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Research article

An integrated Laplace transform and accelerated Adomian decomposition approach for solving time-fractional nonlinear partial differential equations

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Abstract: This study focuses on the efficient and accurate solution of time-fractional nonlinear partial differential equations (PDEs), which arise in various scientific and engineering applications but are often challenging to solve due to their complexity. We propose a novel method that integrates the Laplace transform with the accelerated Adomian decomposition method (AADM), forming the Laplace transform accelerated Adomian decomposition method (LAADM). The key innovation of this approach lies in its ability to handle the fractional time derivative expressed in the Caputo sense, while enhancing convergence speed and reducing computational effort. The methodology is systematically formulated, and several numerical experiments are conducted to validate its performance. The results demonstrate that LAADM provides highly accurate solutions and exhibits superior efficiency when compared to traditional solution techniques for fractional PDEs.

Keywords: fractional Laplace transform; nonlinear partial fractional differential equations; Laplace transform accelerated Adomian; convergence, error analysis uniqueness; accuracy

1. Introduction

A widely recognized field in mathematics called non-integer calculus utilizes operators of fractional order to simulate and explain physical phenomena. The use of non-integer order derivatives to solve integral and differential problems is another aspect of this domain. When the order is zero, the derivative is retrieved. A fractional derivative satisfies certain requirements and has a non-integer order. Podlubny [1] provided a summary of fractional differential equations, fractional derivatives, and some of their applications. In 1695, the conceptualization of a fractional derivative was originally proposed by Leibniz, a famous mathematician. The equations that Leibniz resolved included integrals of non-integer order and derivatives of fractional order. Because fractional calculus has numerous applications and is frequently utilized in a diversity of complex nonlinear fluid dynamics systems, researchers have become increasingly interested in it. Aguilar and Atangana [2] utilized the Hunter-Saxton fractional equation with partial operators. To examine the motion dynamics of a mass-spring system that accelerates using fractional calculus. The traditional equations of motion were derived from the integer-order Euler-Lagrange equations, which yield the corresponding Lagrangian. Additionally, numerical solutions to Euler-Lagrange equations for fractional variables were developed in Defterli et al. In [3], authors used the fractional treatment. to investigate the primary function of the liver by developing mathematical illustrations that faithfully model it. The study used the exponential kernel with the Caputo-Fabrizio fractional derivative to suggest a novel human liver model. Baleanu et al. [4] used Caputo Fabrizio fractional derivative to mathematically simulate the human liver. In several applied scientific disciplines, like fluid mechanics, mathematical biology, quantum physics, chemical kinetics, and linear optics, the purpose of PDEs is to simulate numerous physical phenomena.

In applied mathematics, physics, and engineering-related problems, nonlinear phenomena are significant. Nonlinear issues are extremely challenging to solve, and finding an analytical solution is even more challenging. A number of approaches have been proposed to find approximate solutions to equations. The δ-expansion method, presented in Karmishin et al. [5], provided techniques for testing and calculating the dynamics of thin-walled structures. Hirota bilinear method, presented by Hirota [6], offered exact solutions to the Korteweg-de Vries equation involving solitons colliding. The small parameter method by Lyapunov [7] addressed the problem of overall stability of motion, as further developed by Francis and Taylor. He [8] presented the homotopy perturbation approach. In the field of engineering and physical sciences, Sweilam and Khader [9] applied the homotopy perturbation method to obtain exact solutions for certain paired nonlinear partial differential equations. Nadjafi and Ghorbani [10] demonstrated that He's homotopy perturbation method is a useful technique for solving nonlinear integral and integro-differential equations. The semi-inverse method, explored by Wu and He [11] investigated fractional calculus of variations in fractal spacetime. The Adomian decomposition method (ADM), proposed by Adomian [12] has been successfully used to solve complex physical phenomena. Adomian [13] suggested using deconstruction, computers, and mathematics to solve physical problems. The Laplace decomposition technique, developed by Khan and Austin [14], provides a framework for solving nonlinear homogeneous and non-homogeneous advection equations.

Fractional PDEs of order three can be resolved using a variety of techniques. The classical Riccati equations method, presented by Kocak and Pinar [15], provides solutions for fifth-order dispersive equations with porous medium-type linearity. The fractional variational iteration method (FVIM), suggested by Prakash and Kumar [16], offers a study numerical approach for solving partial differential equations with fractional dispersiveness. The modified fractional differential transform method (MFDTM) and fractional differential transform method (FDTM), introduced by Kanth and Aruna [17], are applied to solve fractional third-order dispersive partial differential equations. The homotopy analysis Sumudu transform method (HASTM), proposed by Pandey and Mishra [18], combines homotopy and Sumudu transform to solve dispersive fractional third-order partial differential equations. The spline method (SM), presented by Sultana et al. [19], proposes a new non-polynomial spline technique for solving third-order dispersive equations, both linear and nonlinear. The Laplace Adomian decomposition method (LADM) by Jafari et al. [20] offers an approach for solving linear and nonlinear fractional diffusion-wave equations using the Laplace decomposition technique. Yang and Li [21] introduced a novel fractional-order financial system that accounts for non-constant elasticity of demand and analyzed its complex dynamics. Also, Araz et al. [22] extended a parametrized approach to find solutions to differential equations with fractal, fractional, fractal-fractional, and piecewise derivatives with the inclusion of a stochastic component. The existence and uniqueness of the solution to the stochastic Atangana-Baleanu fractional differential equation were established using Caratheodory's existence theorem. Guo et al. [23] employed the method of sequence approximation to find minimal and maximal positive solutions for nonlinear singular p-Laplacian Hadamard fractional differential equations.

This paper presents and examines a hybrid methodology utilizing the Laplace integral transform and the accelerated Adomian decomposition method (AADM) for solving nonlinear fractional partial differential equations (FPDEs). The technique does not require a predefined size declaration, making it an ideal technique for equations representing nonlinear models; also, it requires no discretization or linearization and has fewer parameters than other analytical approaches. We successfully attempted to use LAADM to acquire the analytical derivations of dispersive FPDEs in light of the literature mentioned above. The LAADM's findings are intriguing and more closely aligned with precise answers to the issues. Before introducing the technique, some terms and ideas need to be defined.

The remaining word is organized is as follows. Section 2 reviews essential definitions and properties of Riemann-Liouville and Caputo fractional integers. Section 3 presents the proposed numerical methodology and its algorithmic implementation. Section 4 establishes a rigorous convergence analysis of the method. Section 5 demonstrates the efficacy of the approach through five illustrative examples of time-fractional nonlinear PDEs, complete with graphical and tabular comparisons. Finally, Section 6 concludes with remarks and future research directions.

2. Definitions and preliminaries

This section outlines key definitions and essential computational concepts from fractional calculus relevant to this study [24–26]. It includes discussions on the Riemann-Liouville fractional integrals and derivatives, Riesz fractional integrals, Caputo fractional derivatives, the Laplace transform, and the gamma function.

Definition 2.1: The gamma function, represented by Γ, provides a smooth generalization of the factorial function to non-negative integers, according to Abramowitz and Stegun (Chapter 6 in [27]). Then, $\Gamma(\alpha)$ for $\alpha > 0$, has the following expression:

$$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x}\, dx, \text{ where } \alpha > 0. \tag{2.1}$$

Some properties of the gamma function are as follows:

i. $\Gamma(\alpha) = (\alpha-1)\Gamma(\alpha-1)$

ii. $\Gamma(1/2) = \sqrt{\pi}$

iii. $\Gamma(\alpha+1) = \alpha!$ (2.2)

iv. $\Gamma(\alpha+1)=\alpha\Gamma(\alpha)$

v. $\Gamma(\alpha)=\frac{\Gamma(\alpha+1)}{\alpha}$.

Definition 2.2: A real-valued multivariable function $u(x,t)$, with $t>0$, is allegedly in the space C_μ , $\mu \in R$, with respect to t, are as follows: a real number exists $q(>\mu)$, such that $u(x,t)=t^q u_1(x,t)$ and $u_1(x,t)\in C(\omega\times T\ )$, and clearly, $C_\mu\subset C_\varepsilon$ if $\varepsilon\le\mu$.

