\tau_{k,j}}$. By Assumption 3.4, the system coefficients are globally Lipschitz continuous and adapted, and the process $x(t)$ on $[\tau_{k,j}, \tau_{k,j+1})$ thus admits a unique local adapted solution according to the standard theory of G-SDEs (see [22]).

Define the process stopped at σ_N as

$$X^N(t) := x(t \wedge \sigma_N), \qquad t \ge 0.$$

By successively concatenating the unique local solutions on $[\tau_{k,0}, \tau_{k,1}), [\tau_{k,1}, \tau_{k,2}), \dots$ up to σ_N, we construct a piecewise-adapted process $X^N(t)$ which coincides with the original system up to potential accumulation of events or the horizon T.

We now state the main results for Zeno-freeness and global well-posedness.

Lemma 4.1. *Assume that Assumption 3.4 holds. For every $T > 0$, $p \ge 2$, and $0 < r < 1$, there exists a constant $C_{T,p} > 0$, independent of N, k, j, and r, such that the stopped solution satisfies*

$$\hat{\mathbb{E}}[\sup_{\tau_{k,j} \le t \le (\tau_{k,j}+r)\wedge T} |X^N(t) - X^N(\tau_{k,j})|^{2p}] \le C_{T,p} r^{p/2}. \tag{4.3}$$

Proof. Fix N and set

$$\chi_N(t) := \mathbf{1}_{\{t<\sigma_N\}}.$$

Define the stopped coefficients

$$f_N(t) := \chi_N(t)f(t, X^N(t)), \quad h_N(t) := \chi_N(t)h(t, X^N(t)),$$
$$g_N(t) := \chi_N(t)g(t, X^N(t)), \quad u_N(t) := \chi_N(t)u(t).$$

We first establish a finite-horizon moment bound. For $0 \le q \le t \le T$, the stopped integral equation is

$$X^N(q) = x_0 + \int_0^q [f_N(s) + u_N(s)]ds + \int_0^q h_N(s)d\langle B\rangle_s + \int_0^q g_N(s)dB_s.$$

Using $|a_1 + a_2 + a_3 + a_4|^{2p} \le 4^{2p-1}\sum_{i=1}^4 |a_i|^{2p}$, taking the supremum over $q \in [0, t]$ and then applying $\hat{\mathbb{E}}$, we get

$$\begin{aligned}\hat{\mathbb{E}}[\sup_{0\le q\le t}|X^N(q)|^{2p}] \le 4^{2p-1}|x_0|^{2p} &+ 4^{2p-1}\hat{\mathbb{E}}[\sup_{0\le q\le t}|\int_0^q [f_N(s) + u_N(s)]ds|^{2p}] \\ &+ 4^{2p-1}\hat{\mathbb{E}}[\sup_{0\le q\le t}|\int_0^q h_N(s)d\langle B\rangle_s|^{2p}] \\ &+ 4^{2p-1}\hat{\mathbb{E}}[\sup_{0\le q\le t}|\int_0^q g_N(s)dB_s|^{2p}]. \end{aligned} \tag{4.4}$$

On rest windows $u_N(t) = 0$. On active windows, by Assumption 3.4 and formula (3.5), we have

$$|u_N(t)|^2 \le 2L_F^2(1 + \rho)|X^N(t)|^2 + 2L_F^2(\delta_0 + \delta_1).$$

Thus,

$$|u_N(t)|^{2p} \le A_u(1 + |X^N(t)|^{2p}),$$

where $A_u := 2^{2p-1}L_F^{2p}[(1 + \rho)^p + (\delta_0 + \delta_1)^p]$. Moreover, by Assumption 3.4, we have

$$|f_N(t) + u_N(t)|^{2p} \le A_f(1 + |X^N(t)|^{2p}), \quad |h_N(t)|^{2p} \le A_h|X^N(t)|^{2p}, \quad |g_N(t)|^{2p} \le A_g|X^N(t)|^{2p}, \tag{4.5}$$

where $A_f := 2^{2p-1}(L^{2p} + A_u)$, $\quad A_h := L^{2p}$, $\quad A_g := L^{2p}$.

By Hölder's inequality,

$$\hat{\mathbb{E}}[\sup_{0\le q\le t}|\int_0^q [f_N(s) + u_N(s)]ds|^{2p}] \le T^{2p-1}\int_0^t \hat{\mathbb{E}}[|f_N(s) + u_N(s)|^{2p}]ds.$$

Together with (4.5), this implies

$$\hat{\mathbb{E}}[\sup_{0\le q\le t}|\int_0^q [f_N(s) + u_N(s)]ds|^{2p}] \le A_f T^{2p-1}\int_0^t \hat{\mathbb{E}}[1 + |X^N(s)|^{2p}]ds. \tag{4.6}$$

By Lemmas 2.3 and 2.2, and by (4.5), we have

$$\hat{\mathbb{E}}[\sup_{0\le q\le t}|\int_0^q h_N(s)d\langle B\rangle_s|^{2p}] \le C''_{2p}A_h T^{2p-1}\int_0^t \hat{\mathbb{E}}[|X^N(s)|^{2p}]ds \tag{4.7}$$

and

$$\hat{\mathbb{E}}[\sup_{0\le q\le t}|\int_0^q g_N(s)dB_s|^{2p}] \le C'_{2p}A_g T^{p-1}\int_0^t \hat{\mathbb{E}}[|X^N(s)|^{2p}]ds. \tag{4.8}$$

Combining (4.4)–(4.8), we get

$$\hat{\mathbb{E}}[\sup_{0\le q\le t}|X^N(q)|^{2p}] \le 4^{2p-1}(|x_0|^{2p}+A_f T^{2p}) + H_{T,p}\int_0^t \hat{\mathbb{E}}[|X^N(s)|^{2p}]ds,$$

where $H_{T,p} := 4^{2p-1}[A_f T^{2p-1} + C''_{2p}A_h T^{2p-1} + C'_{2p}A_g T^{p-1}]$.

Since $|X^N(s)|^{2p} \le \sup_{0\le q\le s}|X^N(q)|^{2p}$, Gronwall's inequality implies

$$\hat{\mathbb{E}}[\sup_{0\le q\le T}|X^N(q)|^{2p}] \le M_{T,p} := 4^{2p-1}(|x_0|^{2p}+A_f T^{2p})e^{H_{T,p}T} < \infty. \tag{4.9}$$

Next, fix a triggering time $\tau_{k,j} \le T$ and $r \in (0,1)$. For $t \in [\tau_{k,j}, (\tau_{k,j}+r)\wedge T]$,

$$X^N(t) - X^N(\tau_{k,j}) = \int_{\tau_{k,j}}^t [f_N(s)+u_N(s)]ds + \int_{\tau_{k,j}}^t h_N(s)d\langle B\rangle_s + \int_{\tau_{k,j}}^t g_N(s)dB_s.$$

