



*Research article*

## Reduced-order modeling and comparative dynamic analysis of large slender tower crane jib

Mei Liu<sup>1,\*</sup>, Lishan Ma<sup>1</sup>, Huaqiang Ge<sup>1</sup>, Xiaoyun Zhang<sup>2</sup> and Zunbing Sheng<sup>1</sup>

<sup>1</sup> School of Mechanical Electrical Engineering, Heilongjiang University, Harbin 150080, China

<sup>2</sup> School of Mathematics and Statistics, Northwestern Polytechnical University, Xi'an 710129, China

\* **Correspondence:** Email: liumeip@163.com.

**Abstract:** The low structural damping characteristics of a large slender tower crane leads to challenges with respect to vibrations and structural failure risk due to external load excitation. The finite element method is often used for dynamic analysis of tower crane jib, while a model-order reduction technique with advantages is rarely studied and applied to this engineering field. With the reduced-order model, the essential cause of system dynamic characteristics and the physical meaning of key control parameters can be revealed in an intuitive way. Therefore, a novel and competitive reduced-order model of large slender tower crane jib was introduced and was well-suited for the complex truss structure with multiple flexible beams in this study. In this work, we were concerned with the equivalent continuum modeling procedure for the large slender tower crane jib with asymmetric configuration and analysis of coupling vibration characteristics, aiming to focus more on potential studies of nonlinear vibration and vibration control. First, the equivalent anisotropic continuum beam model for tower crane jib with oblique triangular prism configuration was established by means of energy equivalence formulation. Subsequently, an equivalent 6-DOFs coupled beam model, including multiple and complex coupling terms, was solved by the finite element method, leading to reduced-order representation. In the numerical studies, a cantilevered truss structure of a large slender tower crane jib was simulated by commercial finite element software ANSYS to check the accuracy and validity of the proposed reduced-order model. The results, including the natural characteristic and dynamic response of the proposed reduced-order model, were presented by comparison with those of the finite element model for the original tower crane jib, indicating that the established reduced-order model is valid. Moreover, the coupled vibration characteristics were studied between the reduced-order model and original tower crane jib, indicating that the coupled vibration strength is weak.

**Keywords:** tower crane jib; oblique triangular prism configuration; reduced-order modeling; dynamic analysis; coupled vibration characteristics

## 1. Introduction

Tower cranes are integral to various applications, including loading ship cargos, transporting heavy loads in industrial factories, and civil engineering construction works, such as handling nuclear waste, high-rise buildings, bridges, dams, stadiums, and airports, and these transportation processes require low payload oscillation [1,2]. For instance, El Ouni and Jameel [1] designed a smart tower crane system to mitigate turbulent wind loads, while Ramli et al. [2] proposed a neural network-based input shaping method for swing suppression under mass variations. With the rapid increase in size and complexity of construction projects and the requirement of lightweight mechanical structures, tower cranes have the characteristics of complexity, high overall height, and a large slender jib, resulting in large deformations and vibration [3]. Thus, it is imperative to study the vibration behaviors of tower cranes to prevent structural fatigue damage and operator's discomfort [4].

Until now, most researchers focus on the studies of dynamic responses and anti-swing control of tower cranes [5–8]. Specifically, Chen et al. [5] developed an adaptive tracking control method for 4-DOF systems. Kenan and Azeloğlu [6] focused on the design of scaled-down tower crane mast models. Furthermore, Ye and Huang [7] analyzed payload twisting dynamics and its oscillation control, while Zhang et al. [8] applied a sliding mode control based on disturbance employment for payload sway reduction. In the tower crane dynamic analysis field, structural vibration characteristics are one of the main concerns, and the establishment of the dynamic model for a tower crane mainly relies on the finite element method; however, for large slender tower cranes with many structural members, the models established directly using the finite element method will be high-dimensional and introduce significant challenges in vibration suppression [9,10]. Specifically, Liu et al. [9] proposed an equivalent continuum modeling method for the transient response analysis of large space truss structures with nonlinear elastic joints, while Liu et al. [10] developed an equivalent nonlinear beam model to characterize the nonlinear dynamic responses of beam-like trusses. Therefore, achieving low-dimensional dynamic models is key to controlling a large truss structure so that model-order reduction techniques have become a major research direction in this field [11–15]. For example, Cao et al. [11] established an extended continuum model to efficiently evaluate the dynamic characteristics of beam-like truss structures, and Guo et al. [12] conducted dynamic equivalent modeling to facilitate the structural analysis of hoop deployable antennas. Similarly, the equivalent continuum modeling approach has been successfully applied to characterize the complex dynamics of beam-like trusses featuring flexible joints [13], as well as to derive accurate control-oriented models essential for the active vibration control of large space telescope trusses [15]. The pioneer work in this field may date back to the late 1980s, where Noor and Nemeth presented an equivalent modeling method for beam-like trusses with rigid joints [14]. Sun et al. [16] established an orthotropic equivalent Mindlin plate dynamic model and analyzed its natural characteristics. Li et al. [17] explored the role of nonlinear joints in the dynamics of planar repetitive truss structure and presented its equivalent dynamic model based on the multi-harmonic describing function method. An extended nonlinear equivalent micropolar model relied on the linear equivalent micropolar beam model, which was developed for the geometric nonlinearity problem of the space antenna truss structure [18]. Chen et al. [19] studied

nonlinear vibrations of a rotating circular truss antenna subjected to thermal excitation based on the equivalent continuum model. In [20], Li et al. presented a dynamic substitute model for a flexible slender quadrilateral truss and analyzed its natural characteristics. Yuan et al. [21] established a low-dimensional, high-accuracy model of a beamlike truss with geometric nonlinearity using the static condensation method while the high-frequency modes were replaced by the modal derivatives related to the low-frequency bending modes to avoid expensive evaluations.

Most researchers focus on the crane tower positioning and the study of lifting load anti-swinging [22–26]. In this context, advanced control algorithms have been extensively developed to address payload swing and external disturbances. For instance, Martin and Irani [22] proposed a self-tuning anti-sway control strategy for knuckle boom cranes. Roman et al. [23] developed a hybrid data-driven fuzzy active disturbance rejection control, and Samin and Mohamed [24] compared anti-sway control strategies. Moreover, Sun and Ouyang [25] designed a fuzzy adaptive controller for double-pendulum tower cranes, and Zhang et al. [26] proposed an adaptive integral sliding mode control method for payload sway reduction. In addition, the tower crane jib uses large-size triangular prism trusses as supporting and transporting structures, marked by low-frequency and dense-frequency modes. In the field of vibration control, most scholars ignore structural elasticity and regard it as a rigid body. Neglecting the structural elasticity for feedback control design can lead to instability or poor performance. Hence, developing an accurate low-dimensional dynamical model for a tower crane jib structure is critical; to effectively control the system and the loading positioning control. A control-oriented model of top-slewing tower crane is derived as a flexible multibody system, assuming jib and tower to be nonuniform Euler-Bernoulli beams [27]. Static analysis has focused on the truss structure of the tower crane design to make it possible to lift the specified weight [28]. Kenan and Azeloglu [29] investigated the seismic performance of tower crane mast types, incorporating different bracing schemes and acquiring natural frequencies, mode shapes, and transient responses based on the finite element method. Moreover, seismic responses and dynamic characteristics of a boom tower crane under measured strong earthquake excitation were conducted in [30]. Liu et al. [31] analyzed wind-induced vibration of tower cranes under full wind angles and systematically studied the acceleration response and displacement response under random action of fluctuating wind. A full-size three-dimensional finite element model of a tower crane was established using Ansys commercial software, and dynamic analysis was carried out to explore the structural vibration characteristics in [32]. The detailed finite element model is in high-order form, and it is difficult to reveal its physical meaning of key control parameters intuitively. The model-order reduction technique offers a transformative approach to analyzing the dynamic characterization of the large tower crane jib truss, revolutionizing the traditional research methodology. As tower cranes belong to composite truss structures, they can be simplified into composite beam models via the equivalent continuum modeling concept. On this basis, analytical methods can be adopted to further investigate the dynamic characteristics of the tower crane system, and the relevant literature [33,34] is a reference for the research framework of this work. Specifically, these references include the in-depth vibration analysis of L-shaped beams to evaluate their structural dynamics [33] and the nonlinear dynamic modeling of multiple thin plates connected by long hinges, highlighting the significant influence of flexible connections on the overall system responses [34]. Moreover, tower cranes have typical features, such as a large length-diameter ratio and periodic repetition, which are very similar to deployable trusses in the aerospace field and are suitable to the equivalent continuum modeling technique. To our knowledge, the application of reduced-order modeling based on the equivalent continuum modeling technique to the tower crane jib remains limited.