Definition 2.3: The fractional integral of Riemann-Liouville $I^\alpha u$, of function $u(x,t)\in C_\mu$ where $\mu\ge -1$, of order $\alpha\in R$ and $\alpha\ge 0$, has the following definition:

$$I_t^\alpha u(x,t)=\frac{1}{\Gamma(\alpha)}\int_0^t (t-\tau)^{\alpha-1}u(\mathrm{x},\tau)\mathrm{dt},\ \alpha>0, t\in T \tag{2.3}$$

$$I_t^0 \mathrm{u}(x,t)=\mathrm{u}(x,t) \tag{2.4}$$

The operator's I^α properties are given by

$$\begin{aligned}&\text{i. } I^\alpha I^\beta u(x,t)=I^{\alpha+\beta}u(x,t)\\ &\text{ii.} I^\alpha I^\beta u(x,t)=I^\beta I^\alpha u(x,t)\\ &\text{iii. } I^\alpha x^\gamma=\frac{\Gamma(\gamma+1)}{\Gamma(\alpha+\gamma+1)}x^{\alpha+\gamma}\end{aligned} \tag{2.5}$$

Definition 2.4: The Riemann-Liouville of fractional derivative of the order α, where $n-1\le\alpha<n$, and $\mathrm{n}\in Z^+$ of a function $u(x,t)\in C_\mu$, $\mu\ge -1$, is given as:

$${}_0D_t^\alpha u(x,t)=\frac{1}{\Gamma(n-\alpha)}\frac{\partial^n}{\partial t^n}\int_0^t (t-\tau)^{n-\alpha-1}u(x,\tau)\,d\tau, \tag{2.6}$$

$${}_tD_{t_0}^\alpha u(x,t)=\frac{1}{\Gamma(n-\alpha)}\frac{\partial^n}{\partial t^n}\int_t^{t_0} (\tau-t)^{n-\alpha-1}u(x,\tau)\,d\tau,\ t\ \ \in T. \tag{2.7}$$

Lemma 2.1: The integral form of Riemann-Liouville fractional-order derivative of order α, where $0<\alpha<1$, is denoted by:

$$\int_T u(x,t)\ {}_0D_t^\alpha v(x,t)dt=\int_T v(x,t)\ {}_tD_{t_0}^\alpha u(x,t)dt, \tag{2.8}$$

provided that $\mathrm{u},\mathrm{v}\in C(\omega\times \mathrm{T})$, and that for arbitrarily $\mathrm{x}\in\omega$, both ${}_\mathrm{t}\mathrm{D}_{\mathrm{t}_0}^\alpha \mathrm{u}$ and ${}_0D_t^\alpha v$ exist continuously in t and at all points $\mathrm{t}\in \mathrm{T}$.

Definition 2.5: The fractional integral of Riesz of order α, where $n-1\le\alpha<n$, of a function $u\in C_\mu$, $\mu\ge -1$, is defined as follows:

$${}_0^R\mathrm{I}_t^\alpha u(x,t)=\frac{1}{2}\left({}_0\mathrm{I}_t^\alpha u(x,t)+\ {}_tI_{t_0}^\alpha u(x,t)\right)=\frac{1}{2\Gamma(\alpha)}\int_0^{t_0}|t-\tau|^{\alpha-1}u(x,\tau)\mathrm{d}\tau,\ \alpha>0, x>0, \tag{2.9}$$

where the fractional integral operators of Riemann-Liouville on the left and right sides are denoted by ${}_0\mathrm{I}_t^\alpha$ and ${}_tI_{t_0}^\alpha$, respectively.

Definition 2.6: The fractional derivative of Riesz of order α, where $n-1\le\alpha<n$, of a function $u\in C_\mu$, $\mu\ge -1$, is described by

$$ {}_{0}^{R}\mathrm{I}_{t}^{\alpha}u(x,t)=\frac{1}{2}\left({}_{0}\mathrm{D}_{t}^{\alpha}u(x,t)+(-1)^{n}\ {}_{t}D_{t_0}^{\alpha}u(x,t)\right)=\frac{1}{2\Gamma(n-\alpha)}\frac{d^n}{dt^n}\int_0^{t_0}|t-\tau|^{\mathrm{n}-\alpha-1}\,u(x,\tau)\mathrm{d}\tau, \quad (2.10) $$

where ${}_{0}\mathrm{D}_{t}^{\alpha}$ and ${}_{t}D_{t_0}^{\alpha}$ are the fractional differential operators of Riemann-Liouville on the left and right sides, respectively.

Lemma 2.2: Let $\beta>0$, and $\gamma>0$, $r-1\leq\beta<r$, $s-1\leq\boldsymbol{\gamma}<s$, and $\beta+\gamma<r$, and let $\mathrm{u}\in L_1C(\omega\times\mathrm{T})$ and ${}_{0}\mathrm{I}_{t}^{s-\beta}u\in C^{s}(\omega\times\mathrm{T})$. Then, the index rule is as follows:

$$ {}_{0}^{R}\mathrm{D}_{t}^{\beta}\left({}_{0}^{R}\mathrm{D}_{t}^{\gamma}u(x,t)\right)={}_{0}^{R}\mathrm{D}_{t}^{\beta+\gamma}u(x,t)-\sum_{i=1}^{s}{}_{0}^{R}\mathrm{D}_{t}^{\gamma-i}u(x,t)\Big|_{t=0}\frac{t^{-\boldsymbol{\beta}-i}}{\Gamma(1-\boldsymbol{\beta}-i)}. \quad (2.11) $$

Remark 2.1: The fractional differential operator ${}_{0}^{R}\mathrm{D}_{t}^{\alpha-1}$ of Riesz of the order α, where $0\leq\alpha<1$, according to the Riesz fractional integral of the operator ${}_{0}^{R}\mathrm{I}_{t}^{1-\alpha}$, is as follows:

$$ {}_{0}^{R}\mathrm{D}_{t}^{\alpha-1}u(x,t)={}_{0}^{R}\mathrm{I}_{t}^{1-\alpha}u(x,t)\ \ t\in T. \quad (2.12) $$

Definition 2.7: [28] The Caputo operator for a fractional derivative of the order α of a function $u\in C_{\mu}$, $\mu\geq-1$ is denoted as follows:

$$ D_t^{\alpha}u(x,t)=\frac{\partial^{\alpha}u(x,t)}{\partial t^{\alpha}}=\begin{cases}\frac{1}{\Gamma(n-\alpha)}\int_0^t(t-\tau)^{n-\alpha-1}\frac{\partial^{\mathrm{n}}u(x,t)}{\partial\tau^{\mathrm{n}}}d\tau, n-1\leq\alpha<n, n\in\mathrm{N}\\ \frac{\partial^{\mathrm{n}}u(x,t)}{\partial t^{\mathrm{n}}},\alpha=\mathrm{n}.\end{cases} \quad (2.13) $$

Definition 2.8: [29,30] The Laplace transform is a powerful integral transform that changes a function of time into a complex variable's function. Assuming that the function is $u(x,t)$, then the Laplace transform, with respect to the time t defined by $\mathcal{L}\{u(x,t)\}$, is as follows:

$$ \mathcal{L}\{u(x,t)\}=U(x,s)=\int_0^{\infty}e^{-sx}u(x,t)dt, \quad (2.14) $$

given the integral convergence.