Using $|a+b+c|^{2p} \le 3^{2p-1}(|a|^{2p}+|b|^{2p}+|c|^{2p})$, taking the supremum over $t \in [\tau_{k,j}, (\tau_{k,j}+r)\wedge T]$, and applying $\hat{\mathbb{E}}$, we obtain

$$\hat{\mathbb{E}}[\sup_{\tau_{k,j}\le t\le(\tau_{k,j}+r)\wedge T}|X^N(t)-X^N(\tau_{k,j})|^{2p}] \le 3^{2p-1}[I_f+I_h+I_g], \tag{4.10}$$

where

$$\begin{aligned}
I_f &:= \hat{\mathbb{E}}[\sup_{\tau_{k,j}\le t\le(\tau_{k,j}+r)\wedge T}|\int_{\tau_{k,j}}^t [f_N(s)+u_N(s)]ds|^{2p}],\\
I_h &:= \hat{\mathbb{E}}[\sup_{\tau_{k,j}\le t\le(\tau_{k,j}+r)\wedge T}|\int_{\tau_{k,j}}^t h_N(s)d\langle B\rangle_s|^{2p}],\\
I_g &:= \hat{\mathbb{E}}[\sup_{\tau_{k,j}\le t\le(\tau_{k,j}+r)\wedge T}|\int_{\tau_{k,j}}^t g_N(s)dB_s|^{2p}].
\end{aligned}$$

By Hölder's inequality, Lemma 2.2, Lemma 2.3, together with (4.5) and (4.9) , we have

$$\begin{aligned}
I_f &\le r^{2p}\hat{\mathbb{E}}[\sup_{0\le q\le T}|f_N(q)+u_N(q)|^{2p}]\\
&\le A_f r^{2p}\hat{\mathbb{E}}[1+\sup_{0\le q\le T}|X^N(q)|^{2p}] \le A_f r^{2p}(1+M_{T,p}),
\end{aligned}$$

$$I_h \le C''_{2p}r^{2p}A_h M_{T,p},$$

and

$$I_g \le C'_{2p}r^{p}A_g M_{T,p}.$$

Since $0 < r < 1$ and $p \geq 2$, both r^{2p} and r^p are bounded by $r^{p/2}$. Therefore, (4.10) implies

$$\hat{\mathbb{E}}[\sup_{\tau_{k,j} \leq t \leq (\tau_{k,j}+r)\wedge T} |X^N(t) - X^N(\tau_{k,j})|^{2p}] \leq C_{T,p} r^{p/2}. \tag{4.11}$$

Here $C_{T,p} := 3^{2p-1}\{A_f(1 + M_{T,p}) + (C''_{2p}A_h + C'_{2p}A_g)M_{T,p}\}$. The constants $M_{T,p}$ and $C_{T,p}$ are independent of N, k, j, r. Thus this is a uniform estimate on every finite pre-accumulation segment. □

Theorem 4.2. *Assume that Assumptions 3.4 and 3.6 hold. If $\delta_0 > 0$ in* (3.4)*, then the event-triggered intermittent controller* (3.3)–(3.4) *is Zeno-free quasi-surely; that is, its event-time sequence has no finite accumulation point q.s.*

Proof. Fix $T > 0$ and choose $p > 2$. Put

$$K_T := \lfloor T/T_{\min} \rfloor + 1.$$

By Assumption 3.6, at most K_T active windows can meet $[0, T]$. For $N \geq 1$, let

$$E_{N,T} := \{N_T \geq N\}.$$

On $E_{N,T}$, the N-th event occurs before T, so $\sigma_N = \inf\{t \geq 0 : N_t \geq N\}$.

Suppose that $\tau_{k,j+1} \leq \sigma_N$, $\tau_{k,j+1} < s_k$, and $\tau_{k,j+1} - \tau_{k,j} \leq r$ for some $r \in (0, 1)$. The defining threshold condition gives

$$\sup_{\tau_{k,j} \leq t \leq (\tau_{k,j}+r)\wedge T} |x(t \wedge \sigma_N) - x(\tau_{k,j} \wedge \sigma_N)|^2 \geq \delta_0.$$

Therefore, by the capacity Markov inequality and Lemma 4.1,

$$\begin{aligned}
&\mathbb{C}(\tau_{k,j+1} - \tau_{k,j} \leq r,\ \tau_{k,j+1} \leq \sigma_N,\ \tau_{k,j+1} < s_k) \\
&\leq \mathbb{C}(\sup_{\tau_{k,j} \leq t \leq (\tau_{k,j}+r)\wedge T} |x(t \wedge \sigma_N) - x(\tau_{k,j} \wedge \sigma_N)|^2 \geq \delta_0) \\
&\leq C_{T,p,\delta_0} r^{p/2},
\end{aligned} \tag{4.12}$$

where $C_{T,p,\delta_0} := \delta_0^{-p} C_{T,p}$.

Fix an integer $m > T_{\max}$ and set $N := K_T(m + 1)$. On $E_{N,T}$, among the first N nonterminal event times there is an active window containing at least $m + 1$ such times. Among these $m + 1$ event times, the m inter-event intervals have total length at most $s_k - t_k \leq T_{\max}$, so at least one of them is bounded by $T_{\max}/m$. Hence, for some k and some $j \in \{0, \ldots, m - 1\}$,

$$\tau_{k,j+1} \leq \sigma_N, \qquad \tau_{k,j+1} < s_k, \qquad \tau_{k,j+1} - \tau_{k,j} \leq \frac{T_{\max}}{m}.$$

Since $T_{\max}/m \in (0, 1)$, (4.12) and subadditivity of the capacity yield

$$\begin{aligned}
\mathbb{C}(E_{K_T(m+1),T}) &\leq K_T m C_{T,p,\delta_0} \left(\frac{T_{\max}}{m}\right)^{p/2} \\
&= K_T C_{T,p,\delta_0} T_{\max}^{p/2} m^{1-p/2}.
\end{aligned}$$

Since $\{N_T = \infty\} \subseteq E_{N,T}$ for every $N \geq 1$, in particular

$$\{N_T = \infty\} \subseteq E_{K_T(m+1),T}$$

for every $m > T_{\max}$. Letting $m \to \infty$ and using $p > 2$, we obtain

$$\mathbb{C}(\{N_T = \infty\}) = 0.$$

Consequently, for each active window $[t_k, s_k)$, the set $\{j : \tau_{k,j} < s_k\}$ is finite quasi-surely, so the notation $\tau_{k,j}$ is well defined for all k and all relevant j. This proves the asserted quasi-sure absence of finite accumulation points. □

Theorem 4.3 (Global Well-Posedness)**.** *Suppose Assumptions 3.4 and 3.6 are satisfied, and the triggering mechanism guarantees quasi-sure Zeno-freeness (see Theorem 4.2). Then, the event-triggered intermittent closed-loop system admits a unique global adapted solution $x(t)$ for all $t \geq 0$, quasi-surely.*

Proof. Let $T > 0$ be arbitrary. The Zeno-freeness property (Theorem 4.2) implies $N_T < \infty$ quasi-surely, i.e., there are only finitely many events in $[0, T]$ quasi-surely. For each subinterval $[\tau_{k,j}, \tau_{k,j+1})$, as established above, the system admits a unique adapted local solution under the fixed control. By concatenation of these finitely many local solutions, we obtain a unique adapted solution on $[0, T]$. Since T is arbitrary and the concatenation is consistent, the solution extends globally to $[0, \infty)$, quasi-surely. □

Remark 4.4. The assumption of quasi-sure Zeno-freeness (i.e., exclusion of finite event accumulation outside a polar set) is critical for the global-in-time construction. If $\delta_0 > 0$, Theorem 4.2 guarantees this property. If $\delta_0 = 0$, global well-posedness must be separately assumed or established, as event accumulation may occur near the origin.