Thus, it is urgent and attractive to establish a reduced-order model for a large slender tower crane jib with oblique and asymmetric triangular prism configurations. In this work, from the point of view of the equivalent continuum modeling technique, a new reduced-order model is established for a large slender tower crane jib, providing a fresh reference to achieve a higher precision, efficiency equivalent model and facilitate structural vibration suppression. Most notably, according to the equivalent stiffness and mass of the tower crane jib, the couple vibration is investigated to identify the effect of coupling effects caused by the structural size and configuration asymmetry. The reduced-order model is further evaluated by comparative dynamic analysis of the finite element simulation results of a tower crane jib prototype, including natural frequencies, modal shapes, and dynamic responses. The work in this study paves a way for structural vibration control and anti-swing control of large slender tower crane jibs. In conclusion, the novelty of this work lies in the extended application of this modeling approach to tower crane jibs composed of oblique triangular prisms with intrinsic asymmetry, which gives rise to multi-mode coupling vibration behaviors. For structures featuring such multi-mode coupling characteristics, finite-element-based solutions are implemented, and the strength of coupling vibration characteristics is further analyzed.

The following sections are structured as follows: In Section 2, an equivalent continuum model of the tower crane jib with asymmetric configuration is formulated via the energy equivalence principle. In Section 3, derived from the equivalent stiffness and mass matrix of the tower crane jib, the finite element method is implemented to establish reduced-order model and analyze its vibration characteristics. In Section 4, the tower crane jib prototype's dynamic characteristics are reported alongside the full-scale finite element model and theoretical results by reduced-order model to validate the effectiveness. Furthermore, the coupling vibration effect is investigated. The primary conclusions are summarized in Section 5.

## 2. Equivalent continuum modeling

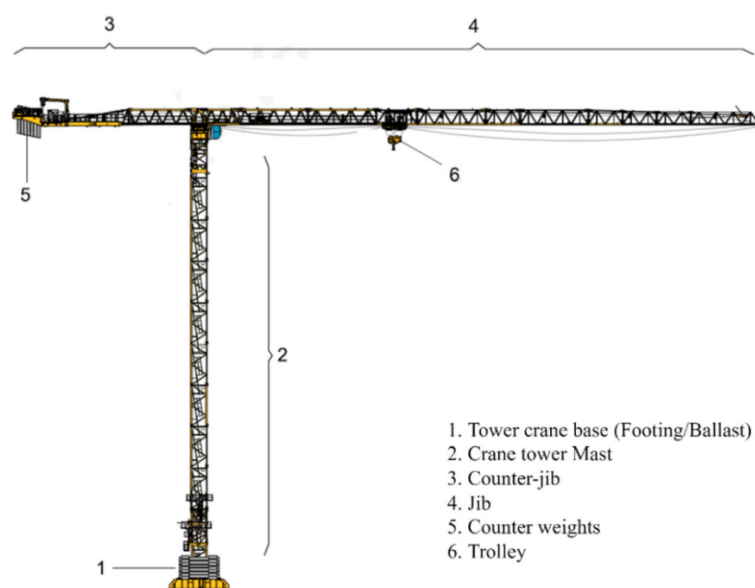


**Figure 1.** The prototype of the tower crane system [35].

Tower cranes are widely used in modern construction projects and are affected by diverse dynamics and working conditions. The prototype of tower crane is shown in Figure 1, which illustrates its key components and includes a tower crane base, crane tower mast, counter-jib, jib counter weights, and a trolley [35]. It should be noted that the jib is chosen as the research object in this study.

### 2.1. Jib structure

Unlike the regular triangular prism configuration, our focus of this study is the large slender tower crane jib with oblique and asymmetric triangular prism configurations. The overall configuration of the tower crane jib is shown in Figure 2(a). The jib is assembled from a steel truss containing 36 typical lattice elements. A typical lattice element is given in Figure 2(b), which is constructed from three longitudinal beams and nine inclined beams. The yellow and purple lines indicate longitudinal and inclined beams, respectively. The triangular sections on both sides have an inclination angle  $\alpha$  with the repetition direction. It should be noted that the structure has the characteristic of section size and configuration asymmetries. In this study, the length-diameter ratio of the tower crane jib reaches about 50 magnitudes, so the flexibility of the tower crane jib cannot be ignored and deserves to be further investigated when carrying heavy cargos under large-amplitude three-dimensional motion. In this paper, the large slender tower crane jib is considered as a cantilevered state and is modeled as a continuum beam model of equal length.

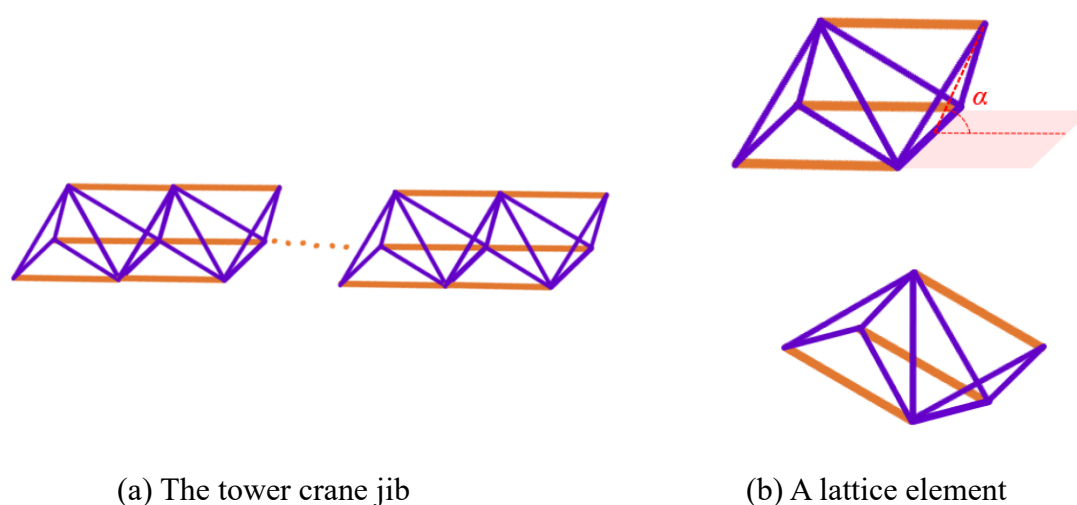


**Figure 2.** Structure of the large slender tower crane jib.

### 2.2. Energy method

The tower crane jib is modeled as 3D space beam elements. Figure 3(a) shows the tower crane jib lattice element, which is welded by 12 beam components, including three longitudinal beams and nine inclined beams. The Cartesian coordinate system of the lattice whose origin is at the centroid of the lattice volume is shown in Figure 3. As a special kind of geometrical configuration, the cross-section ABC is an isosceles triangle, as shown in Figure 3(b).

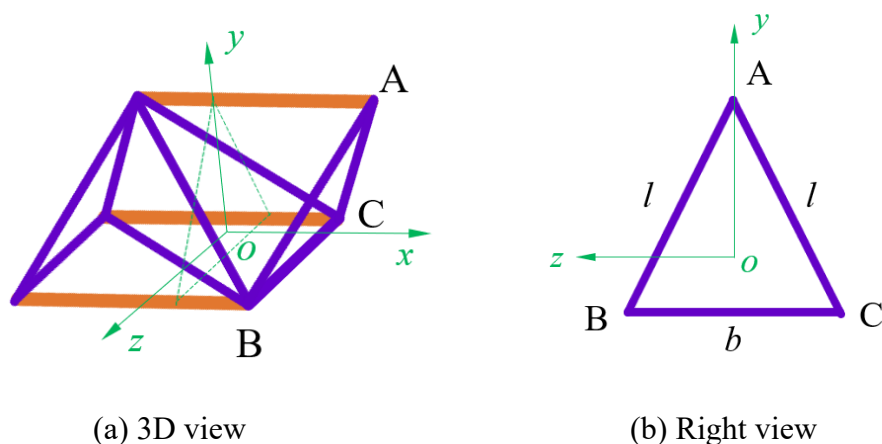


**Figure 3.** Cartesian coordinate system in the lattice element.

Accordingly, the displacement field [10] of the lattice element can be written as its central displacement, as follows:

$$\begin{cases} u(x, y, z) \approx u_0 - y\varphi_{z0} + z\varphi_{y0} + x\varepsilon_{x0} - xy\kappa_{y0} + xz\kappa_{z0}, \\ v(x, y, z) \approx v_0 + y\varepsilon_{y0} + z\left(-\varphi_{x0} + \frac{1}{2}\gamma_{yz0}\right) + x(\varphi_{z0} + \gamma_{xy0}) - xz\kappa_{x0} + \frac{1}{2}x^2\kappa_{y0}, \\ w(x, y, z) \approx w_0 + y\left(\varphi_{x0} + \frac{1}{2}\gamma_{yz0}\right) + z\varepsilon_{z0} + x(\gamma_{xz0} - \varphi_{y0}) + xy\kappa_{x0} - \frac{1}{2}x^2\kappa_{z0}, \end{cases} \quad (1)$$