The inverse Laplace transform is defined as follows:

$$ \mathcal{L}^{-1}U(x,s)=\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}e^{sx}U(x,s)ds. \quad (2.15) $$

Definition 2.9: [25] The following series notation defines the Mittag-Leffler function $E_{\alpha}(t)$ with $\alpha>0$, which is valid throughout the complex plane.

$$ E_{\alpha}(t)=\sum_{i=0}^{\infty}\frac{t^i}{\Gamma(i\alpha+1)}. \quad (2.16) $$

Definition 2.10: [31] The Laplace transform $\mathcal{L}$, with respect to the time t applied to the Riemann-Liouville time-fractional integral I_t^{α} on a function $u(x,t)$ takes the following form:

$$ \mathcal{L}[I_t^{\alpha}u(x,t)]=s^{-\alpha}\mathcal{L}[u(x,t)]. \quad (2.17) $$

Definition 2.11: [31] The Laplace transform $\mathcal{L}$, with respect to the time t of fractional derivative D_t^{α} in the sense of Caputo on a function $u(x,t)$ takes the form:

$$ \mathcal{L}[D_t^{\alpha}u(x,t)]=s^{\alpha}\mathcal{L}[u(x,t)]-\sum_{i=0}^{n-1}s^{\alpha-\mathrm{i}-1}\,u^{(i)}(x,0),\ n-1\leq\alpha<n \quad (2.18) $$

where $u^{(i)}(x,0)$ is the initial condition.

Definition 2.12: [32] With the continuous differentiable function $f(t): R^{+} \longrightarrow R$, the short memory fractional derivative can be defined as:

$$D_t^{\alpha} u(x,t) = \frac{1}{\Gamma(n-\alpha)} \int_{\varphi_\ell(t)}^{t} (t-\tau)^{n-\alpha-1} u^n(x,\tau) d\tau, \tag{2.19}$$

where $n-1 \leq \alpha < n$, and $\varphi_\ell(t)$ is denoted as

$$\varphi_\ell(t) = \begin{cases} t_0, t_0 \leq t \leq t_0 + \ell, \\ t-\ell, \ t > t_0 + \ell, \end{cases} \tag{2.20}$$

and ℓ is the length of the chosen memory.

3. Methodological analysis

The Laplace-Adomian decomposition method (LADM) is a powerful analytical technique that combines the Adomian decomposition method with the Laplace transform to address nonlinear fractional partial differential equations. Its flexibility allows it to be applied across various mathematical models, including algebraic, differential, integral, and integro-differential equations. Recent studies have highlighted LADM's capability in handling complex nonlinear systems. For instance, Gaxiola [33] applied it to the Kundu-Eckhaus equation, while Shah et al. [34] used it to analytically solve third-order dispersive fractional PDEs. A more efficient version, the LAADM, integrates the accelerated Adomian technique with the Laplace transform, as shown by Ramadan and Abd El-Latif [35] in their work on nonlinear fractional differential equations.

In this work, we apply both LADM and LAADM for solving a nonlinear partial differential equation of fractional order in the following form:

$$\frac{\partial^{\alpha} u(x,t)}{\partial t^{\alpha}} + Ru(x,t) + N\frac{\partial^{n} u(x,t)}{\partial x^{n}} = f(x,t), \text{where } x,t \geq 0, n-1 < \alpha \leq n, n \in \mathcal{N}. \tag{3.1}$$

where ${}_0^cD_t^{\alpha} = \frac{\partial^{\alpha}}{\partial t^{\alpha}}$ represents the Caputo fractional derivative operator, R is restricted to linear differential operators with bounded coefficients, and N are nonlinear terms that can be handled by accelerated Adomian polynomials and Adomian polynomials. f is the source term, and u depends on x, t, which can be obtained by utilizing the initial and boundary conditions listed below.

$$u(x,0) = g(x), \tag{3.2}$$

$$u(0,t) = h_0(t), \frac{\partial u(0,t)}{\partial x} = h_1(t), \frac{\partial^2 u(0,t)}{\partial x^2} = h_2(t), \dots, \frac{\partial^{n-1} u(0,t)}{\partial x^{n-1}} = h_{n-1}(t), t > 0. \tag{3.3}$$

3.1. Solution approach via the LADM

To discuss the solution of Eq (3.1) using LADM, we apply the following steps:

Step1: By applying the Laplace transform, with respect to the time t for each side of Eq (3.1) and the linearity of Laplace transform, we find:

$$\mathcal{L}[\frac{\partial^{\alpha}u(x,t)}{\partial t^{\alpha}}] = \mathcal{L}[f(x,t)] - \mathcal{L}\left[Ru(x,t) + N\frac{\partial^{n}u(x,t)}{\partial x^{n}})\right]. \tag{3.4}$$

Step 2. By assuming the concept of Caputo-sense's Laplace transform for fractional derivatives Eq (2.18), we observe that:

$$s^{\alpha}\mathcal{L}[u(x,t)] - \sum_{i=0}^{n-1} s^{\alpha-\mathrm{i}-1}\, u^{(i)}(x,0) = \mathcal{L}[f(x,t)] - \mathcal{L}\left[Ru(x,t) + N\frac{\partial^{n}u(x,t)}{\partial x^{n}}\right]. \tag{3.5}$$

Then, we have

$$U(x,s) = \frac{g(x)}{s} + \frac{1}{s^{\alpha}}\sum_{i=1}^{n-1} s^{\alpha-\mathrm{i}-1}\, u^{(i)}(x,0) + \frac{1}{s^{\alpha}}\mathcal{L}[f(x,t)] - \frac{1}{s^{\alpha}}\mathcal{L}\left[Ru(x,t) + N\frac{\partial^{n}u(x,t)}{\partial x^{n}}\right]. \tag{3.6}$$

Step 3. So, the solution $u(x,t)$, is determined by the series

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t), \tag{3.7}$$

and the nonlinear term's decomposition is

$$N\frac{\partial^{n}u(x,t)}{\partial x^{n}} = \sum_{n=0}^{\infty} A_n, \tag{3.8}$$

such that, A_1 , $A_1, A_2, A_3, \ldots$ are Adomian polynomials, and A_n may be formulated as follows:

$$A_n = \frac{1}{n\,!}\,(\frac{d^n}{d\lambda^n}\left[N\sum_{i=0}^{n}\left(\lambda^i u_i\right)\right])_{\lambda=0} \quad , n = 0, 1, 2, 3 \ldots. \tag{3.9}$$

Step 4. After applying the inverse Laplace transform to Eq (3.6), substitute Eqs (3.7) and (3.8) in Eq (3.6), to obtain:

$$\sum_{n=0}^{\infty} u_n(x,t) = \mathcal{L}^{-1}\left[\frac{g(x,0)}{s}\right] + \mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\sum_{i=1}^{n-1} s^{\alpha-\mathrm{i}-1}\, u^{(i)}(x,0)\right] + \mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\mathcal{L}[f(x,t)]\right] - \mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\mathcal{L}[\sum_{n=0}^{\infty} u_n(x,t) + \sum_{n=0}^{\infty} A_n]\right]. \tag{3.10}$$

Step 5. For Eq (3.10), and by comparing each side, the iterative algorithm takes the form:

$$u_0(x,t) = g(x,t) + \mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\sum_{i=1}^{n-1} s^{\alpha-\mathrm{i}-1}\, u^{(i)}(x,0)\right] + \mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\mathcal{L}[f(x,t)]\right], \tag{3.11}$$

$$u_1(x,t) = -\mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\mathcal{L}[Ru_0(x,t) + A_0]\right], \tag{3.12}$$

$$u_2(x,t) = -\mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\mathcal{L}[Ru_1(x,t) + A_1]\right], \tag{3.13}$$

$$\vdots$$

$$u_n(x,t) = -\mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\mathcal{L}[Ru_{n-1}(x,t) + A_{n-1}]\right], n \geq 1. \tag{3.14}$$

3.2. Solution approach via the LAADM

To discuss the solution of Eq (3.1) using LADM, we apply the following steps:

Step1: By applying the Laplace transform, with respect to the time t for each side of Eq (3.1) and the linearity of Laplace transform, we find:

$$\mathcal{L}[\frac{\partial^{\alpha}u(x,t)}{\partial t^{\alpha}}] = \mathcal{L}[f(x,t)] - \mathcal{L}\left[Ru(x,t) + N\frac{\partial^{n}u(x,t)}{\partial x^{n}}\right]. \tag{3.15}$$