5. Practical exponential stability in G-mean square

Lemma 5.1. *Suppose that Assumptions 3.4 and 3.5 hold. For a given $\varepsilon > 0$, define*

$$\mu_\varepsilon := \alpha_c - \frac{\varepsilon c_3}{2} - \frac{L_F^2 \rho}{2\varepsilon c_1}, \qquad \bar{\beta} := \min\{\mu_\varepsilon, \beta\}. \tag{5.1}$$

If $\mu_\varepsilon > 0$, then, for $t \in [t_k, s_k)$,

$$\hat{\mathbb{E}}[V(t, x(t))] \leq e^{-\mu_\varepsilon (t - t_k)} \hat{\mathbb{E}}[V(t_k, x(t_k))] + \frac{L_F^2 \delta_0}{2\varepsilon \mu_\varepsilon} + \frac{L_F^2 \delta_1}{2\varepsilon \bar{\beta}} e^{-\beta t_k}. \tag{5.2}$$

Moreover, for $t \in [s_k, t_{k+1})$,

$$\hat{\mathbb{E}}[V(t, x(t))] \leq e^{\alpha_r (t - s_k)} \hat{\mathbb{E}}[V(s_k, x(s_k))]. \tag{5.3}$$

Proof. Consider first the case $t \in [t_k, s_k)$. Let

$$\Delta_F(s) := F(x(\eta(s))) - F(x(s)).$$

For every non-triggering time $s \in [t_k, t]$, there is an index j such that $s \in [\tau_{k,j}, \tau_{k,j+1})$ and the transmitted state is $x(\eta(s)) = x(\tau_{k,j})$. Then, the Lipschitz continuity of F and the triggering rule imply

$$|\Delta_F(s)|^2 \leq L_F^2 |e(s)|^2 \leq L_F^2 (\rho |x(s)|^2 + \delta_0 + \delta_1 e^{-\beta s}). \tag{5.4}$$

Applying G-Itô formula to the process $e^{\mu_\varepsilon(r-t_k)}V(r,x(r))$ on $[t_k,t]$, we obtain

$$\begin{aligned}&e^{\mu_\varepsilon(t-t_k)}V(t,x(t))-V(t_k,x(t_k))\\&=\int_{t_k}^{t}e^{\mu_\varepsilon(s-t_k)}[\mathcal{L}_cV(s,x(s))+\mu_\varepsilon V(s,x(s))+V_x(s,x(s))^T\Delta_F(s)]ds+M_t^{t_k}+Q_t^{t_k},\end{aligned}$$

where

$$\begin{aligned}M_t^{t_k}&:=\int_{t_k}^{t}e^{\mu_\varepsilon(s-t_k)}V_x(s,x(s))^Tg(s,x(s))dB_s,\\Q_t^{t_k}&:=\frac{1}{2}\int_{t_k}^{t}e^{\mu_\varepsilon(s-t_k)}[2V_x(s,x(s))^Th(s,x(s))+g(s,x(s))^TV_{xx}(s,x(s))g(s,x(s))]d\langle B\rangle_s\\&\quad-\int_{t_k}^{t}e^{\mu_\varepsilon(s-t_k)}G(2V_x(s,x(s))^Th(s,x(s))+g(s,x(s))^TV_{xx}(s,x(s))g(s,x(s)))ds.\end{aligned}$$

The stochastic integral $M_t^{t_k}$ is a G-martingale with $\hat{\mathbb{E}}[M_t^{t_k}]=0$, while $Q_t^{t_k}$ is a non-increasing G-martingale with $\hat{\mathbb{E}}[Q_t^{t_k}]\le 0$. By Young's inequality, Assumption 3.5, and (5.4), we have

$$\begin{aligned}V_x(s,x(s))^T\Delta_F(s)&\le\frac{\varepsilon}{2}|V_x(s,x(s))|^2+\frac{1}{2\varepsilon}|\Delta_F(s)|^2\\&\le\frac{\varepsilon c_3}{2}V(s,x(s))+\frac{L_F^2}{2\varepsilon}(\rho|x(s)|^2+\delta_0+\delta_1e^{-\beta s})\\&\le(\frac{\varepsilon c_3}{2}+\frac{L_F^2\rho}{2\varepsilon c_1})V(s,x(s))+\frac{L_F^2\delta_0}{2\varepsilon}+\frac{L_F^2\delta_1}{2\varepsilon}e^{-\beta s}.\end{aligned}$$

Using Assumption 3.5 and the definition of μ_ε, we get

$$\mathcal{L}_cV(s,x(s))+\mu_\varepsilon V(s,x(s))+V_x(s,x(s))^T\Delta_F(s)\le\frac{L_F^2\delta_0}{2\varepsilon}+\frac{L_F^2\delta_1}{2\varepsilon}e^{-\beta s}.$$

Consequently,

$$\begin{aligned}e^{\mu_\varepsilon(t-t_k)}V(t,x(t))&\le V(t_k,x(t_k))+\frac{L_F^2\delta_0}{2\varepsilon}\int_{t_k}^{t}e^{\mu_\varepsilon(s-t_k)}ds\\&\quad+\frac{L_F^2\delta_1}{2\varepsilon}\int_{t_k}^{t}e^{\mu_\varepsilon(s-t_k)}e^{-\beta s}ds+M_t^{t_k}+Q_t^{t_k}.\end{aligned}$$

Taking G-expectation on both sides, we obtain

$$e^{\mu_\varepsilon(t-t_k)}\hat{\mathbb{E}}[V(t,x(t))]\le\hat{\mathbb{E}}[V(t_k,x(t_k))]+\frac{L_F^2\delta_0}{2\varepsilon}\int_{t_k}^{t}e^{\mu_\varepsilon(s-t_k)}ds+\frac{L_F^2\delta_1}{2\varepsilon}\int_{t_k}^{t}e^{\mu_\varepsilon(s-t_k)}e^{-\beta s}ds.$$

Multiplying both sides by $e^{-\mu_\varepsilon(t-t_k)}$ leads to

$$\hat{\mathbb{E}}[V(t,x(t))]\le e^{-\mu_\varepsilon(t-t_k)}\hat{\mathbb{E}}[V(t_k,x(t_k))]+\frac{L_F^2\delta_0}{2\varepsilon}\int_{t_k}^{t}e^{-\mu_\varepsilon(t-s)}ds$$

$$+\frac{L_F^2\delta_1}{2\varepsilon}\int_{t_k}^{t} e^{-\mu_\varepsilon(t-s)}e^{-\beta s}ds. \tag{5.5}$$

The first integral satisfies

$$\int_{t_k}^{t} e^{-\mu_\varepsilon(t-s)}ds = \frac{1-e^{-\mu_\varepsilon(t-t_k)}}{\mu_\varepsilon} \le \frac{1}{\mu_\varepsilon}.$$

For the second integral, by the definition of $\bar{\beta}$,

$$\begin{aligned}\int_{t_k}^{t} e^{-\mu_\varepsilon(t-s)}e^{-\beta s}ds &= e^{-\beta t_k}\int_{t_k}^{t} e^{-\mu_\varepsilon(t-s)}e^{-\beta(s-t_k)}ds\\ &\le e^{-\beta t_k}(t-t_k)e^{-\bar{\beta}(t-t_k)}\\ &\le \frac{1}{e\bar{\beta}}e^{-\beta t_k} \le \frac{1}{\bar{\beta}}e^{-\beta t_k}.\end{aligned}$$

Here the second inequality follows from the fact that $xe^{-ax} \le 1/(ea)$ for $x, a > 0$.