where  $u_0$ ,  $v_0$ , and  $w_0$  are the displacements along the  $x$ ,  $y$ , and  $z$  axes of the lattice element center;  $\varphi_{x0}$ ,  $\varphi_{y0}$ , and  $\varphi_{z0}$  are the rotations along the  $x$ ,  $y$ , and  $z$  axes of the lattice element center;  $\varepsilon_{x0}$ ,  $\varphi_{y0}$ ,  $\varphi_{z0}$ ,  $\gamma_{xy0}$ ,  $\gamma_{yz0}$ , and  $\gamma_{xz0}$  are the tension and shear strains of the lattice element center;  $\kappa_{x0}$ ,  $\kappa_{y0}$ , and  $\kappa_{z0}$  are the curvatures of lattice element center. The corresponding expressions are:

$$\begin{cases} \varphi_{y0} = -\left(\frac{dw^0(x)}{dx}\right)\Big|_{x=0} - \gamma_{xz0}, \varphi_{z0} = \left(\frac{dv^0(x)}{dx}\right)\Big|_{x=0} - \gamma_{xy0}, \varepsilon_{x0} = \frac{du^0(x)}{dx}\Big|_{x=0}, \\ \kappa_{x0} = \frac{d\theta_x^0(x)}{dx}\Big|_{x=0}, \kappa_{y0} = \frac{d\theta_z^0(x)}{dx}\Big|_{x=0}, \kappa_{z0} = \frac{d\theta_y^0(x)}{dx}\Big|_{x=0}. \end{cases} \quad (2)$$

The lattice element total potential energy is the summation of elastic potential energy of every component. For the lattice element, the strain energy is quantified by the following equation, which integrates the strain energy due to axial, torsional, and bending deformations in all beam members (longitudinal and inclined). Similarly, the total kinetic energy of the lattice element is defined as the summation of the kinetic energies of all its components, as

$$S_e = \frac{1}{2} \sum_{k=1}^{12} \int_0^{l^{(k)}} \left[ E^{(k)} A^{(k)} u_x^{(k)2} + E^{(k)} I_z^{(k)} \theta_z^{(k)2} + E^{(k)} I_y^{(k)} \theta_y^{(k)2} + G^{(k)} J^{(k)} \theta_x^{(k)2} \right] dx_l, \quad (3)$$

$$T_e = \frac{1}{2} \sum_{k=1}^{12} \int_0^{l^{(k)}} \left[ \rho^{(k)} A^{(k)} \dot{u}_y^{(k)2} + \rho^{(k)} A^{(k)} \dot{u}_z^{(k)2} + J_x^{(k)} \dot{\theta}_x^{(k)2} \right] dx_l, \quad (4)$$

where  $k$  stands for the member number from 1 to 12;  $E^{(k)} A^{(k)}$ ,  $E^{(k)} I_z^{(k)}$ ,  $E^{(k)} I_y^{(k)}$ , and  $G^{(k)} J^{(k)}$  are

extension, bending, and torsional stiffnesses, respectively;  $\rho^{(k)}A^{(k)}$  and  $J_x^{(k)}$  denote mass and rotary inertia per unit length, respectively.  $x_l$  is the coordinate along the axis of the member. The prime symbol “'” indicates differentiation with respect to the  $x_l$  coordinate; “.” is the derivative versus time.

Classical beam theory posits that no interlayer extrusion occurs within the cross-section. Under this assumption, the beam model and lattice element are equivalent, i.e.,

$$\frac{\partial S_e}{\partial \varepsilon_{y0}} = \frac{\partial S_e}{\partial \varepsilon_{z0}} = \frac{\partial S_e}{\partial \gamma_{yz0}} = 0. \quad (5)$$

Further, the potential energy of the lattice element can be written as the function of  $e = [\varepsilon_{x0}, \kappa_{y0}, \kappa_{z0}, \kappa_{x0}, \gamma_{xz0}, \gamma_{xy0}]^T$ . The potential energy  $S_e$  of the lattice element has the form

$$S_e = \frac{1}{2} \varepsilon^T \mathbf{D}_e \varepsilon, \quad (6)$$

where  $\mathbf{D}_e$  is the elastic matrix.

The kinetic energy of the lattice element can be given by the function of  $u = [u_0, v_0, w_0, \varphi_{x0}, \varphi_{y0}, \varphi_{z0}]^T$  and can be derived as follows:

$$T_e = \frac{1}{2} \dot{u}^T \mathbf{M}_e \dot{u}, \quad (7)$$

where  $\mathbf{M}_e$  is the inertia matrix.

Once the elastic matrix  $\mathbf{D}_e$  and inertia matrix  $\mathbf{M}_e$  of the lattice element are obtained, the equivalent continuum model stiffness matrix and mass matrix can be calculated. The lattice element is equivalent to an anisotropic Timoshenko beam having a length equal to that of the lattice element.

### 3. Reduced-order modeling and computation strategy

#### 3.1. Equivalent parameters

According to the elastic matrix  $\mathbf{D}_e$  in the preceding section, it is possible to achieve model order reduction for the equivalent continuum model by applying the finite element method. Its stiffness matrix can be derived and further used to calculate and analyze the dynamic characteristics. Here,  $\mathbf{D}_e \in \mathbf{R}^{6 \times 6}$  is given by

$$\mathbf{D}_e = \begin{bmatrix} D_{11} & v_{12} & v_{13} & v_{14} & v_{15} & v_{16} \\ & D_{22} & v_{23} & v_{24} & v_{25} & v_{26} \\ & & D_{33} & v_{34} & v_{35} & v_{36} \\ & & & D_{44} & v_{45} & v_{46} \\ & sym & & & D_{55} & v_{56} \\ & & & & & D_{66} \end{bmatrix}, \quad (8)$$

where  $D_{ii} (i = 1, 2, 3, \dots, 6)$  are equivalent extension, bending, shear, and torsion stiffness;  $v_{ij}$  are the

coupling stiffnesses, which may be zero depending on the structural symmetry feature. In this study, the equivalent stiffness includes 6 coupling items  $v_{13}$ ,  $v_{14}$ ,  $v_{16}$ ,  $v_{34}$ ,  $v_{36}$ , and  $v_{46}$ , namely, extension-bending, extension-torsion, extension-shear, bending-torsion, bending-shear, and torsion-shear coupling terms, respectively, and these coupling terms are caused by the structural size and configuration asymmetries, which make it difficult to solve the dynamic behavior for the complex equivalent continuum model in the analytic method.

On the basis of inertia matrix  $\mathbf{M}_e$ , the mass matrix of the equivalent continuum model is achieved, and the  $\mathbf{M}_e \in \mathbf{R}^{6 \times 6}$  is

$$\mathbf{M}_e = \begin{bmatrix} M_{11} & v_{12} & v_{13} & v_{14} & v_{15} & v_{16} \\ & M_{22} & v_{23} & v_{24} & v_{25} & v_{26} \\ & & M_{33} & v_{34} & v_{35} & v_{36} \\ & & & M_{44} & v_{45} & v_{46} \\ & sym & & & M_{55} & v_{56} \\ & & & & & M_{66} \end{bmatrix}, \quad (9)$$

where  $M_{ii} (i = 1, 2, 3, \dots, 6)$  are equivalent mass and rotation inertia; and  $v_{ij}$  are the coupling coefficients. Similarly, the coupling mass contains  $v_{15}$ ,  $v_{24}$ ,  $v_{26}$ ,  $v_{35}$ ,  $v_{45}$ , and  $v_{46}$ , 6 items due to the structural asymmetry.

### 3.2. Reduced-order modeling

The finite element method is employed to derive the reduced-order model stiffness and mass matrices. It is worth noting that the “reduced-order model” herein refers to the lower-order dynamic structural model obtained by establishing an equivalent continuum model for the tower crane jib and performing subsequent finite element discretization. The large slender tower crane jib is considered a cantilever structure combined with the characteristic of a practical engineering structure, and its constraint condition is

$$\begin{aligned} u(0) = 0, v(0) = 0, w(0) = 0, \varphi_x(0) = 0, \varphi_y(0) = 0, \varphi_z(0) = 0, \\ v''(l_l) = 0, w''(l_l) = 0, v'''(l_l) = 0, w'''(l_l) = 0, \end{aligned} \quad (10)$$

where  $l_l$  is the total length of the equivalent continuum model.