Step 2. By assuming the concept of Caputo-sense's Laplace transform for fractional derivatives Eq (2.18), we observe that:

$$s^{\alpha}\mathcal{L}[u(x,t)] - \sum_{i=0}^{n-1} s^{\alpha-\mathrm{i}-1}\, u^{(i)}(x,0) = \mathcal{L}[f(x,t)] - \mathcal{L}\left[Ru(x,t) + N\frac{\partial^{n}u(x,t)}{\partial x^{n}}\right]. \tag{3.16}$$

Then, we have

$$U(x,s) = \frac{g(x)}{s} + \frac{1}{s^{\alpha}}\sum_{i=1}^{n-1} s^{\alpha-\mathrm{i}-1}\, u^{(i)}(x,0) + \frac{1}{s^{\alpha}}\mathcal{L}[f(x,t)] - \frac{1}{s^{\alpha}}\mathcal{L}\left[Ru(x,t) + N\frac{\partial^{n}u(x,t)}{\partial x^{n}}\right]. \tag{3.17}$$

Step 3. So, the solution $u(x,t)$, is determined by the series

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t), \tag{3.18}$$

and the nonlinear term's decomposition is

$$N\frac{\partial^{n}u(x,t)}{\partial x^{n}} = \sum_{n=0}^{\infty} \bar{A}_n, \tag{3.19}$$

such that, $\bar{A}_0$, $\bar{A}_1, \bar{A}_2, \bar{A}_3, \ldots$ are accelerated Adomian polynomials, and $\bar{A}_n$ may be formulated as

$$\bar{A}_n = N(s_n) - \sum_{i=0}^{n-1} \bar{A}_i, \text{ where } (s_n) = u_0 + u_1 + \cdots + u_n. \tag{3.20}$$

Step 4. After applying the inverse Laplace transform to Eq (3.17), substitute Eqs (3.18) and (3.19) in Eq (3.17), to obtain:

$$\sum_{n=0}^{\infty} u_n(x,t) = \mathcal{L}^{-1}\left[\frac{g(x,0)}{s}\right] + \mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\sum_{i=1}^{n-1} s^{\alpha-\mathrm{i}-1}\, u^{(i)}(x,0)\right] + \mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\mathcal{L}[f(x,t)]\right] - \mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\mathcal{L}[\sum_{n=0}^{\infty} u_n(x,t) + \sum_{n=0}^{\infty} \bar{A}_n]\right]. \tag{3.21}$$

Step 5. For Eq (3.21), and by comparing each side, the iterative algorithm takes the form:

$$u_0(x,t) = g(x,t) + \mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\sum_{i=1}^{n-1} s^{\alpha-\mathrm{i}-1}\, u^{(i)}(x,0)\right] + \mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\mathcal{L}[f(x,t)]\right], \tag{3.22}$$

$$u_1(x,t) = -\mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\mathcal{L}[Ru_0(x,t) + \bar{A}_0]\right], \tag{3.23}$$

$$u_2(x,t) = -\mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\mathcal{L}[Ru_1(x,t) + \bar{A}_1]\right], \tag{3.24}$$

$$\vdots$$

$$u_n(x,t) = -\mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\mathcal{L}[Ru_{n-1}(x,t) + \bar{A}_{n-1}]\right], n \geq 1. \tag{3.25}$$

Then, the fractional nonlinear partial differential equation's approximate solution is

$$\tilde{u}(x,t) = u_0(x,t) + u_1(x,t) + u_2(x,t) + u_3(x,t) + \cdots. \quad (3.26)$$

The residual error is obtained by substituting Eq (3.26) in Eq (3.1) as follows

$$Res = \frac{\partial^\alpha \tilde{u}(x,t)}{\partial t^\alpha} + R\tilde{u}(x,t) + N\frac{\partial^n \tilde{u}(x,t)}{\partial x^n} - f(x,t). \quad (3.27)$$

The standard and accelerated formulations of Adomian polynomials for nonlinear functions are presented. For instance, Table 1 shows both the traditional and accelerated Adomian polynomials for the nonlinear term u^2, while Table 2 provides the corresponding expressions for u^3. It is evident that the accelerated Adomian polynomials generate terms more rapidly than the standard approach.

Table 1. Standard and accelerated Adomian polynomials for the nonlinear term u^2.

Adomian polynomials of $\boldsymbol{u^2}$	Accelerated Adomian polynomials of $\boldsymbol{u^2}$
$A_0 = u_0^2$	$\bar{A}_0 = u_0^2$
$A_1 = 2u_0u_1$	$\bar{A}_1 = 2u_0u_1 + u_1^2$
$A_2 = 2u_0u_2 + u_1^2$	$\bar{A}_2 = 2u_0u_2 + 2u_1u_2 + u_2^2$
$A_3 = 2u_0u_3 + 2u_1u_2$	$\bar{A}_3 = 2u_0u_3 + 2u_1u_3 + 2u_2u_3 + u_3^2$

Table 2. Standard and accelerated Adomian polynomials for the nonlinear term u^3.

Adomian polynomials of $\boldsymbol{u^3}$	Accelerated Adomian polynomials of $\boldsymbol{u^3}$
$A_0 = u_0^3$	$\bar{A}_0 = u_0^3$
$A_1 = 3u_0^2u_1$	$\bar{A}_1 = 3u_1u_0^2 + 3u_0u_1^2 + u_1^3$
$A_2 = 3u_0^2u_2 + 3u_0u_1^2$	$\bar{A}_2 = 3u_2u_0^2 + 6u_0u_1u_2 + 3u_0u_2^2 + 3u_2u_1^2 + 3u_1u_2^2 + u_2^3$
$A_3 = 6u_0u_1u_2 + u_1^3 + 3u_0^2u_3$	$\bar{A}_3 = 3u_3y_0^2 + 6u_0u_1u_3 + 6u_0u_2u_3 + 3u_0u_3^2 + 3u_3u_1^2 + 6u_1u_2u_3 + 3u_1u_3^2 + 3u_3u_2^2 + 3u_2u_3^2 + u_3^3$

Indeed, Eq (3.1) is based on the classical Caputo fractional derivative, which inherently assumes the full-memory effect. While our current approach focuses on this traditional formulation, we acknowledge the importance of short-memory principles in improving computational efficiency. Incorporating such approaches represents a promising future direction, and we will consider this importance of short-memory in our forthcoming research. The significance of short-memory effects was demonstrated in Sumelka et al. [36], where the modelling of AAA was presented within the framework of time-fractional damage hyperelasticity.

4. Convergence analysis

In this study, we analyze the error estimation and convergence of the previously obtained series solution, which rapidly approaches the exact solution. To examine the series' convergence in Eq (3.26), we apply classical methods to establish adequate conditions for the convergence and error bounds of the LAADM utilized to solve the time-fractional equation with nonlinear terms in Eq (3.1).

Theorem 4.1: [37] Let us assume that $(\mathbb{k}[0,\mathfrak{J}],\|.\|)$ is a Banach space and consider that $u_i(x,t)$ and $u(x,t)$ are denoted in it. Where ς is a constant, and $0<\varsigma<1$, the series solution of Eq (3.26) converges to the solution in Eq (3.1).