Substituting these bounds into the preceding inequality, we obtain

$$\hat{\mathbb{E}}[V(t,x(t))] \le e^{-\mu_\varepsilon(t-t_k)}\hat{\mathbb{E}}[V(t_k,x(t_k))] + \frac{L_F^2\delta_0}{2\varepsilon\mu_\varepsilon} + \frac{L_F^2\delta_1}{2\varepsilon\bar{\beta}}e^{-\beta t_k}. \tag{5.6}$$

It remains to deal with the rest interval. Let $t \in [s_k, t_{k+1})$. Applying G-Itô formula to $e^{-\alpha_r(r-s_k)}V(r,x(r))$ on $[s_k,t]$, we obtain

$$\begin{aligned}&e^{-\alpha_r(t-s_k)}V(t,x(t)) - V(s_k,x(s_k))\\ &= \int_{s_k}^{t} e^{-\alpha_r(r-s_k)}[\mathcal{L}_r V(r,x(r)) - \alpha_r V(r,x(r))]dr + \widetilde{M}_t^{s_k} + \widetilde{Q}_t^{s_k},\end{aligned}$$

where

$$\begin{aligned}\widetilde{M}_t^{s_k} &:= \int_{s_k}^{t} e^{-\alpha_r(r-s_k)}V_x(r,x(r))^T g(r,x(r))dB_r,\\ \widetilde{Q}_t^{s_k} &:= \frac{1}{2}\int_{s_k}^{t} e^{-\alpha_r(r-s_k)}[2V_x(r,x(r))^T h(r,x(r)) + g(r,x(r))^T V_{xx}(r,x(r))g(r,x(r))]d\langle B\rangle_r\\ &\quad - \int_{s_k}^{t} e^{-\alpha_r(r-s_k)}G(2V_x(r,x(r))^T h(r,x(r)) + g(r,x(r))^T V_{xx}(r,x(r))g(r,x(r)))dr.\end{aligned}$$

Here, $\widetilde{M}_t^{s_k}$ is a symmetric G-martingale with $\hat{\mathbb{E}}[\widetilde{M}_t^{s_k}] = 0$, while $\widetilde{Q}_t^{s_k}$ is a non-increasing G-martingale with $\hat{\mathbb{E}}[\widetilde{Q}_t^{s_k}] \le 0$.

By Assumption 3.5, we have

$$e^{-\alpha_r(t-s_k)}V(t,x(t)) \le V(s_k,x(s_k)) + \widetilde{M}_t^{s_k} + \widetilde{Q}_t^{s_k}.$$

Taking G-expectation on both sides, we obtain

$$\hat{\mathbb{E}}[V(t,x(t))] \le e^{\alpha_r(t-s_k)}\hat{\mathbb{E}}[V(s_k,x(s_k))].$$

□

Theorem 5.2. *Suppose that Assumptions 3.4–3.6 hold. Assume that there exists $\varepsilon > 0$ such that $\mu_\varepsilon > 0$, where μ_ε is defined by* (5.1)*, and define*

$$\lambda_* := \theta\mu_\varepsilon - (1-\theta)\alpha_r. \tag{5.7}$$

If $\lambda_* > 0$, then, for every globally defined closed-loop solution of (3.2) *and every $\gamma \in (0, \min\{\lambda_*, \beta\})$, the estimate*

$$\hat{\mathbb{E}}[|x(t)|^2] \le C_\gamma e^{-\gamma t}(|x_0|^2 + \delta_1) + D\delta_0, \qquad t \ge 0, \tag{5.8}$$

where

$$\begin{aligned} C_\gamma &:= \max\{\frac{c_2 e^{(\alpha_r+\gamma)T_{\max}}}{c_1}, \frac{L_F^2}{2\varepsilon c_1 \bar{\beta}}[\frac{e^{2(\alpha_r+\gamma)T_{\max}}}{1-e^{-(\beta-\gamma)T_{\min}}} + e^{(\alpha_r+\gamma)T_{\max}}]\}, \\ D &:= \frac{L_F^2}{2\varepsilon c_1 \mu_\varepsilon}[\frac{e^{2\alpha_r T_{\max}}}{1-e^{-\lambda_* T_{\min}}} + e^{\alpha_r T_{\max}}]. \end{aligned} \tag{5.9}$$

In particular,

$$\limsup_{t\to\infty} \hat{\mathbb{E}}[|x(t)|^2] \le D\delta_0.$$

If $\delta_1 = 0$, then the system is practically exponentially stable in G-mean square with practical bound $D\delta_0$. If $\delta_0 = \delta_1 = 0$, then the system is exponentially stable in G-mean square.

Proof. By Lemma 5.1, we have

$$\hat{\mathbb{E}}[V(s_k, x(s_k))] \le e^{-\mu_\varepsilon(s_k - t_k)}\hat{\mathbb{E}}[V(t_k, x(t_k))] + \frac{L_F^2\delta_0}{2\varepsilon\mu_\varepsilon} + \frac{L_F^2\delta_1}{2\varepsilon\bar{\beta}}e^{-\beta t_k} \tag{5.10}$$

and

$$\hat{\mathbb{E}}[V(t_{k+1}, x(t_{k+1}))] \le e^{\alpha_r(t_{k+1} - s_k)}\hat{\mathbb{E}}[V(s_k, x(s_k))]. \tag{5.11}$$

Combining these two estimates, we obtain

$$\begin{aligned} \hat{\mathbb{E}}[V(t_{k+1}, x(t_{k+1}))] \le{}& e^{-\mu_\varepsilon(s_k - t_k) + \alpha_r(t_{k+1} - s_k)}\hat{\mathbb{E}}[V(t_k, x(t_k))] \\ &+ \frac{L_F^2 e^{\alpha_r T_{\max}}}{2\varepsilon\mu_\varepsilon}\delta_0 + \frac{L_F^2 e^{\alpha_r T_{\max}}}{2\varepsilon\bar{\beta}}\delta_1 e^{-\beta t_k}. \end{aligned} \tag{5.12}$$

By Assumption 3.6,

$$s_k - t_k \ge \theta(t_{k+1} - t_k), \qquad t_{k+1} - s_k \le (1-\theta)(t_{k+1} - t_k).$$

By the definition of λ_*,

$$-\mu_\varepsilon(s_k - t_k) + \alpha_r(t_{k+1} - s_k) \le -\lambda_*(t_{k+1} - t_k).$$

Iterating (5.12) over $[t_0, t_1], [t_1, t_2], \ldots, [t_{k-1}, t_k]$, for every $k \ge 1$,

$$\hat{\mathbb{E}}[V(t_k, x(t_k))] \le e^{-\lambda_* t_k}\hat{\mathbb{E}}[V(0, x_0)] + \frac{L_F^2 e^{\alpha_r T_{\max}}}{2\varepsilon\mu_\varepsilon}\delta_0 \sum_{i=0}^{k-1} e^{-\lambda_*(t_k - t_{i+1})}$$

$$+ \frac{L_F^2 e^{\alpha_r T_{\max}}}{2\varepsilon\bar{\beta}}\delta_1 \sum_{i=0}^{k-1} e^{-\lambda_*(t_k - t_{i+1})} e^{-\beta t_i}. \tag{5.13}$$

Since $t_{i+1} - t_i \geq T_{\min}$, the first sum is bounded by

$$\sum_{i=0}^{k-1} e^{-\lambda_*(t_k - t_{i+1})} \leq \sum_{i=0}^{k-1} e^{-\lambda_* i T_{\min}} \leq \frac{1}{1 - e^{-\lambda_* T_{\min}}}.$$