Figure 4 shows the reduced-order finite element model of the equivalent continuum model adopting a two-nodal beam element. The large slender tower crane jib is composed of 36 lattice elements and thus the finite element discretization of the equivalent continuum model has 37 nodes in contrast to the original structural full-scale finite element model with 1314 nodes. As such, it is observed that the degree of freedom is greatly reduced after model-order reduction, which has the great advantage of saving computing resources for the large slender tower crane jib. Timoshenko beam theory models the transverse vibration displacement as the sum of contributions from bending and shear, namely

$$\begin{aligned} v &= v_s + v_b, \\ w &= w_s + w_b. \end{aligned} \quad (11)$$

We employ independent interpolations for bending and shear displacements, and then

$$\begin{aligned}
u &= N_1 u_i + N_2 u_j, \\
v_b &= N_3 v_{bi} + N_4 \varphi_{zi} + N_5 v_{bj} + N_6 \varphi_{zj}, \\
v_s &= N_1 v_{si} + N_2 v_{sj}, \\
w_b &= N_3 w_{bi} + N_4 \varphi_{yi} + N_5 w_{bj} + N_6 \varphi_{yj}, \\
w_s &= N_1 w_{si} + N_2 w_{sj}, \\
\varphi_x &= N_1 \varphi_{xi} + N_2 \varphi_{xj},
\end{aligned} \tag{12}$$

where  $N_1, N_2, \dots, N_6$  are the corresponding shape functions. Their mathematical expressions are:

$$\begin{aligned}
N_1 &= 1 - \zeta, \\
N_2 &= \zeta, \\
N_3 &= 1 - 3\zeta^2 + 2\zeta^3, \\
N_4 &= (\zeta - 2\zeta^2 + \zeta^3)l_0, \\
N_5 &= 3\zeta^2 - 2\zeta^3, \\
N_6 &= (\zeta^3 - \zeta^2)l_0,
\end{aligned} \tag{13}$$

where  $\zeta = x/l_0$ ,  $l_0$  is the meshed element length of the equivalent continuum model.

Within the framework of the finite element method, the beam element stiffness matrix and mass matrix are constructed in the following form:

$$\begin{aligned}
\mathbf{K} &= \int_0^{l_0} \mathbf{B}^T \mathbf{D}_e \mathbf{B} dx, \\
\mathbf{M} &= \int_0^{l_0} \mathbf{N}^T \mathbf{M}_e \mathbf{N} dx,
\end{aligned} \tag{14}$$

where  $\mathbf{B}$  and  $\mathbf{N}$  are the expressions of  $N_1, N_2, \dots, N_6$ .

The global stiffness matrix and mass matrix of the whole reduced-order model are obtained by assembling the element-level stiffness and mass matrices, respectively, as

$$\begin{aligned}
\mathbf{K}_T &= \sum \mathbf{K}, \\
\mathbf{M}_T &= \sum \mathbf{M}.
\end{aligned} \tag{15}$$

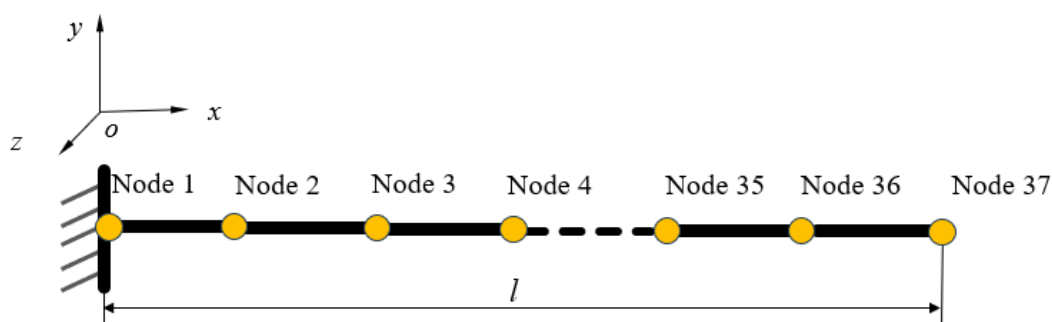


**Figure 4.** Finite element discretization of the equivalent continuum model.

### 3.3. Dynamic characteristic analysis

The finite element method is employed to construct the reduced-order model of the tower crane jib. The material and size parameters are inserted into the relevant expressions of the element stiffness and mass matrices, respectively. The displacement field of the two-node equivalent continuum beam model is defined by 16 DOFs, with each node's displacement  $\delta$  contributing 8 DOFs:

$$\delta_i = [u_i \ v_{bi} \ v_{si} \ w_{bi} \ w_{si} \ \varphi_{xi} \ \varphi_{yi} \ \varphi_{zi}]^T. \quad (16)$$

The governing equation of the reduced-order model of the tower crane jib, without considering the structural damping, can be expressed as

$$\mathbf{M}_T \ddot{\delta} + \mathbf{K}_T \delta = 0. \quad (17)$$

From the boundary conditions of the cantilever reduced-order beam model, the characteristic equation takes the following form:

$$(\mathbf{K}_T - \omega^2 \mathbf{M}_T) \delta = 0. \quad (18)$$

From Eq (18), the natural frequencies and modal shapes of the reduced-order model can be achieved to analyze structural dynamic characterizations. For the large slender tower crane jib, its first few modes are the dominant contributors to the structural vibration. The first 6 mode shapes are analyzed to verify the constructed reduced-order model of the tower crane jib.

In the next section, the natural frequencies and mode shapes obtained from the full-scale tower crane jib prototype are presented as the objective values. Furthermore, the validity of the equivalent parameters are checked by comparisons of natural frequencies and mode shapes with the original tower crane jib. In addition, the coupled vibration characteristics of the system is studied.

## 4. Comparative dynamic analysis

### 4.1. Equivalent parameters of the tower crane jib

The large slender tower crane jib is shown in Figure 2(a), which consists of 36 lattice elements. Referring to the member cross-section parameters of the actual tower crane jib, the equivalent parameters can be obtained using the reduced-order modeling proposed in this paper. The material parameters and geometric sizes of the members in the lattice element are provided in Table 1. The cross sections of longitudinal beams and inclined beams are solid circle and hollow circle, respectively.

Using the proposed reduced-order modeling method, the equivalence of the large slender tower crane jib in Section 4.1 is conducted. As shown in Figure 3, the coordinates of points A, B, and C are  $\left(\frac{3}{4}L_L, \frac{2}{3}\sqrt{l^2 - \frac{1}{4}b^2}, 0\right)$ ,  $\left(\frac{1}{4}L_L, -\frac{1}{3}\sqrt{l^2 - \frac{1}{4}b^2}, \frac{1}{2}b\right)$ , and  $\left(\frac{1}{4}L_L, -\frac{1}{3}\sqrt{l^2 - \frac{1}{4}b^2}, -\frac{1}{2}b\right)$ , respectively, based on the defined coordinate system and structural geometric dimensions. The coordinates of the remaining points on the lattice element can be derived via such conversion. The elastic matrix and inertia matrix are obtained, and the specific forms are given below:

$$\mathbf{D}_e = \begin{bmatrix} 3371942334 & 0 & -68405646 & -36483011 & 0 & -109449034 \\ 0 & 353055454 & 0 & 0 & 0 & 0 \\ -68405646 & 0 & 738444817 & 12213642 & 0 & 36483011 \\ -36483011 & 0 & 12213642 & 23491087 & 0 & -388814 \\ 0 & 0 & 0 & 0 & 138458403 & 0 \\ -109449034 & 0 & 36483011 & -388814 & 0 & 66197897 \end{bmatrix}, \quad (19)$$

$$\mathbf{M}_e = \begin{bmatrix} 182.08 & 0 & 0 & 0 & 0.34 & 0 \\ 0 & 182.08 & 0 & -0.34 & 0 & -22.50 \\ 0 & 0 & 182.08 & 0 & 22.50 & 0 \\ 0 & -0.34 & 0 & 49.52 & 1.39 & -22.30 \\ 0.34 & 0 & 22.50 & 1.39 & 61.46 & 0 \\ 0 & -22.50 & 0 & -22.30 & 0 & 44.11 \end{bmatrix}. \quad (20)$$

**Table 1.** Material parameters and geometric sizes of the tower crane jib.

| Parameters                                     | Values |
|------------------------------------------------|--------|
| Elasticity modulus $E$ (GPa)                   | 210    |
| Poisson ratio $\mu$                            | 0.3    |
| Mass density $\rho$ (kg/m <sup>3</sup> )       | 7850   |
| Longitudinal beam length $L_L$ (mm)            | 1500   |
| Inclined beam length $b$ (mm)                  | 800    |
| Inclined beam length $l$ (mm)                  | 1312   |
| Diameter of longitudinal beam $D_L$ (mm)       | 80     |
| Outer diameter of inclined beam $D_I$ (mm)     | 60     |
| Inner diameter of inclined beam $d_I$ (mm)     | 40     |
| Angle of the oblique triangular prism $\alpha$ | 53.13° |

#### 4.2. Natural frequency of the tower crane jib and reduced-order model

The equivalent continuum model with 36 beam elements along the x-direction is investigated. The vibration natural frequency is calculated using Eq (18). To validate the results, the conventional finite element model of the large slender tower crane jib is also established by implementing the commercial finite element software. The BEAM4 element is adopted when establishing the full-order finite element model, which consists of 4572 elements and 4356 nodes. Modal analysis is performed using the LANB calculation method. The results calculated using the reduced-order model and original tower crane jib finite element model are listed and compared in Table 2. The error is defined as follows:

$$\text{Error} = \frac{f_i^{RM} - f_i^{TCJ}}{f_i^{TCJ}} \times 100\%, \quad (21)$$

where  $f_i^{RM}$  and  $f_i^{TCJ}$  stand for the  $i$ th order natural frequencies calculated by the reduced-order model and finite element simulation of tower crane jib, respectively.