Proof: To demonstrate that $s_i(x,t)$ is a Cauchy sequence in $(\mathbb{k}[0,\mathfrak{J}],\|.\|)$, let $\{s_i\}$ be a partial sum sequence of Eq (3.26). Assume that

$$\begin{aligned}\|s_{i+1}(x,t)-s_i(x,t)\| &= \|u_{i+1}(x,t)\| \\ &\leq \varsigma\|u_i(x,t)\| \\ &\leq \varsigma^2\|u_{i-1}(x,t)\| \\ &\leq \cdots \leq \varsigma^{i+1}\|u_0(x,t)\|.\end{aligned} \tag{4.1}$$

For a partial sum s_i and s_j where $i,j \in \mathcal{N}$ and $i\geq j$, by using triangle inequality, we find that

$$\|s_i-s_j\| = \|\big(s_i(x,t)-s_{i-1}(x,t)\big)+\big(s_{i-1}(x,t)-s_{i-2}(x,t)\big)+\cdots+(s_{j+2}(x,t)-s_{j+1}(x,t))+ (s_{j+1}(x,t)-s_j(x,t))\| \tag{4.2}$$

$$\leq \|s_i(x,t)-s_{i-1}(x,t)\|+\|s_{i-1}(x,t)-s_{i-2}(x,t)\|+\cdots+ \|s_{j+2}(x,t)-s_{j+1}(x,t)\| + \|(s_{j+1}(x,t)-s_j(x,t)\|.$$

Applying Eq (4.1), we find

$$\begin{aligned}\|s_i-s_j\| &\leq \varsigma^i\|u_0(x,t)\|+\varsigma^{i-1}\|u_0(x,t)\|+\cdots+\varsigma^{j+2}\|u_0(x,t)\|+\varsigma^{j+1}\|u_0(x,t)\| \\ &\leq (\varsigma^i+\varsigma^{i-1}+\cdots+\varsigma^{j+2}+\varsigma^{j+1})\|u_0(x,t)\| \\ &\leq \varsigma^{j+1}(\varsigma^{i-j-1}+\varsigma^{i-j-2}+\cdots+\varsigma+1)\|u_0(x,t)\| \\ &\leq \varsigma^{j+1}\left(\frac{1-\varsigma^{i-j}}{1-\varsigma}\right)\|u_0(x,t)\|,\end{aligned} \tag{4.3}$$

since $0<\varsigma<1$, and therefore, $1-\varsigma^{i-j}<1$. Then, we have

$$\|s_i-s_j\| \leq \frac{\varsigma^{j+1}}{1-\varsigma} max|u_0(x,t)|, \ \forall \ t\in[0,\mathfrak{J}]. \tag{4.4}$$

And since u_0 is bounded, then

$$\lim_{i,j\to\infty}\|s_i(x,t)-s_j(x,t)\| = 0. \tag{4.5}$$

Thus, $s_i(x,t)$ represents a Cauchy sequence in $(\mathbb{k}[0,\mathfrak{J}],\|.\|)$. Consequently, the series solution in Eq (3.26) converges to the solution of Eq (3.1). Hence, this ends the proof.

Theorem 4.2: [37] The maximum absolute truncation error of the solution of Eq (3.26) assuming Eq (3.1) is obtained as follows

$$\left|u(x,t) - \sum_{h=0}^{j} u_h(x,t)\right| \leq \frac{\varsigma^{j+1}}{1-\varsigma} \|u_0(x,t)\|. \tag{4.6}$$

Proof: From Eq (4.3), we obtain

$$\left|u(x,t) - s_j\right| \leq \varsigma^{j+1} \left(\frac{1-\varsigma^{i-j}}{1-\varsigma}\right) \|u_0(x,t)\|, \tag{4.7}$$

since $0 < \varsigma < 1$, and therefore, $1 - \varsigma^{i-j} < 1$. Then, we have

$$\left|u(x,t) - \sum_{h=0}^{j} u_h(x,t)\right| \leq \frac{\varsigma^{j+1}}{1-\varsigma} \|u_0(x,t)\|. \tag{4.8}$$

Hence, the proof is completed.

5. Numerical results and discussions

In this paper, four examples of time-fractional nonlinear partial differential equations are solved using the LAADM. The accuracy and effectiveness of the proposed hybrid technique (LAADM) are demonstrated through the analysis of its approximate solutions and absolute errors compared to recent available methods. Additionally, figures are presented for different fractional orders α. For each example, a MATHEMATICA program was developed to address the problems posed by the suggested combined method.

Example 5.1: [38] Examine the partial differential equation for nonlinear time-fractional advection

$$D_t^{\alpha} u(x,t) + u(x,t) u_x(x,t) = x + x t^2, \; t > 0, \; 0 < \alpha \leq 1, \tag{5.1}$$

with initial condition $u(x,0) = 0$, and the exact solution for $\alpha = 1$ is

$$u(x,t) = xt. \tag{5.2}$$

Several researchers have examined this example. The latest were Singh and Pippal [38], who investigated the solution of nonlinear partial differential equations of integer and non-integer order using the Shehu transform Adomian decomposition approach.

In this example, we compare the accuracy of the regular Adomian decomposition method and the proposed accelerated Adomian utilizing the Laplace transform with respect to time t.

5.1. Analytical solution via Laplace transform combined with Adomian decomposition method

Utilizing the Laplace transform, with respect to the time t, and its fractional derivative characteristics for Eq (5.1), we obtain the following:

$$s^{\alpha} U(x,s) - s^{\alpha-1} u(x,0) = \mathcal{L}[x + x t^2] - \mathcal{L}[u(x,t) u_x(x,t)], \tag{5.3}$$

$$U(x,s) = \frac{1}{s^{\alpha}} \mathcal{L}[x + x t^2] - \frac{1}{s^{\alpha}} \mathcal{L}[u(x,t) u_x(x,t)]. \tag{5.4}$$

Then, by applying the inverse Laplace transform to each side of Eq (5.4), we find:

$$u(x,t)=\mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\ \mathcal{L}[x+xt^{2}]\right]-\mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\ \mathcal{L}[u(x,t)u_{x}(x,t)]\right]. \quad (5.5)$$

The solution is then shown as an infinite series, and the nonlinear term may be formulated as described below:

$$u(x,t)=\sum_{n=0}^{\infty}u_{n}(x,t), uu_{x}=\sum_{n=0}^{\infty}A_{n}(u), \quad (5.6)$$

where, A_n are Adomian polynomials obtained using the following formula:

$$A_{n}=\frac{1}{n\ !}(\frac{d^{n}}{d\lambda^{n}}\left[N\sum_{i=0}^{n}\left(\lambda^{i}u_{i}u_{ix}\right)\right])_{\lambda=0}, i=0,1,2,3\ .... \quad (5.7)$$

Substitute Eq (5.6) in Eq (5.5), and then we have:

$$\sum_{n=0}^{\infty}u_{n}(x,t)=\mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\ \mathcal{L}[x+xt^{2}]\right]-\mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\ \mathcal{L}[\sum_{n=0}^{\infty}A_{n}(u)]\right]. \quad (5.8)$$

Then, from Eq (5.8), we get:

$$u_{0}=\mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\ \mathcal{L}[x+xt^{2}]\right], \quad (5.9)$$

$$u_{1}=-\mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\ \mathcal{L}[A_{0}]\right], \quad (5.10)$$

$$u_{2}=-\mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\ \mathcal{L}[A_{1}]\right], \quad (5.11)$$

$$u_{3}=-\mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\ \mathcal{L}[A_{2}]\right], \quad (5.12)$$

$$\vdots$$

$$u_{n}(x,t)=-\mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\ \mathcal{L}[A_{n-1}(u)]\right], n=1,2,3,\dots\ . \quad (5.13)$$

The primary components of A_n are easily provided by

$$\left.\begin{array}{c} A_{0}=u_{0}u_{0x}, \\ A_{1}=u_{0}u_{1x}+u_{1}u_{0x},, \\ A_{2}=u_{0}u_{2x}+u_{1}u_{1x}+u_{2}u_{0x}, \\ \vdots \end{array}\right\} \quad (5.14)$$