Moreover, since $t_{i+1} - t_i \leq T_{\max}$, $t_i \geq iT_{\min}$, and $\gamma \leq \lambda_*$,

$$\begin{aligned} e^{-\lambda_*(t_k - t_{i+1})} e^{-\beta t_i} &\leq e^{-\gamma t_k} e^{\gamma T_{\max}} e^{-(\beta-\gamma)t_i} \\ &\leq e^{-\gamma t_k} e^{\gamma T_{\max}} e^{-(\beta-\gamma) i T_{\min}}. \end{aligned}$$

Consequently,

$$\sum_{i=0}^{k-1} e^{-\lambda_*(t_k - t_{i+1})} e^{-\beta t_i} \leq e^{-\gamma t_k} e^{\gamma T_{\max}} \sum_{i=0}^{k-1} e^{-(\beta-\gamma) i T_{\min}} \leq \frac{e^{-\gamma t_k} e^{\gamma T_{\max}}}{1 - e^{-(\beta-\gamma)T_{\min}}}.$$

Substituting these estimates into (5.13) and using $V(0, x_0) \leq c_2 |x_0|^2$, we obtain

$$\hat{\mathbb{E}}[V(t_k, x(t_k))] \leq c_2 e^{-\gamma t_k} |x_0|^2 + \frac{L_F^2 e^{(\alpha_r+\gamma)T_{\max}}}{2\varepsilon\bar{\beta}[1 - e^{-(\beta-\gamma)T_{\min}}]} e^{-\gamma t_k}\delta_1 + \frac{L_F^2 e^{\alpha_r T_{\max}}}{2\varepsilon\mu_\varepsilon(1 - e^{-\lambda_* T_{\min}})}\delta_0. \tag{5.14}$$

For $t \in [t_k, s_k)$, Lemma 5.1 implies

$$\hat{\mathbb{E}}[V(t, x(t))] \leq \hat{\mathbb{E}}[V(t_k, x(t_k))] + \frac{L_F^2 \delta_0}{2\varepsilon\mu_\varepsilon} + \frac{L_F^2 \delta_1}{2\varepsilon\bar{\beta}} e^{-\beta t_k},$$

whereas for $t \in [s_k, t_{k+1})$,

$$\hat{\mathbb{E}}[V(t, x(t))] \leq e^{\alpha_r T_{\max}} (\hat{\mathbb{E}}[V(t_k, x(t_k))] + \frac{L_F^2 \delta_0}{2\varepsilon\mu_\varepsilon} + \frac{L_F^2 \delta_1}{2\varepsilon\bar{\beta}} e^{-\beta t_k}).$$

Since $e^{\alpha_r T_{\max}} \geq 1$, these two cases are covered by

$$\hat{\mathbb{E}}[V(t, x(t))] \leq e^{\alpha_r T_{\max}} (\hat{\mathbb{E}}[V(t_k, x(t_k))] + \frac{L_F^2 \delta_0}{2\varepsilon\mu_\varepsilon} + \frac{L_F^2 \delta_1}{2\varepsilon\bar{\beta}} e^{-\beta t_k}). \tag{5.15}$$

The relation $t \leq t_k + T_{\max}$ implies

$$e^{-\gamma t_k} \leq e^{\gamma T_{\max}} e^{-\gamma t}. \tag{5.16}$$

Substituting (5.14) into (5.15), using (5.16) and Assumption 3.5, we get

$$\hat{\mathbb{E}}[|x(t)|^2] \leq C_\gamma e^{-\gamma t}(|x_0|^2 + \delta_1) + D\delta_0, \tag{5.17}$$

where

$$C_\gamma := \max\{\frac{c_2 e^{(\alpha_r+\gamma)T_{\max}}}{c_1}, \frac{L_F^2}{2\varepsilon c_1 \bar{\beta}}[\frac{e^{2(\alpha_r+\gamma)T_{\max}}}{1 - e^{-(\beta-\gamma)T_{\min}}} + e^{(\alpha_r+\gamma)T_{\max}}]\},$$

$$D := \frac{L_F^2}{2\varepsilon c_1 \mu_\varepsilon}[\frac{e^{2\alpha_r T_{\max}}}{1 - e^{-\lambda_* T_{\min}}} + e^{\alpha_r T_{\max}}].$$

Letting $t \to \infty$ in (5.17), we obtain

$$\limsup_{t\to\infty} \hat{\mathbb{E}}[|x(t)|^2] \le D\delta_0.$$

If $\delta_1 = 0$, then the system is practically exponentially stable in G-mean square with practical bound $D\delta_0$. If $\delta_0 = 0$, this estimate is of the form required in the definition of exponential stability in G-mean square. □

Remark 5.3. The quantity

$$\lambda_* = \theta\mu_\varepsilon - (1-\theta)\alpha_r$$

is the balance between control and rest. The active windows contribute the decay term $\theta\mu_\varepsilon$, while the rest windows contribute the possible growth term $(1-\theta)\alpha_r$. Thus, $\lambda_* > 0$ means that the decay gained during control dominates the growth allowed during rest. A larger threshold ρ reduces transmissions but also decreases μ_ε, so the duty ratio θ must be chosen large enough to keep $\lambda_* > 0$.

6. Quasi-sure exponential stability

Lemma 6.1. *Under Assumption 3.4, for every $T > 0$ there exists a positive constant C_T, independent of ℓ, such that, for every $\ell \ge 0$,*

$$\hat{\mathbb{E}}[\sup_{\ell\le t\le \ell+T} |x(t)|^2] \le C_T(\hat{\mathbb{E}}[|x(\ell)|^2] + \delta_0 + \delta_1 e^{-\beta\ell}). \tag{6.1}$$

Proof. Fix $\ell \ge 0$ and $t \in [\ell, \ell+T]$. For $\ell \le r \le t$,

$$x(r) = x(\ell) + \int_\ell^r [f(s,x(s)) + u(s)]ds + \int_\ell^r h(s,x(s))d\langle B\rangle_s + \int_\ell^r g(s,x(s))dB_s.$$

The elementary inequality $(a+b+c+d)^2 \le 4(a^2+b^2+c^2+d^2)$ implies

$$\begin{aligned}\hat{\mathbb{E}}[\sup_{\ell\le r\le t} |x(r)|^2] \le{}& 4\hat{\mathbb{E}}[|x(\ell)|^2] + 4\hat{\mathbb{E}}[\sup_{\ell\le r\le t} |\int_\ell^r (f(s,x(s)) + u(s))ds|^2] \\ &+ 4\hat{\mathbb{E}}[\sup_{\ell\le r\le t} |\int_\ell^r h(s,x(s))d\langle B\rangle_s|^2] + 4\hat{\mathbb{E}}[\sup_{\ell\le r\le t} |\int_\ell^r g(s,x(s))dB_s|^2].\end{aligned} \tag{6.2}$$

By Hölder's inequality and Assumption 3.4, we have

$$\begin{aligned}&\hat{\mathbb{E}}[\sup_{\ell\le r\le t} |\int_\ell^r (f(s,x(s)) + u(s))ds|^2] \le T\int_\ell^t \hat{\mathbb{E}}[|f(s,x(s)) + u(s)|^2]ds \\ &\le 2TL^2\int_\ell^t \hat{\mathbb{E}}[|x(s)|^2]ds + 2T\int_\ell^t \hat{\mathbb{E}}[|u(s)|^2]ds.\end{aligned}$$