The data indicate that the natural frequencies obtained from the reduced-order model agrees well with that obtained from traditional finite element method for the original tower crane jib. The maximum error of the first 6 frequencies between the reduced-order model and the tower crane jib is 3.84%, which

corresponds to the first bending in  $v$ , and the vibration modes, including bending vibrations in two directions and torsion vibration, indicating that the proposed reduced-order model can effectively characterize the vibration in three dimensions; the vibration is dominated by the bending mode. Furthermore, the results in Table 2 verify that the proposed reduced-order model is applicable for the natural frequency calculation of the tower crane jib structures. According to the frequency results, it is evident that the mode order of bending vibration in the  $y$  direction is lower than those of the bending vibration in the  $z$  direction, since the  $z$  direction stiffness is stronger than the  $y$  direction stiffness, which is caused by the structural size asymmetry.

**Table 2.** Comparisons of the frequencies between the reduced-order model and the tower crane jib.

| Mode                  | Reduced-order model (Hz) | Tower crane jib (Hz) | Error (%) |
|-----------------------|--------------------------|----------------------|-----------|
| First bending in $v$  | 0.2676                   | 0.2577               | 3.84      |
| First bending in $w$  | 0.3850                   | 0.3787               | 1.74      |
| Second bending in $v$ | 1.6455                   | 1.5871               | 3.68      |
| Second bending in $w$ | 2.3699                   | 2.3335               | 1.56      |
| First torsion         | 3.1622                   | 3.1607               | 0.05      |
| Third bending in $v$  | 4.4764                   | 4.3271               | 3.45      |

#### 4.3. Mode shapes of the tower crane jib and the reduced-order model

In Section 4.2, the applicability of the established reduced-order model for calculating the dynamic behavior of the large slender tower crane jib is verified through frequency comparison. In the following section, as a numerical verification example, the proposed reduced-order model is applied to the tower crane jib for the prediction of mode shape.

The comparisons of the first 6 mode shapes are carried out between the tower crane jib and reduced-order model, as shown in Figure 5. The broken lines and full lines represent the results obtained from the tower crane jib and reduced-order model, respectively. The fifth order mode shape is torsional mode (Figure 5(e)), and the other orders are bending modes. It can be seen that the mode shapes of the reduced-order model match well with those of the tower crane jib in three directions. As such, the established reduced-order model is effective when imitating three-direction vibrations of the tower crane jib.

#### 4.4. Dynamic responses of the tower crane jib and reduced-order model

In the previous section, the natural characteristics of the full-scale tower crane jib and reduced-order model are analyzed through numerical simulations. To further validate the reduced-order model's applicability in practical engineering, dynamic responses of deflections in the  $y$ -direction and  $z$ -direction are investigated under typical harmonic excitations  $F = F_0 \sin(2\pi ft)$ . The same full-order finite element model as in Section 4.2 is employed, and the transient dynamic analysis is carried out to compute the dynamic responses of the tower crane jib.

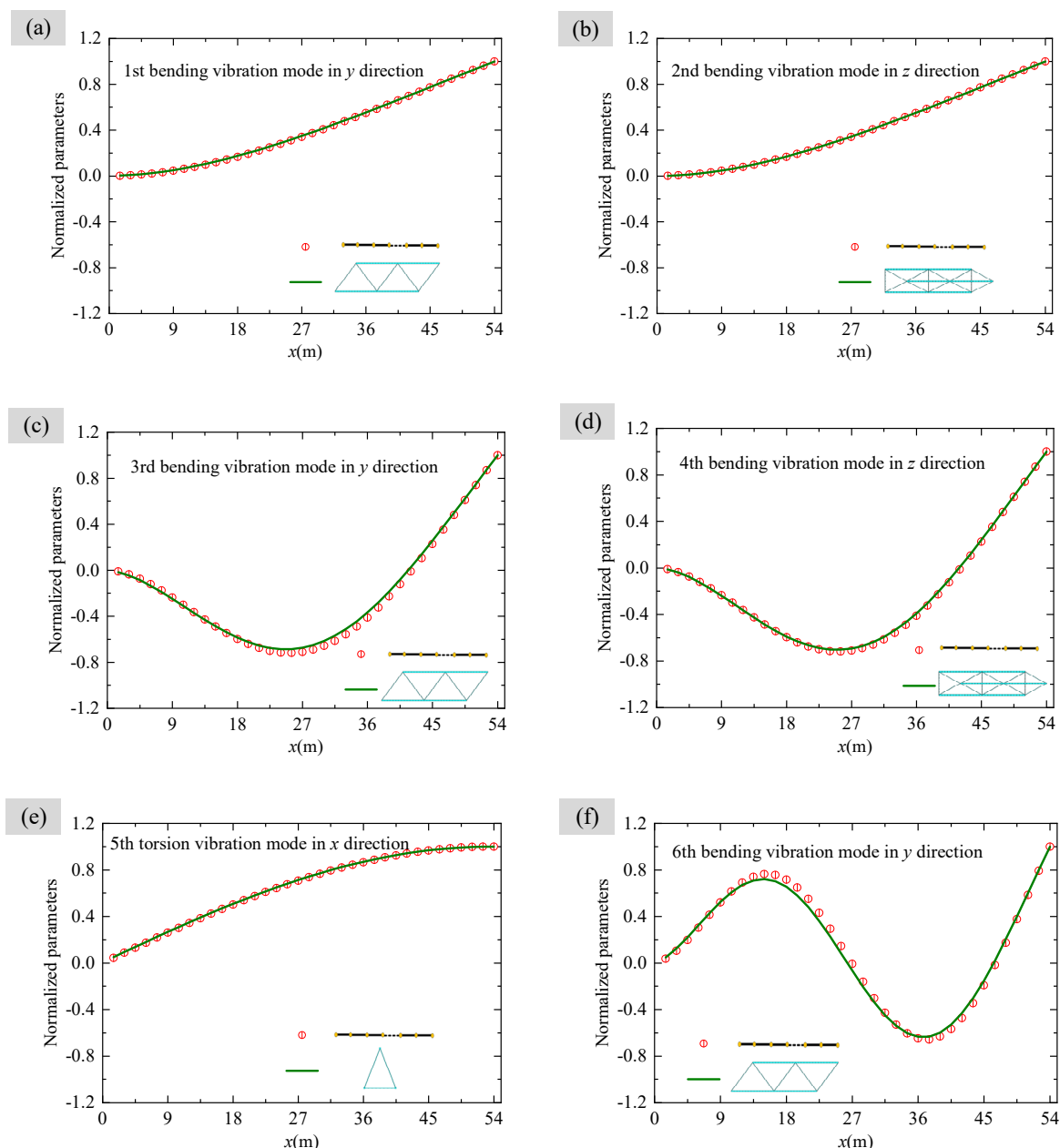


**Figure 5.** Mode shape comparisons of the tower crane jib and its reduced-order model: (a) 1st order bending mode, (b) 2nd order bending mode, (c) 3rd order bending mode, (d) 4th order bending mode, (e) 5th order torsional mode, and (f) 6th order bending mode.

Figure 6 presents the  $z$ -direction dynamic response of the tower crane jib at the free end, which is calculated using the reduced-order model and compared with full finite element model predictions. It should be noted that Rayleigh damping is adopted for the systems in this study. Subjected to harmonic external excitations with different parameters, Figure 6 shows good agreement between the time-displacement response and finite element results in terms of a waveform trend and phase for transient and steady-state stages. To fully validate the performance of the reduced-order model, comparative case studies involving varying load position and magnitude are conducted, and detailed parameter information is presented in Figure 6(a)–(d). The peak value of a steady state of tower crane

jib is close to that predicted by the reduced-order model, with a steady amplitude error of 1.9% to 3.9%, indicating that the reduced-order model accurately captures the dynamic behavior of the tower crane jib.