Then, from Eq (5.9), Eq (5.10), and Eq (5.14), we get

$$u_{0}(x,t)=\mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\ \mathcal{L}[x+xt^{2}]\right]=\mathcal{L}^{-1}\left[\frac{2x}{s^{\alpha+3}}+\frac{x}{s^{\alpha+1}}\right]=t^{\alpha}x\left(\frac{1}{\Gamma(1+\alpha)}+\frac{2t^{2}}{\Gamma(3+\alpha)}\right) \quad (5.15)$$

$$u_{1}(x,t)=-\mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\ \mathcal{L}[A_{0}]\right]=-\mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\ \mathcal{L}\left[t^{2\alpha}x\left(\frac{1}{\Gamma(1+\alpha)}+\frac{2t^{2}}{\Gamma(3+\alpha)}\right)^{2}\right]\right]$$

$$=-\mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\left(x\left(\frac{s^{-1-2\alpha}\Gamma(1+2\alpha)}{\Gamma(1+\alpha)^{2}}+\frac{4s^{-3-2\alpha}\Gamma(3+2\alpha)}{\Gamma(1+\alpha)\Gamma(3+\alpha)}+\frac{4s^{-5-2\alpha}\Gamma(5+2\alpha)}{\Gamma(3+\alpha)^{2}}\right)\right)\right]$$

$$= -x\left(\frac{t^{3\alpha}\Gamma(1+2\alpha)}{\Gamma(1+\alpha)^2\Gamma(1+3\alpha)} + \frac{4t^{2+3\alpha}\Gamma(3+2\alpha)}{\Gamma(1+\alpha)\Gamma(3+\alpha)\Gamma(3+3\alpha)} + \frac{4t^{4+3\alpha}\Gamma(5+2\alpha)}{\Gamma(3+\alpha)^2\Gamma(5+3\alpha)}\right). \tag{5.16}$$

Similarly, we obtain $u_2, u_3, \ldots$ as follows:

$$u_2 = \frac{2t^{5\alpha}x\Gamma(1+2\alpha)\Gamma(1+4\alpha)}{\Gamma(1+\alpha)^3\Gamma(1+3\alpha)\Gamma(1+5\alpha)} + \frac{4t^{2+5\alpha}x\Gamma(1+2\alpha)\Gamma(3+4\alpha)}{\Gamma(1+\alpha)^2\Gamma(3+\alpha)\Gamma(1+3\alpha)\Gamma(3+5\alpha)} + \frac{8t^{2+5\alpha}x\Gamma(3+2\alpha)\Gamma(3+4\alpha)}{\Gamma(1+\alpha)^2\Gamma(3+\alpha)\Gamma(3+3\alpha)\Gamma(3+5\alpha)} + \frac{16t^{4+5\alpha}x\Gamma(3+2\alpha)\Gamma(5+4\alpha)}{\Gamma(1+\alpha)\Gamma(3+\alpha)^2\Gamma(3+3\alpha)\Gamma(5+5\alpha)} + \frac{8t^{4+5\alpha}x\Gamma(5+2\alpha)\Gamma(5+4\alpha)}{\Gamma(1+\alpha)\Gamma(5+3\alpha)\Gamma(5+5\alpha)} + \frac{16t^{6+5\alpha}x\Gamma(5+2\alpha)\Gamma(7+4\alpha)}{\Gamma(3+\alpha)^3\Gamma(5+3\alpha)\Gamma(7+5\alpha)}, \tag{5.17}$$

$$\vdots$$

Hence, the approximate series solution is provided by

$$u(x,t) = t^{\alpha}x\left(\frac{1}{\Gamma(1+\alpha)} + \frac{2t^2}{\Gamma(3+\alpha)}\right) - x\left(\frac{t^{3\alpha}\Gamma(1+2\alpha)}{\Gamma(1+\alpha)^2\Gamma(1+3\alpha)} + \frac{4t^{2+3\alpha}\Gamma(3+2\alpha)}{\Gamma(1+\alpha)\Gamma(3+\alpha)\Gamma(3+3\alpha)} + \frac{4t^{4+3\alpha}\Gamma(5+2\alpha)}{\Gamma(3+\alpha)^2\Gamma(5+3\alpha)}\right) + \ldots \tag{5.18}$$

Following computation at $\alpha = 1$ and three iterations, the approximate solution takes the following form:

$$u(x,t) \approx tx - \frac{t^9\left(930930+352716t^2+49420t^4+2535t^6\right)x}{42567525}. \tag{5.19}$$

5.2. Analytical solution via Laplace transform combined with accelerated Adomian decomposition method

In the same way, utilizing the Laplace transform, with respect to the time t, and its fractional derivative characteristics for Eq (5.1), we obtain the following:

$$s^{\alpha}U(x,s) - s^{\alpha-1}u(x,0) = \mathcal{L}[x + xt^2] - \mathcal{L}[u(x,t)u_x(x,t)], \tag{5.20}$$

$$U(x,s) = \frac{1}{s^{\alpha}}\ \mathcal{L}[x + xt^2] - \frac{1}{s^{\alpha}}\ \mathcal{L}[u(x,t)u_x(x,t)]. \tag{5.21}$$

Then, by applying the inverse Laplace transform to each side of Eq (5.21), we find:

$$u(x,t) = \mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\ \mathcal{L}[x + xt^2]\right] - \mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\ \mathcal{L}[u(x,t)u_x(x,t)]\right]. \tag{5.22}$$

The solution is then shown as an infinite series, and the nonlinear term may be described as shown below:

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t), uu_x = \sum_{n=0}^{\infty} \bar{A}_n(u). \tag{5.23}$$

The accelerated Adomian polynomials, denoted by $\bar{A}_n$, can be generated by applying the following formula

$$\bar{A}_n = N(s_n) - \sum_{i=0}^{n-1} \bar{A}_i, \text{ where } s_n = u_0u_{0x} + u_1u_{1x} + \cdots + u_nu_{nx}. \tag{5.24}$$

Substitute Eq (5.23) in Eq (5.22), then we have:

$$\sum_{n=0}^{\infty} u_n(x,t) = \mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\ \mathcal{L}[x + xt^2]\right] - \mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\ \mathcal{L}[\sum_{n=0}^{\infty} \bar{A}_n(u)]\right]. \tag{5.25}$$

Then, from Eq (5.25), we get

$$u_0 = \mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\ \mathcal{L}[x + xt^2]\right], \tag{5.26}$$

$$u_1 = -\mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\ \mathcal{L}[\bar{A}_0]\right], \tag{5.27}$$

$$u_2 = -\mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\ \mathcal{L}[\bar{A}_1]\right], \tag{5.28}$$

$$u_3 = -\mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\ \mathcal{L}[\bar{A}_2]\right], \tag{5.29}$$

$$\vdots$$

$$u_n(x,t) = -\mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\ \mathcal{L}[\bar{A}_{n-1}(u)]\right],\ n = 1, 2, 3,\ \dots\ . \tag{5.30}$$

The primary components of $\bar{A}_n$ are easily provided by

$$\left.\begin{array}{c} \bar{A}_0 = u_0 u_{0x}, \\ \bar{A}_1 = u_0 u_{1x} + u_1 u_{0x} + u_1 u_{1x}, \\ \bar{A}_2 = u_0 u_{2x} + u_1 u_{2x} + u_2 u_{0x} + u_2 u_{1x} + u_2 u_{2x}, \\ \vdots \end{array}\right\} \tag{5.31}$$

from Eq (5.26), Eq (5.27), and Eq (5.31), it is easy to show that

$$u_0(x,t) = \mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\ \mathcal{L}[x + xt^2]\right] = \mathcal{L}^{-1}\left[\frac{2x}{s^{\alpha+3}} + \frac{x}{s^{\alpha+1}}\right] = t^{\alpha}x\left(\frac{1}{\Gamma(1+\alpha)} + \frac{2t^2}{\Gamma(3+\alpha)}\right), \tag{5.32}$$

$$u_1(x,t) = -\mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\ \mathcal{L}[\bar{A}_0]\right] = -\mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\ \mathcal{L}\left[t^{2\alpha}x\left(\frac{1}{\Gamma(1+\alpha)} + \frac{2t^2}{\Gamma(3+\alpha)}\right)^2\right]\right]$$

$$= -\mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\ \left(x\left(\frac{s^{-1-2\alpha}\Gamma(1+2\alpha)}{\Gamma(1+\alpha)^2} + \frac{4s^{-3-2\alpha}\Gamma(3+2\alpha)}{\Gamma(1+\alpha)\Gamma(3+\alpha)} + \frac{4s^{-5-2\alpha}\Gamma(5+2\alpha)}{\Gamma(3+\alpha)^2}\right)\right)\right]$$

$$= -x\left(\frac{t^{3\alpha}\Gamma(1+2\alpha)}{\Gamma(1+\alpha)^2\Gamma(1+3\alpha)} + \frac{4t^{2+3\alpha}\Gamma(3+2\alpha)}{\Gamma(1+\alpha)\Gamma(3+\alpha)\Gamma(3+3\alpha)} + \frac{4t^{4+3\alpha}\Gamma(5+2\alpha)}{\Gamma(3+\alpha)^2\Gamma(5+3\alpha)}\right). \tag{5.33}$$