On rest windows $u(s) = 0$. On active windows, by (3.5),

$$|u(s)|^2 \le 2L_F^2|x(s)|^2 + 2L_F^2|e(s)|^2 \le 2L_F^2(1+\rho)|x(s)|^2 + 2L_F^2(\delta_0 + \delta_1 e^{-\beta s}).$$

Therefore,

$$\hat{\mathbb{E}}[\sup_{\ell\le r\le t}|\int_\ell^r (f(s,x(s))+u(s))ds|^2]$$
$$\le T[2L^2+4L_F^2(1+\rho)]\int_\ell^t \hat{\mathbb{E}}[|x(s)|^2]ds+4T^2L_F^2(\delta_0+\delta_1e^{-\beta\ell}). \tag{6.3}$$

Moreover, by Lemmas 2.2 and 2.3, we have

$$\hat{\mathbb{E}}[\sup_{\ell\le r\le t}|\int_\ell^r h(s,x(s))d\langle B\rangle_s|^2]\le C_2''T\int_\ell^t \hat{\mathbb{E}}[|h(s,x(s))|^2]ds\le C_2''TL^2\int_\ell^t \hat{\mathbb{E}}[|x(s)|^2]ds, \tag{6.4}$$

and

$$\hat{\mathbb{E}}[\sup_{\ell\le r\le t}|\int_\ell^r g(s,x(s))dB_s|^2]\le C_2'\int_\ell^t \hat{\mathbb{E}}[|g(s,x(s))|^2]ds\le C_2'L^2\int_\ell^t \hat{\mathbb{E}}[|x(s)|^2]ds. \tag{6.5}$$

Substituting (6.3)–(6.5) into (6.2), we have

$$\hat{\mathbb{E}}[\sup_{\ell\le r\le t}|x(r)|^2]\le 4\hat{\mathbb{E}}[|x(\ell)|^2]+R_T(\ell)+K_T\int_\ell^t \hat{\mathbb{E}}[|x(s)|^2]ds,$$

where $R_T(\ell):=16T^2L_F^2(\delta_0+\delta_1e^{-\beta\ell})$ and $K_T:=4\{T[2L^2+4L_F^2(1+\rho)]+C_2''TL^2+C_2'L^2\}$.

Since $|x(s)|^2\le\sup_{\ell\le q\le s}|x(q)|^2$, then

$$\hat{\mathbb{E}}[\sup_{\ell\le r\le t}|x(r)|^2]\le 4\hat{\mathbb{E}}[|x(\ell)|^2]+R_T(\ell)+K_T\int_\ell^t \hat{\mathbb{E}}[\sup_{\ell\le q\le s}|x(q)|^2]ds.$$

Then, by Gronwall's inequality, we obtain

$$\hat{\mathbb{E}}[\sup_{\ell\le r\le t}|x(r)|^2]\le(4\hat{\mathbb{E}}[|x(\ell)|^2]+R_T(\ell))e^{K_TT}\le C_T(\hat{\mathbb{E}}[|x(\ell)|^2]+\delta_0+\delta_1e^{-\beta\ell}),$$

where $C_T:=\max\{4,16T^2L_F^2\}\cdot e^{K_TT}$. □

Theorem 6.2. *Suppose that the assumptions of Theorem 5.2 hold, that $\delta_0=0$, and that the corresponding closed-loop solution is globally defined q.s. Then, for every $\gamma\in(0,\min\{\lambda_*,\beta\})$,*

$$\limsup_{t\to\infty}\frac{1}{t}\log|x(t)|\le-\frac{\gamma}{4},\qquad q.s. \tag{6.6}$$

Proof. When $\delta_0=0$, Theorem 5.2 implies

$$\hat{\mathbb{E}}[|x(t)|^2]\le C_\gamma e^{-\gamma t}(|x_0|^2+\delta_1),\qquad t\ge0, \tag{6.7}$$

where C_γ is defined in Theorem 5.2.

Let

$$I_m=[mT_{\min},(m+1)T_{\min}],\qquad m\in\mathbb{N}.$$

Applying Lemma 6.1 with $\ell = mT_{\min}$ and $T = T_{\min}$, and using (6.7), we get

$$\hat{\mathbb{E}}[\sup_{t\in I_m}|x(t)|^2] \le C_{T_{\min}}(\hat{\mathbb{E}}[|x(mT_{\min})|^2] + \delta_1 e^{-\beta mT_{\min}}) \le C_{T_{\min}}(C_\gamma|x_0|^2 + C_\gamma\delta_1 + \delta_1)e^{-\gamma mT_{\min}},$$

where we used $\gamma < \beta$ in the last inequality.

Define

$$A_m := \{\sup_{t\in I_m}|x(t)|^2 \ge e^{-\gamma mT_{\min}/2}\}.$$

The capacity Markov inequality implies

$$\mathbb{C}(A_m) \le e^{\gamma mT_{\min}/2}\hat{\mathbb{E}}[\sup_{t\in I_m}|x(t)|^2] \le C_{T_{\min}}[C_\gamma|x_0|^2 + (C_\gamma + 1)\delta_1]e^{-\gamma mT_{\min}/2}.$$

Since the right-hand side is summable in m, the capacity Borel–Cantelli lemma implies that, outside a polar set, there exists $m_0(\omega)$ such that for all $m \ge m_0(\omega)$,

$$\sup_{t\in I_m}|x(t)|^2 < e^{-\gamma mT_{\min}/2}.$$

Equivalently, for all such m and all $t \in I_m$,

$$|x(t)| < e^{-\gamma mT_{\min}/4}.$$

If $t \in I_m$, then $t \le (m+1)T_{\min}$. Taking logarithms, we obtain

$$\frac{1}{t}\log|x(t)| \le -\frac{\gamma}{4}\frac{mT_{\min}}{(m+1)T_{\min}}.$$

Letting $t \to \infty$ and hence $m \to \infty$, we obtain

$$\limsup_{t\to\infty}\frac{1}{t}\log|x(t)| \le -\frac{\gamma}{4}, \qquad \text{q.s.} \tag{6.8}$$

□

7. Example

Consider the following uncontrolled system driven by G-Brownian motion:

$$dx(t) = f(t,x(t))dt + h(t,x(t))d\langle B\rangle_t + g(t,x(t))dB_t, \tag{7.1}$$

where the volatility uncertainty interval is $[\underline{\sigma}^2, \overline{\sigma}^2] = [\frac{1}{2}, 1]$ and

$$f(t,x) = x + \frac{1}{2}\arctan x, \qquad h(t,x) = \frac{\arctan x}{1+t^2}, \qquad g(t,x) = e^{-t}\sin x.$$

Take $V(t,x) = (1+e^{-t})x^2$. Then

$$V_t = -e^{-t}x^2, \qquad V_x = 2(1+e^{-t})x, \qquad V_{xx} = 2(1+e^{-t}).$$

Since $1 \le 1 + e^{-t} \le 2$ for all $t \ge 0$, we have

$$c_1|x|^2 \le V(t,x) \le c_2|x|^2, \qquad c_1 = 1,\ c_2 = 2,$$

and

$$|V_x|^2 = 4(1+e^{-t})^2x^2 = 4(1+e^{-t})V \le 8V,$$

so the third condition of Assumption 3.5 holds with $c_3 = 8$.