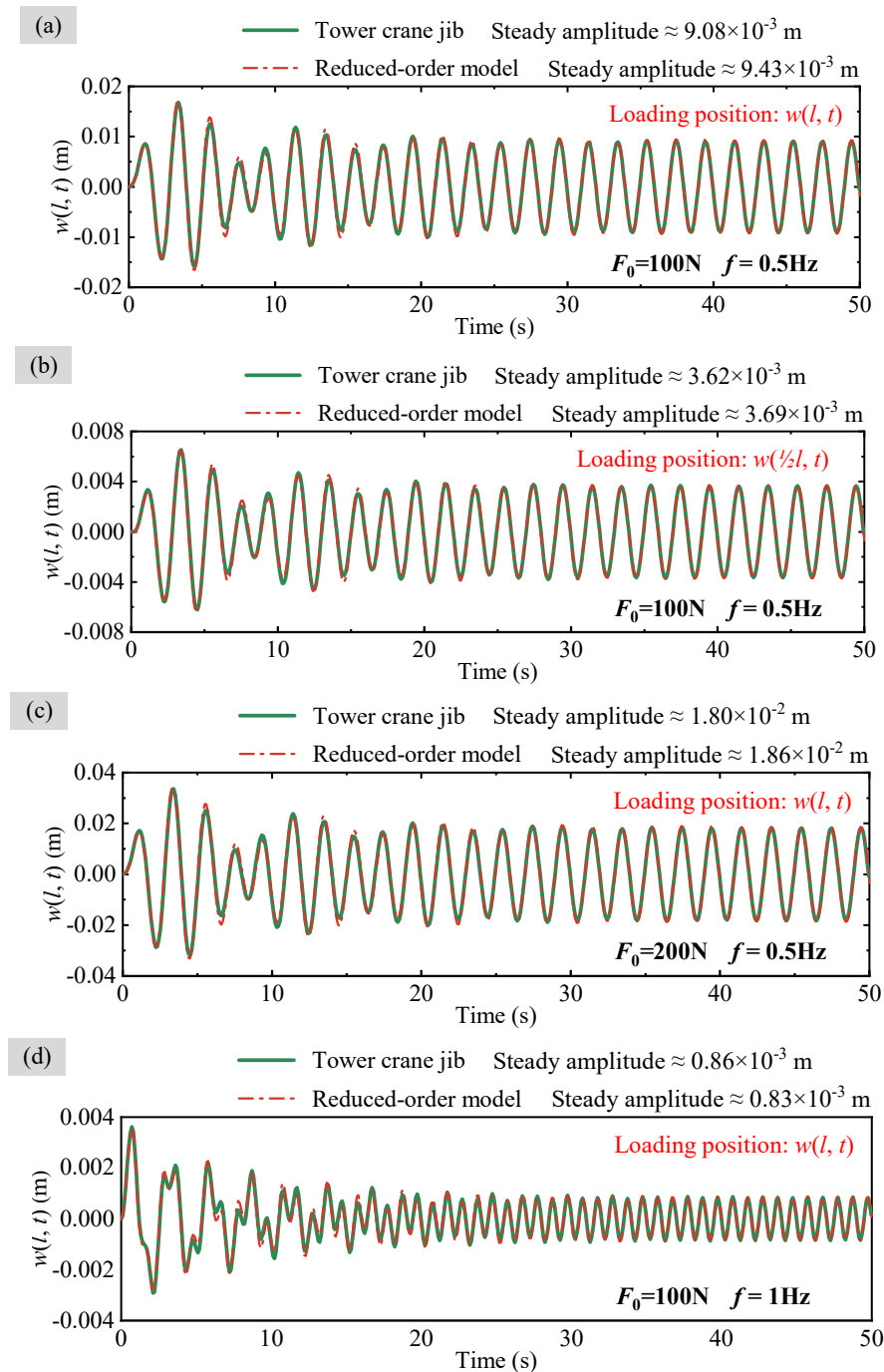


**Figure 6.** Z-direction dynamic response comparisons of the tower crane jib and its reduced-order model: (a) External excitation with magnitude  $F_0 = 100$  N,  $f = 0.5$  Hz, and position  $x = l$ , (b) External excitation with magnitude  $F_0 = 100$  N,  $f = 0.5$  Hz, and position  $x = \frac{1}{2}l$ , (c) External excitation with magnitude  $F_0 = 200$  N,  $f = 0.5$  Hz, and position  $x = l$ , and (d) External excitation with magnitude  $F_0 = 100$  N,  $f = 1$  Hz, and position  $x = l$ .

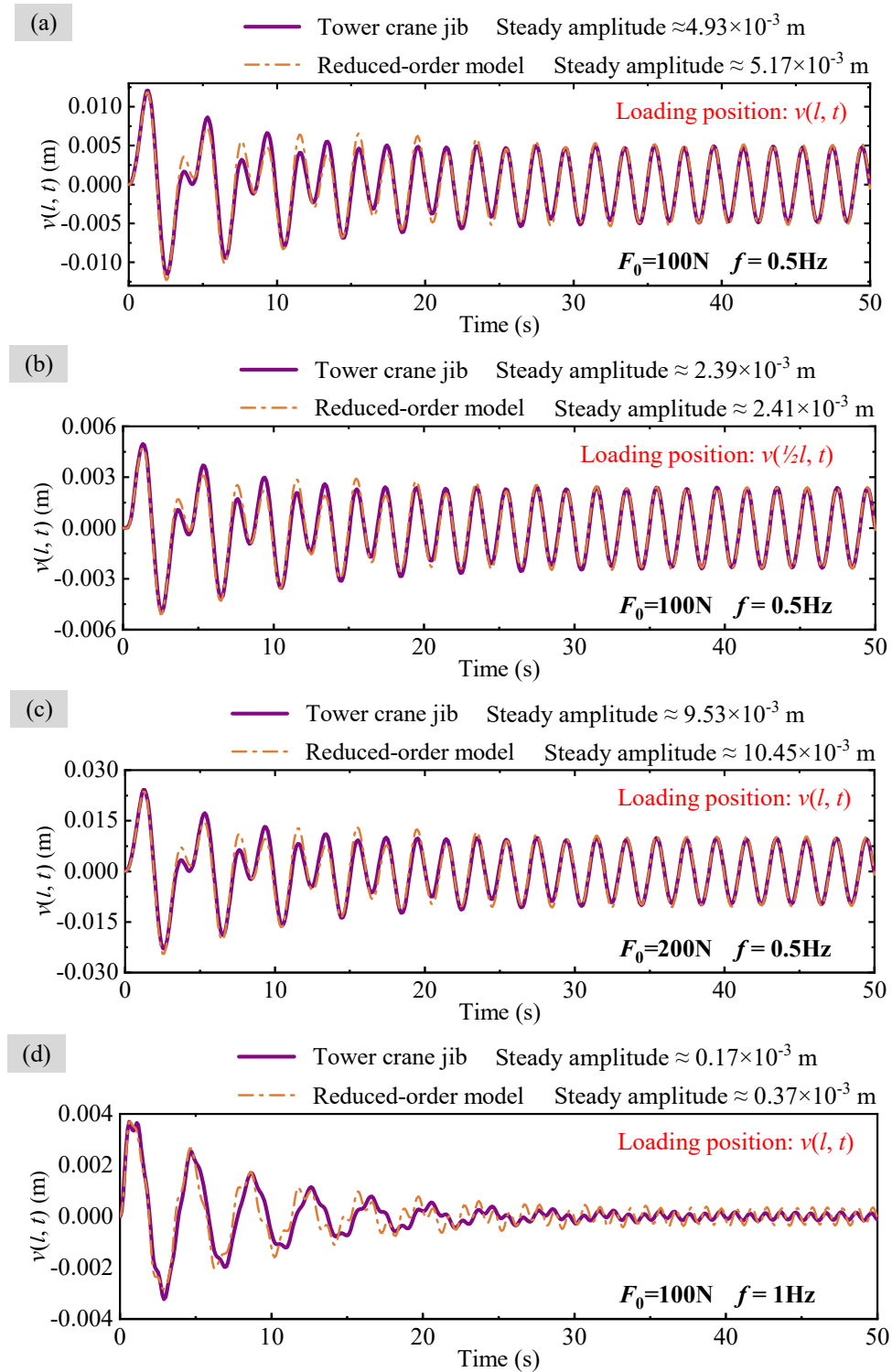


**Figure 7.** Y-direction dynamic response comparisons of the tower crane jib and its reduced-order model: (a) External excitation with magnitude  $F_0 = 100$  N,  $f = 0.5$  Hz, and position  $x = l$ , (b) External excitation with magnitude  $F_0 = 100$  N,  $f = 0.5$  Hz, and position  $x = \frac{1}{2}l$ , (c) External excitation with magnitude  $F_0 = 200$  N,  $f = 0.5$  Hz, and position  $x = l$ , and (d) External excitation with magnitude  $F_0 = 100$  N,  $f = 1$  Hz, and position  $x = l$ .