Similarly, we obtain u_2, u_3 ... as follows:

$$u_2 = \frac{2t^{5\alpha}x\Gamma(1+2\alpha)\Gamma(1+4\alpha)}{\Gamma(1+\alpha)^3\Gamma(1+3\alpha)\Gamma(1+5\alpha)} + \frac{4t^{2+5\alpha}x\Gamma(1+2\alpha)\Gamma(3+4\alpha)}{\Gamma(1+\alpha)^2\Gamma(3+\alpha)\Gamma(1+3\alpha)\Gamma(3+5\alpha)} +$$
$$\frac{8t^{2+5\alpha}x\Gamma(3+2\alpha)\Gamma(3+4\alpha)}{\Gamma(1+\alpha)^2\Gamma(3+\alpha)\Gamma(3+3\alpha)\Gamma(3+5\alpha)} + \frac{16t^{4+5\alpha}x\Gamma(3+2\alpha)\Gamma(5+4\alpha)}{\Gamma(1+\alpha)\Gamma(3+\alpha)^2\Gamma(3+3\alpha)\Gamma(5+5\alpha)} +$$
$$\frac{8t^{4+5\alpha}x\Gamma(5+2\alpha)\Gamma(5+4\alpha)}{\Gamma(1+\alpha)\Gamma(3+\alpha)^2\Gamma(5+3\alpha)\Gamma(5+5\alpha)} + \frac{16t^{6+5\alpha}x\Gamma(5+2\alpha)\Gamma(7+4\alpha)}{\Gamma(3+\alpha)^3\Gamma(5+3\alpha)\Gamma(7+5\alpha)} -$$
$$\frac{t^{7\alpha}x\Gamma(1+2\alpha)^2\Gamma(1+6\alpha)}{\Gamma(1+\alpha)^4\Gamma(1+3\alpha)^2\Gamma(1+7\alpha)} - \frac{8t^{2+7\alpha}x\Gamma(1+2\alpha)\Gamma(3+2\alpha)\Gamma(3+6\alpha)}{\Gamma(1+\alpha)^3\Gamma(3+\alpha)\Gamma(1+3\alpha)\Gamma(3+3\alpha)\Gamma(3+7\alpha)} -$$
$$\frac{16t^{4+7\alpha}x\Gamma(3+2\alpha)^2\Gamma(5+6\alpha)}{\Gamma(1+\alpha)^2\Gamma(3+\alpha)^2\Gamma(3+3\alpha)^2\Gamma(5+7\alpha)} - \frac{8t^{4+7\alpha}x\Gamma[1+2\alpha]\Gamma[5+2\alpha]\Gamma[5+6\alpha]}{\Gamma(1+\alpha)^2\Gamma(3+\alpha)^2\Gamma(1+3\alpha)\Gamma(5+3\alpha)\Gamma(5+7\alpha)} -$$
$$\frac{32t^{6+7\alpha}x\Gamma[3+2\alpha]\Gamma[5+2\alpha]\Gamma[7+6\alpha]}{\Gamma(1+\alpha)\Gamma(3+\alpha)^3\Gamma(3+3\alpha)\Gamma(5+3\alpha)\Gamma(7+7\alpha)} - \frac{16t^{8+7\alpha}x\Gamma(5+2\alpha)^2\Gamma(9+6\alpha)}{\Gamma(3+\alpha)^4\Gamma(5+3\alpha)^2\Gamma(9+7\alpha)}, \tag{5.34}$$

⋮

Hence, the approximate series solution is provided by

$$u(x,t) = t^{\alpha} x \left(\frac{1}{\Gamma(1+\alpha)} + \frac{2t^2}{\Gamma(3+\alpha)}\right) - x\left(\frac{t^{3\alpha}\Gamma(1+2\alpha)}{\Gamma(1+\alpha)^2\Gamma(1+3\alpha)} + \frac{4t^{2+3\alpha}\Gamma(3+2\alpha)}{\Gamma(1+\alpha)\Gamma(3+\alpha)\Gamma(3+3\alpha)} + \frac{4t^{4+3\alpha}\Gamma(5+2\alpha)}{\Gamma(3+\alpha)^2\Gamma(5+3\alpha)}\right) + \dots \quad (5.35)$$

Following computation at $\alpha = 1$ and three iterations, the approximate solution takes the following form:

$$u(x,t) \approx tx - \frac{8t^9x}{945} - \frac{4t^{11}x}{6237} + \frac{8t^{13}x}{32175} - \frac{344t^{15}x}{6449625} - \frac{2t^{17}x}{144585} + \frac{48932t^{19}x}{8398900125} + \frac{4624t^{21}x}{2681754075} + \frac{711512t^{23}x}{16962094524375} - \frac{112708t^{25}x}{3016973334375} - \frac{3992t^{27}x}{672354057375} - \frac{8t^{29}x}{21210236775} - \frac{t^{31}x}{109876902975} \quad (5.36)$$

It is necessary to note that the suggested approach provides a more precise approximation compared to the one presented by Singh and Pippal [38], where nonlinear partial differential equations were solved using the Shehu transform Adomian decomposition method. The researchers stated that as the number of terms increases and when $\alpha = 1$, an approximate solution can be obtained; however, no computational results were presented.

Table 3. Absolute errors obtained by applying Laplace transform coupled with the accelerated Adomian using two iterations with higher-precision (e.g., 34 digits), when $\alpha = 1$, for Example 5.1.

Laplace with Accelerated Adomian $u(x,t) = u_0 + u_1 + u_2$			
T	Exact solution	Approximate solution	Absolute error $\alpha = 1$
0.2	0.04000000000000001	0.04000009787833556	$9.78783355642943\times10^{-8}$
0.4	0.08000000000000002	0.0800001957566711	$1.957566711285885\times10^{-7}$
0.6	0.12	0.1200002936350067	$2.936350066928828\times10^{-7}$
0.8	0.16	0.1600003915133422	$3.915133422571771\times10^{-7}$
1	0.2	0.2000004893916778	$4.893916778214713\times10^{-7}$

Table 4. Approximate solutions, absolute errors, and experimental rates of convergence obtained by using the Laplace transform coupled with the accelerated Adomian with various fractional order α for Example 5.1.

Laplace with Accelerated Adomian: $u(x,t) = u_0 + u_1 + u_2 + u_3$								
t	X	Exact sol.	$\alpha = 0.7$	$\alpha = 0.8$	$\alpha = 0.9$	$\alpha = 1$	Absolute error ($\alpha = 1$)	Exp. Rate
0.2	0.2	0.04	0.0682253039	0.0579452062	0.0484477294	0.0399999991	8.69465×10^{-10}	–
	0.4	0.08	0.1364506080	0.1158904120	0.0968954588	0.0799999983	1.73893×10^{-9}	1
	0.6	0.12	0.2046759120	0.1738356190	0.1453431880	0.1199999970	2.60839×10^{-9}	0.999995
	0.8	0.16	0.2729012150	0.2317808250	0.1937909180	0.1599999970	3.47786×10^{-9}	1.000007
	1	0.2	0.3411265190	0.2897260310	0.2422386470	0.1999999960	4.34732×10^{-9}	0.999995

Table 5. Approximate solutions, absolute errors, and experimental rates of convergence obtained by using Laplace transform coupled with the Adomian with various fractional order α for Example 5.1.