Since $G(\cdot) = \frac{1}{2}(\cdot)$, then for the open-loop generator, we have

$$\mathcal{L}_0 V = (1+e^{-t})[-\nu(t)x^2 + 2x^2 + x\arctan x + \frac{2x\arctan x}{1+t^2} + e^{-2t}\sin^2 x],$$

where $\nu(t) := \dfrac{e^{-t}}{1+e^{-t}} \in (0, \frac{1}{2}]$. Since $x\arctan x \ge 0$ and $\nu(t) \le \frac{1}{2}$, we obtain

$$\mathcal{L}_0 V \ge (1+e^{-t})[2-\nu(t)]x^2 \ge (1+e^{-t})x^2 \ge V.$$

Hence, the open-loop generator is nonnegative for all $t \ge 0$, and by the Lyapunov instability criterion in [7], the trivial solution of (46) is exponentially unstable.

We design an event-triggered intermittent control in the drift term:

$$dx(t) = [x(t) + \tfrac{1}{2}\arctan x(t) + u(t)]dt + \frac{\arctan x(t)}{1+t^2}d\langle B\rangle_t + e^{-t}\sin x(t)dB_t, \tag{7.2}$$

where the implemented control is

$$u(t) = \begin{cases} F(x(\eta(t))), & t \in [t_k, s_k), \\ 0, & t \in [s_k, t_{k+1}), \end{cases}$$

with

$$F(x) = -12x - 2\arctan x.$$

For $x, y \in \mathbb{R}$, the following Lipschitz estimates hold:

$$|f(t,x) - f(t,y)| \le \tfrac{3}{2}|x-y|, \qquad |h(t,x) - h(t,y)| \le |x-y|,$$
$$|g(t,x) - g(t,y)| \le |x-y|, \qquad |F(x) - F(y)| \le 14|x-y|.$$

Hence Assumption 3.4 holds with

$$L = \tfrac{3}{2}, \qquad L_F = 14.$$

Choose an aperiodic intermittent schedule satisfying

$$P_k \in [1,2], \qquad s_k - t_k \ge 0.6P_k, \qquad k \ge 0.$$

Then, Assumption 3.6 holds with

$$T_{\min} = 1, \qquad T_{\max} = 2, \qquad \theta = 0.6.$$

During active windows, the continuously updated feedback satisfies

$$\begin{aligned}\mathcal{L}_c V &= V_t + V_x(f+F) + G(2V_x h + g^\top V_{xx} g)\\ &= (1+e^{-t})[-\nu(t)x^2 - 22x^2 - 3x\arctan x + \frac{2x\arctan x}{1+t^2} + e^{-2t}\sin^2 x].\end{aligned}$$

Since $x\arctan x \geq 0$, $\dfrac{2x\arctan x}{1+t^2} \leq 2x^2$, and $e^{-2t}\sin^2 x \leq x^2$, we obtain

$$\mathcal{L}_c V \leq (1+e^{-t})[-(\nu(t)+22)+2+1]x^2 = -(\nu(t)+19)V \leq -19V.$$

Hence, $\alpha_c = 19$.

During rest windows, $u = 0$, and

$$\begin{aligned}\mathcal{L}_r V &= V_t + V_x f + G(2V_x h + g^\top V_{xx} g)\\ &= (1+e^{-t})[-\nu(t)x^2 + 2x^2 + x\arctan x + \frac{2x\arctan x}{1+t^2} + e^{-2t}\sin^2 x]\\ &\leq (1+e^{-t})[6-\nu(t)]x^2 \leq 6V.\end{aligned}$$

Thus, $\alpha_r = 6$.

We conclude that Assumption 3.5 is satisfied with

$$c_1 = 1, \quad c_2 = 2, \quad \alpha_c = 19, \quad \alpha_r = 6, \quad c_3 = 8.$$

Choose

$$\varepsilon = 1, \qquad \rho = 10^{-2}, \qquad \delta_0 = 10^{-3}, \qquad \delta_1 = 10^{-2}, \qquad \beta = 10.$$

Then,

$$\mu_\varepsilon = \alpha_c - \frac{\varepsilon c_3}{2} - \frac{L_F^2 \rho}{2\varepsilon c_1} = 19 - 4 - 0.98 = 14.02 > 0,$$

and

$$\lambda_* = \theta\mu_\varepsilon - (1-\theta)\alpha_r = 0.6 \times 14.02 - 0.4 \times 6 = 8.412 - 2.4 = 6.012 > 0.$$

Hence, all the conditions of Theorem 5.2 are satisfied. For every $\gamma \in (0, \min\{\lambda_*, \beta\}) = (0, 6.012)$, Theorem 5.2 implies that the controlled system (7.2) satisfies

$$\hat{\mathbb{E}}[|x(t)|^2] \leq C_\gamma e^{-\gamma t}(|x_0|^2 + \delta_1) + D\delta_0, \qquad t \geq 0.$$

If $\delta_1 = 0$, the controlled system is practically exponentially stable in G-mean square with practical bound $D\delta_0$. If $\delta_0 = 0$ and the corresponding closed-loop solution is globally defined q.s., Theorem 6.2 further provides a quasi-sure exponential estimate. If $\delta_0 = \delta_1 = 0$, the controlled system is exponentially stable in G-mean square. If $\delta_0 > 0$, Theorem 4.2 ensures that the triggering law is Zeno-free q.s. on every finite horizon.

For the numerical simulation, we use exactly the same parameters as above. The G-Brownian motion is approximated by sampling the volatility σ uniformly from $[\underline{\sigma}, \overline{\sigma}] = [1/\sqrt{2}, 1]$. We use the Euler scheme with step size $\Delta t = 10^{-3}$ on $[0, T]$ with $T = 20$, and $M = 500$ independent samples. The initial condition is $x_0 = 1$. The intermittent schedule is $P_k = 1.5$ and $s_k - t_k = 0.9$, which satisfies

Assumption 3.6 with $T_{\min} = 1$, $T_{\max} = 2$, and $\theta = 0.6$. The triggering parameters are $\rho = 10^{-2}, \beta = 10$, $\delta_0 = 10^{-3}$, and $\delta_1 = 10^{-2}$. The stability parameter is chosen as $\gamma = 4 \in (0, \lambda_*) = (0, 6.012)$.

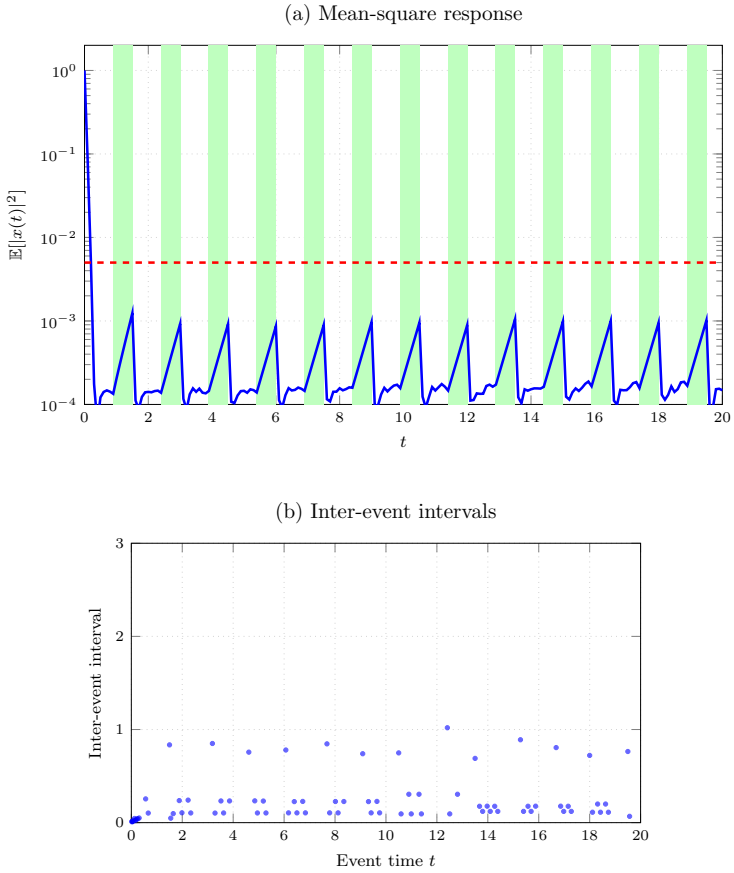


Figure 2. Numerical simulation results for the event-triggered intermittent control scheme: (a) mean-square trajectory; the red dashed line indicates the empirical ultimate bound; (b) inter-event intervals.