Figure 7 presents a comparison between the y-direction displacement responses of the full-order and reduced-order models at the free end when harmonic excitation with variable parameters similar to those in the z-direction is applied in the y-direction. Subfigures (a) and (b) depict the displacement responses under varying excitation position and the load application positions with  $l$  and  $\frac{1}{2}l$ , respectively. The results show that the displacement responses of the full tower crane jib model and the reduced-order model are generally consistent, with the maximum relative error at 4.9%. Figure 7(c),(d) compare the dynamic responses of the full-order model and its reduced-order when external excitation  $F_0 = 200$  N,  $f = 0.5$  Hz and  $F_0 = 100$  N,  $f = 1$  Hz are applied in the y-direction at the free end. The two responses exhibit good agreement.

Overall, the dynamic responses of the reduced-order model show good agreement with the full-order model in the main response frequency and amplitude, verifying the accuracy and reliability of the proposed model in predicting the vibration response of the tower crane jib. The reduced-order model and the full-order finite element model of the tower crane jib possess 216 and 26,136 degrees of freedom, respectively, demonstrating that the established reduced-order model exhibits a significant reduction in degrees of freedom.

#### 4.5. Coupled vibration characteristic analysis

In this subsection, the coupled vibration characteristic, which is presented in Eqs (8) and (9), is investigated to confirm the effect caused by the structural asymmetry. We analyze the natural frequency of the reduced-order model and reduced-order model without considering coupling terms, as illustrated in Table 3. The results of the actual large slender tower crane jib mode shapes are depicted in Figure 5. The rate is calculated by

$$\text{Rate} = \frac{f_i^{\text{uncoupled}} - f_i^{\text{coupled}}}{f_i^{\text{coupled}}} \times 100\%, \quad (22)$$

where  $f_i^{\text{uncoupled}}$  and  $f_i^{\text{coupled}}$  denote the  $i$ th order natural frequencies calculated by the reduced-order model without coupling terms and reduced-order model with coupling terms, respectively.

The highest rate of the first 6 order natural frequencies regarding the two models is 0.84%. It is evident that the natural frequency rate is small enough between the reduced-order model and the reduced-order model without considering coupling terms according to rate analysis.

**Table 3.** Comparisons of frequencies between the reduced-order model and the reduced-order model without considering coupling terms.

| Mode                  | Reduced-order model<br>with coupling terms (Hz) | Reduced-order model<br>without coupling terms (Hz) | Rate (%) |
|-----------------------|-------------------------------------------------|----------------------------------------------------|----------|
| First bending in $v$  | 0.2676                                          | 0.2664                                             | -0.45    |
| First bending in $w$  | 0.3850                                          | 0.3853                                             | 0.08     |
| Second bending in $v$ | 1.6455                                          | 1.6384                                             | -0.43    |
| Second bending in $w$ | 2.3699                                          | 2.3735                                             | 0.15     |
| First torsion         | 3.1622                                          | 3.1889                                             | 0.84     |
| Third bending in $v$  | 4.4764                                          | 4.4588                                             | -0.39    |

It is also interesting to investigate the mode shapes of the tower crane jib using the full-scale finite element model to analyze dominant mode shape forms, which are presented in Figure 8. Moreover, the vibration is dominated by bending mode shapes, except for the 5<sup>th</sup> order mode shape, and the coupled vibration feature caused by asymmetry of the tower crane jib is weak, which contrasts with the simple triangular truss without a coupled effect.

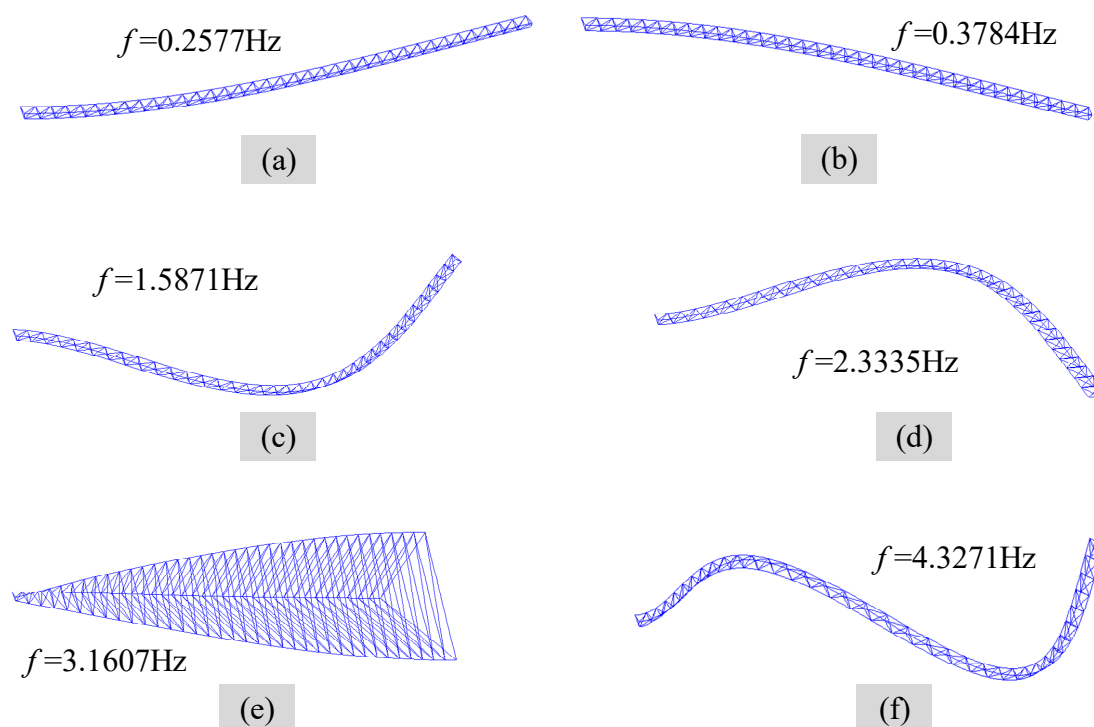


**Figure 8.** First six orders of the tower crane jib: (a) 1<sup>st</sup> order bending mode, (b) 2<sup>nd</sup> order bending mode, (c) 3<sup>rd</sup> order bending mode, (d) 4<sup>th</sup> order bending mode, (e) 5<sup>th</sup> order torsional mode, and (f) 6<sup>th</sup> order bending mode.

## 5. Conclusions

In this paper, a novel reduced-order model for a large slender tower crane jib that possesses oblique and asymmetric triangular prism configurations is proposed. Versus the conventional detailed finite element model, this reduced-order model can efficiently predict the dynamic characteristics of the tower crane jib with satisfactory accuracy. The tower crane jib structure is equivalent to anisotropic continuum beam considering three-direction vibration and includes multiple and complex coupling terms due to structural size and configuration asymmetry. The dynamic characteristics of the cantilevered equivalent continuum model are solved by adopting the finite element method. To check the accuracy and efficiency of the proposed reduced-order model, the natural characteristics and dynamic responses are compared between the original full-scale structure and its reduced-order model. Furthermore, the coupled vibration characteristic is investigated.

Numerical results show that the error of the first 6 order natural frequencies between the reduced-order model and the tower crane jib is no more than 3.84%. Additionally, the mode shape comparisons have good consistency. Moreover, under harmonic excitation loads, the dynamic responses of the full-

order model, and the proposed reduced-order model exhibit good consistency. Parametric studies varying load amplitude and action location validate the precision and reliability of the presented reduced-order model. The results indicate that the proposed reduced-order model can provide a sufficient prediction accuracy on dynamic characteristics for the large slender tower crane jib. Comparing this with the reduced-order model without considering the coupling terms, it can be concluded that the coupled vibration is weak to a certain extent and its effect can be ignored under some circumstances. These insights offer a reference for researchers when addressing a truss-type structural nonlinear vibration mechanism and active vibration control problems.

### Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

### Acknowledgments

This research was supported by the National Natural Science Foundation of China (Grant No. 12302009) and the Natural Science Foundation of Heilongjiang Province (Grant No. LH2022A007).

### Conflict of interest

The authors declare there is no conflict of interest.