Figure 2 reports the numerical results. In panel (a), the mean-square response decreases rapidly at the initial stage and then settles into a periodic sawtooth pattern, where the green shaded regions mark the rest windows. The red dashed line indicates a conservative upper bound for the steady-state response. In panel (b), all inter-event intervals are bounded away from zero, confirming the absence of Zeno behavior established in Theorem 4.2.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

Acknowledgments

This work is supported in part by the NSFC Grant Nos. 12171084 and 12571164, the Research Foundation of Jinling Institute of Technology Grant No. Jit-b-201836, the Incubation Project of

Jinling Institute of Technology Grant No. Jit-fhxm-2021, the Industry-University-Research Cooperation Project of Jiangsu Provincial Department of Science and Technology under Grant No. jit-h-2025-040.

Conflict of interest

The authors declare there is no conflicts of interest.

References

1. S. Peng, *G*-expectation, *G*-Brownian motion and related stochastic calculus of Itô type, in *Stochastic Analysis and Applications*, Springer, (2007), 541–567. https://doi.org/10.1007/978-3-540-70847-6_25
2. S. Peng, Multi-dimensional *G*-Brownian motion and related stochastic calculus under *G*-expectation, *Stoch. Process. Appl.*, **118** (2008), 2223–2253. https://doi.org/10.1016/j.spa.2007.10.015
3. S. Peng, Survey on normal distributions, central limit theorem, Brownian motion and the related stochastic calculus under sublinear expectation, *Sci. China Ser. A Math.*, **52** (2009), 1391–1411. https://doi.org/10.1007/s11425-009-0121-8
4. L. Denis, M. Hu, S. Peng, Function spaces and capacity related to a sublinear expectation: application to *G*-Brownian motion paths, *Potential Anal.*, **34** (2011), 139–161. https://doi.org/10.1007/s11118-010-9185-x
5. F. Gao, Pathwise properties and homeomorphic flows for stochastic differential equations driven by *G*-Brownian motion, *Stoch. Process. Appl.*, **119** (2009), 3356–3382. https://doi.org/10.1016/j.spa.2009.05.010
6. X. Bai, Y. Lin, On the existence and uniqueness of solutions to stochastic differential equations driven by *G*-Brownian motion with integral-Lipschitz coefficients, *Acta Math. Appl. Sin. Engl. Ser.*, **30** (2014), 589–610. https://doi.org/10.1007/s10255-014-0405-9
7. X. Li, X. Lin, Y. Lin, Lyapunov-type conditions and stochastic differential equations driven by *G*-Brownian motion, *J. Math. Anal. Appl.*, **439** (2016), 235–255. https://doi.org/10.1016/j.jmaa.2016.02.042
8. X. Mao, Almost sure exponential stabilization by discrete-time stochastic feedback control, *IEEE Trans. Autom. Control*, **61** (2016), 1619–1624. https://doi.org/10.1109/TAC.2015.2471696
9. L. Hu, Y. Ren, T. Xu, *p*-moment stability of solutions to stochastic differential equations driven by *G*-Brownian motion, *Appl. Math. Comput.*, **230** (2014), 231–237. https://doi.org/10.1016/j.amc.2013.12.111
10. W. Yin, J. Cao, Y. Ren, Quasi-sure exponential stabilization of stochastic systems induced by *G*-Brownian motion with discrete time feedback control, *J. Math. Anal. Appl.*, **474** (2019), 276–289. https://doi.org/10.1016/j.jmaa.2019.01.045
11. Y. Ren, W. Yin, R. Sakthivel, Stabilization of stochastic differential equations driven by *G*-Brownian motion with feedback control based on discrete-time state observation, *Automatica*, **95** (2018), 146–151. https://doi.org/10.1016/j.automatica.2018.05.039

12. W. Yin, J. Cao, Y. Ren, G. Zheng, Improved result on stabilization of G-SDEs by feedback control based on discrete-time observations, *SIAM J. Control Optim.*, **59** (2021), 1927–1950. https://doi.org/10.1137/20M1338720

13. B. Wang, H. Gao, M. Yuan, A note for discrete time feedback control for stochastic systems driven by G-Brownian motion, *Probab. Uncertain. Quant. Risk*, **10** (2025), 559–568. https://doi.org/10.3934/puqr.2025024

14. B. Wang, H. Gao, M. Yuan, Exponential stabilization via discrete-time feedback control for stochastic systems driven by G-Lévy process, *Statist. Probab. Lett.*, **232** (2026), 1–7. https://doi.org/10.1016/j.spl.2026.110639

15. Q. Zhu, T. Huang, Stability analysis for a class of stochastic delay nonlinear systems driven by G-Brownian motion, *Syst. Control Lett.*, **140** (2020), 1–9. https://doi.org/10.1016/j.sysconle.2020.104699

16. X. Yang, Q. Zhu, Event-triggered control of stochastic functional differential systems via degenerate Lyapunov functionals, *IEEE Trans. Syst. Man Cybern. Syst.*, **53** (2023), 2492–2499. https://doi.org/10.1109/TSMC.2022.3212340

17. W. P. M. H. Heemels, M. C. F. Donkers, A. R. Teel, Periodic event-triggered control for linear systems, *IEEE Trans. Autom. Control*, **58** (2013), 847–861. https://doi.org/10.1109/TAC.2012.2220443

18. H. Yang, Y. Ren, W. Lu, Stabilisation of stochastic differential equations driven by G-Brownian motion via aperiodically intermittent control, *Int. J. Control*, **93** (2020), 565–574. https://doi.org/10.1080/00207179.2018.1480836

19. S. Li, B. Zhang, W. Li, Stabilization of multi-weights stochastic complex networks with time-varying delay driven by G-Brownian motion via aperiodically intermittent adaptive control, *Int. J. Control*, **94** (2021), 7–20. https://doi.org/10.1080/00207179.2019.1577562

20. J. Ma, J. Hu, S. Li, Y. Li, Stability in dynamic event-triggered control of stochastic nonlinear systems: A G-Brownian motion scenario, *ISA Trans.*, **167** (2025), 1119–1136. https://doi.org/10.1016/j.isatra.2025.10.007

21. X. Li, S. Peng, Stopping times and related Itô's calculus with G-Brownian motion, *Stoch. Process. Appl.*, **121** (2011), 1492–1508. https://doi.org/10.1016/j.spa.2011.03.008

22. S. Peng, *Nonlinear Expectations and Stochastic Calculus under Uncertainty*, Springer, Berlin, 2019. https://doi.org/10.1007/978-3-662-59903-7



AIMS Press

© 2026 the Author(s), licensee AIMS Press. This is an open access article distributed under the terms of the Creative Commons Attribution License (https://creativecommons.org/licenses/by/4.0)