### References

1. M. H. El Ouni, N. Ben Kahla, S. Islam, M. Jameel, A smart tower crane to mitigate turbulent wind loads, *Struct. Eng. Int.*, **31** (2021), 18–29. <https://doi.org/10.1080/10168664.2019.1668334>
2. L. Ramli, Z. Mohamed, H. I. Jaafar, A neural network-based input shaping for swing suppression of an overhead crane under payload hoisting and mass variations, *Mech. Syst. Signal Process.*, **107** (2018), 484–501. <https://doi.org/10.1016/j.ymssp.2018.01.029>
3. H. Jiang, Y. Li, Dynamic reliability analysis of tower crane with wind loading, *IOP Conf. Ser. Mater. Sci. Eng.*, **677** (2019), 052031. <https://doi.org/10.1088/1757-899X/677/5/052031>
4. W. Chen, X. Qin, Z. Yang, P. Zhan, Wind-induced tower crane vibration and safety evaluation, *J. Low Freq. Noise Vib. Act. Control*, **39** (2020), 297–312. <https://doi.org/10.1177/1461348419847306>
5. H. Chen, Y. Fang, N. Sun, An adaptive tracking control method with swing suppression for 4-DOF tower crane systems, *Mech. Syst. Signal Process.*, **123** (2019), 426–442. <https://doi.org/10.1016/j.ymssp.2018.11.018>
6. H. Kenan, O. Azeloğlu, Design of scaled down model of a tower crane mast by using similitude theory, *Eng. Struct.*, **220** (2020), 110985. <https://doi.org/10.1016/j.engstruct.2020.110985>
7. J. Ye, J. Huang, Analytical analysis and oscillation control of payload twisting dynamics in a tower crane carrying a slender payload, *Mech. Syst. Signal Process.*, **158** (2021), 107763. <https://doi.org/10.1016/j.ymssp.2021.107763>
8. M. Zhang, X. Jing, Z. Zhu, Disturbance employment-based sliding mode control for 4-DOF tower crane systems, *Mech. Syst. Signal Process.*, **161** (2021), 107946. <https://doi.org/10.1016/j.ymssp.2021.107946>

9. F. Liu, D. Jin, X. Li, G. Wei, Equivalent continuum modeling method for transient response analysis of large space truss structures with nonlinear elastic joints, *Acta Mech.*, **234** (2023), 3499–3517. <https://doi.org/10.1007/s00707-023-03565-8>
10. M. Liu, D. Cao, X. Zhang, J. Wei, D. Zhu, Nonlinear dynamic responses of beamlike truss based on the equivalent nonlinear beam model, *Int. J. Mech. Sci.*, **194** (2021), 106197. <https://doi.org/10.1016/j.ijmecsci.2020.106197>
11. S. Cao, M. Huo, N. Qi, C. Zhao, D. Zhu, L. Sun, Extended continuum model for dynamic analysis of beam-like truss structures with geometrical nonlinearity, *Aerosp. Sci. Technol.*, **103** (2020), 105927. <https://doi.org/10.1016/j.ast.2020.105927>
12. H. Guo, C. Shi, M. Li, Z. Deng, R. Liu, Design and dynamic equivalent modeling of double-layer hoop deployable antenna, *Int. J. Aerosp. Eng.*, **2018** (2018), 1–15. <https://doi.org/10.1155/2018/2941981>
13. F. Liu, L. Wang, D. Jin, H. Wen, Equivalent continuum modeling of beam-like truss structures with flexible joints, *Acta Mech. Sin.*, **35** (2019), 1067–1078. <https://doi.org/10.1007/s10409-019-00872-z>
14. A. K. Noor, M. P. Nemeth, Micropolar beam models for lattice grids with rigid joints, *Comput. Methods Appl. Mech. Eng.*, **21** (1980), 249–263. [https://doi.org/10.1016/0045-7825\(80\)90034-1](https://doi.org/10.1016/0045-7825(80)90034-1)
15. Y. Tang, X. Liu, G. Cai, X. Liu, C. You, S. Yao, Active vibration control based on the equivalent dynamic model of a large space telescope truss structure, *Int. J. Dyn. Control*, **11** (2023), 1718–1735. <https://doi.org/10.1007/s40435-022-01098-x>
16. J. Sun, J. Sun, D. Zhu, X. Yan, Equivalent plate dynamic modeling of space periodic truss structures, *Aerosp. Sci. Technol.*, **138** (2023), 108315. <https://doi.org/10.1016/j.ast.2023.108315>
17. X. Li, G. Wei, J. Guo, F. Liu, D. Jin, Multi-harmonic equivalent dynamic model of the gyroelastic repetitive truss with nonlinear joints, *Acta Mech.*, **234** (2023), 3361–3375. <https://doi.org/10.1007/s00707-023-03560-z>
18. S. Jiang, G. P. Cai, Equivalent beam model and improved structure design of large space antenna truss with geometric nonlinearity, *J. Comput. Nonlinear Dyn.*, **18** (2023), 051003. <https://doi.org/10.1115/1.4057043>
19. J. Chen, W. Zhang, Y. F. Zhang, Equivalent continuum model and nonlinear breathing vibrations of rotating circular truss antenna subjected to thermal excitation, *Thin-Walled Struct.*, **157** (2020), 107127. <https://doi.org/10.1016/j.tws.2020.107127>
20. B. Li, Y. Wang, Y. Liu, J. Tao, H. Ren, H. Yang, Equivalent continuum modeling for flexible slender quadrilateral truss structure, *Machines*, **12** (2024), 111. <https://doi.org/10.3390/machines12020111>
21. T. Yuan, W. Fan, H. Ren, Y. Liu, Efficient reduced order modeling for nonlinear dynamic analysis of beamlike truss structures, *Aerosp. Sci. Technol.*, **159** (2025), 109952. <https://doi.org/10.1016/j.ast.2025.109952>
22. I. A. Martin, R. A. Irani, Dynamic modeling and self-tuning anti-sway control of a seven degree of freedom shipboard knuckle boom crane, *Mech. Syst. Signal Process.*, **153** (2021), 107441. <https://doi.org/10.1016/j.ymssp.2020.107441>
23. R. C. Roman, R. E. Precup, E. M. Petriu, Hybrid data-driven fuzzy active disturbance rejection control for tower crane systems, *Eur. J. Control*, **58** (2021), 373–387. <https://doi.org/10.1016/j.ejcon.2020.08.001>
24. R. E. Samin, Z. Mohamed, Comparative assessment of anti-sway control strategy for tower crane system, *AIP Conf. Proc.*, **1883** (2017), 020035. <https://doi.org/10.1063/1.5002053>

25. Z. Sun, H. Ouyang, Fuzzy adaptive controller design for double-pendulum tower crane with distribute mass payload, preprint, (2021), 1–8. <https://doi.org/10.21203/rs.3.rs-911408/v1>
26. M. Zhang, Y. Zhang, H. Ouyang, C. Ma, X. Cheng, Adaptive integral sliding mode control with payload sway reduction for 4-DOF tower crane systems, *Nonlinear Dyn.*, **99** (2020), 2727–2741. <https://doi.org/10.1007/s11071-020-05471-3>
27. F. Rauscher, O. Sawodny, Modeling and control of tower cranes with elastic structure, *IEEE Trans. Control Syst. Technol.*, **29** (2021), 64–79. <https://doi.org/10.1109/TCST.2019.2961639>
28. S. Hric, J. Tkac, D. Matiskova, M. Micko, D. Mandulak, Preliminary analysis of tower crane as a type of truss structure, *MATEC Web Conf.*, **299** (2019), 03003. <https://doi.org/10.1051/mateconf/201929903003>
29. H. Kenan, C. O. Azeloglu, Seismic performance analysis of tower crane mast types with different bracing configurations, *Structures*, **34** (2021), 286–302. <https://doi.org/10.1016/j.istruc.2021.07.052>
30. G. Yao, H. Li, Y. Yang, W. Pu, Seismic responses and dynamic characteristics of boom tower crane basing on measured strong earthquake excitation, *J. Vibroeng.*, **21** (2019), 154–169. <https://doi.org/10.21595/jve.2018.19626>
31. Q. Liu, R. Zhang, S. Sun, J. Zhang, Aerodynamic load variation and wind-induced vibration analysis of tower cranes under full wind angles, *Int. J. Acoust. Vib.*, **29** (2024), 101–115. <https://doi.org/10.20855/ijav.2024.29.22016>
32. S. Sun, Q. Zhang, J. Zhang, X. Zhang, Q. Meng, Tower cranes mixed H2/H $\infty$  under wind load research on active vibration control, *Proc. Inst. Mech. Eng., Part E J. Process Mech. Eng.*, **240** (2026), 4200–4214. <https://doi.org/10.1177/09544089241302963>
33. Y. Cao, D. Cao, G. He, X. Ge, Y. Hao, Vibration analysis and distributed piezoelectric energy harvester design for the L-shaped beam, *Eur. J. Mech. A Solids*, **87** (2021), 104214. <https://doi.org/10.1016/j.euromechsol.2021.104214>
34. Y. Cao, D. Cao, G. He, Y. Hao, X. Ge, Nonlinear dynamic modelling and analysis of multiple thin plates connected by long hinges, *Nonlinear Dyn.*, **110** (2022), 1199–1222. <https://doi.org/10.1007/s11071-022-07726-7>
35. M. Z. Akber, W. K. Chan, H. H. Lee, G. A. Anwar, TPE-optimized DNN with attention mechanism for prediction of tower crane payload moving conditions, *Mathematics*, **12** (2024), 3006. <https://doi.org/10.3390/math12193006>



AIMS Press

©2026 the Author(s), licensee AIMS Press. This is an open access article distributed under the terms of the Creative Commons Attribution License (<http://creativecommons.org/licenses/by/4.0>)