Laplace with Adomian:	$u(x,t) = u_0 + u_1 + u_2 + u_3$							
t	X	Exact sol.	α = 0.7	α = 0.8	α = 0.9	α = 1	Absolute error	Exp. Rate
0.2	0.2	0.04	0.0682228160	0.0579449814	0.0484477109	0.0399999977	2.27357×10^{-9}	–
	0.4	0.08	0.1364456320	0.1158899630	0.0968954218	0.0799999955	4.54713×10^{-9}	0.999997
	0.6	0.12	0.2046684480	0.1738349440	0.1453431330	0.1199999930	6.8207×10^{-9}	1.000002
	0.8	0.16	0.2728912640	0.2317799260	0.1937908440	0.1599999910	9.09426×10^{-9}	0.999997
	1	0.2	0.3411140800	0.2897249070	0.2422385550	0.1999999890	1.13678×10^{-8}	0.999990

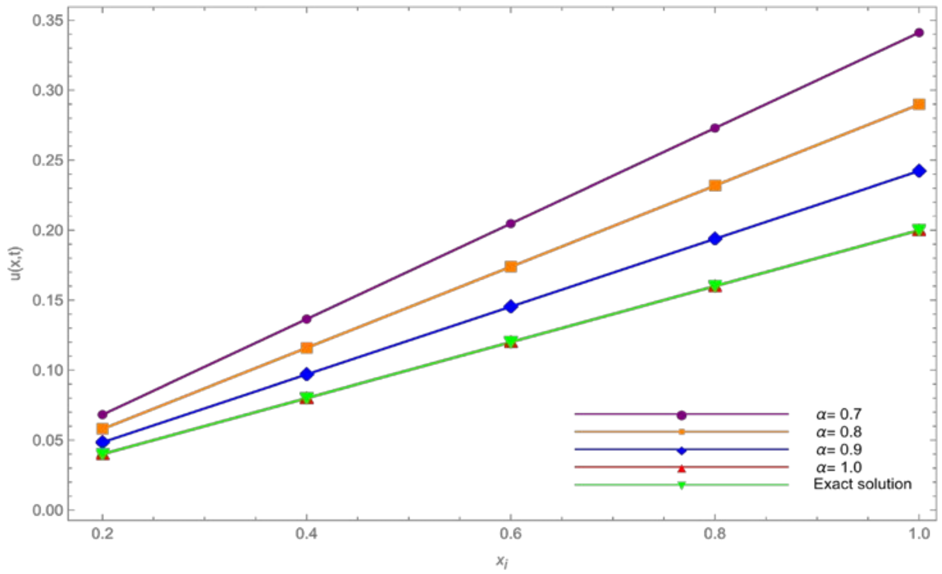

Figure 1. Comparison of approximate solutions produced for several fractional values of α, using Laplace transform paired with accelerated Adomian for Example 5.1.

Tables 3 and 4 clearly show that using three iterations results in a lower absolute error compared to using only two iterations. This indicates that the accuracy and convergence of the Laplace transform with the accelerated Adomian method improve as the number of iterations increases, thereby demonstrating its effectiveness and reliability.

Tables 4 and 5 demonstrate that integrating the Laplace transform with the accelerated Adomian method yields improved accuracy and lower absolute errors compared to the traditional Adomian decomposition method with the Laplace transform, using three iterations.

Figure 1 shows a set of curves resulting from gradually changing the parameter α from 0.7 to 1.0 $(0.7, 0.8, 0.9$, and finally $1)$. It is noticeable that the curves are very close to each other, indicating the stability and consistency of the numeric solution with varying α. This closeness reflects the superior accuracy and effectiveness of the applied approximation technique, as the results are not significantly affected by small changes in α, which enhances the reliability of the employed model.

Example 5.2: [39] Examine the nonlinear homogeneous time fractional KdV equation

$$D_t^{\alpha}u(x,t) + 6u(x,t)u_x(x,t) + u_{xxx}(x,t) = 0,\ \ t > 0, 0 < \alpha \leq 1, \tag{5.37}$$

with initial condition $u(x,0) = \frac{1}{2}sech^2(\frac{x}{2})$, and the exact solution of the KdV equation for $\alpha = 1$ is

$$u(x,t) = \frac{1}{2}sech^2\left(\frac{x+t}{2}\right). \tag{5.38}$$

This example is widely referenced by numerous researchers, with the most recent being Khirsariya et al. [39], who applied a semi-analytic solution for Kdv equations using the fractional residual power series technique.

Here, we employ Laplace transform with respect to the time t with the proposed accelerated Adomian for comparing the accuracy of both methods.

5.3. Analytical solution via Laplace transform combined with accelerated Adomian decomposition method

Utilizing the Laplace transform, with respect to the time t, and its fractional derivatives characteristics for Eq (5.37), we obtain the following:

$$s^{\alpha}U(x,s) - s^{\alpha-1}u(x,0) = -\mathcal{L}[6u(x,t)u_x(x,t) + u_{xxx}(x,t)], \tag{5.39}$$

$$U(x,s) = \frac{1}{2s}sech^2\left(\frac{x}{2}\right) - \frac{1}{s^{\alpha}}\mathcal{L}[6u(x,t)u_x(x,t) + u_{xxx}(x,t)]. \tag{5.40}$$

For Eq (5.40), by applying the inverse Laplace transform to each side, we find:

$$u(x,t) = \frac{1}{2}sech^2\left(\frac{x}{2}\right) - \mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\mathcal{L}[6u(x,t)u_x(x,t) + u_{xxx}(x,t)]\right]. \tag{5.41}$$

The solution is then shown as an infinite series, and the nonlinear term may be formulated as described below:

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t), u_{xxx}(x,t) = \sum_{n=0}^{\infty} u_{nxxx}(x,t), uu_x = \sum_{n=0}^{\infty} \bar{A}_n(u). \tag{5.42}$$

The accelerated Adomian polynomials, denoted by $\bar{A}_n$, can be obtained by applying the following formula

$$\bar{A}_n = N(s_n) - \sum_{i=0}^{n-1} \bar{A}_i,\ \text{where}\ \ s_n = u_0u_{0x} + u_1u_{1x} + \cdots + u_nu_{nx}. \tag{5.43}$$

By substituting Eq (5.42) in Eq (5.41), we get

$$\sum_{n=0}^{\infty} u_n(x,t) = \frac{1}{2}sech^2\left(\frac{x}{2}\right) - \mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\mathcal{L}[6\sum_{n=0}^{\infty} \bar{A}_n(u) + \sum_{n=0}^{\infty} u_{nxxx}(x,t)]\right], \tag{5.44}$$

Then, from Eq (5.44), we get:

$$u_0 = \frac{1}{2}Sech[\frac{x}{2}]^2, \tag{5.45}$$

$$u_1 = -\mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\ \ \mathcal{L}[6\bar{A}_0 + u_{0xxx}]\right], \tag{5.46}$$

$$u_2 = -\mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\ \mathcal{L}[6\bar{A}_1 + u_{1xxx}]\right], \tag{5.47}$$

$$u_3 = -\mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\ \mathcal{L}[6\bar{A}_2 + u_{2xxx}]\right], \tag{5.48}$$

$$\vdots$$

$$u_n(x,t) = -\mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\mathcal{L}\left[6\bar{A}_{n-1}(u) + u_{(n-1)xxx}\right]\right], n = 1,2,3,\ \dots\ . \tag{5.49}$$

Easily, we can obtain the primary components of $\bar{A}_n$ as follows:

$$\left.\begin{array}{c}\bar{A}_0 = u_0 u_{0x},\\ \bar{A}_1 = u_0 u_{1x} + u_1 u_{0x} + u_1 u_{1x},\\ \bar{A}_2 = u_0 u_{2x} + u_1 u_{2x} + u_2 u_{0x} + u_2 u_{1x} + u_2 u_{2x},\\ \vdots\end{array}\right\} \tag{5.50}$$

From Eq (5.46) and Eq (5.50), it is easy to show that

$$u_1 = -\mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\ \mathcal{L}[6\bar{A}_0 + u_{0xxx}]\right]$$

$$= -\mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\left(\frac{-\frac{3}{2}Sech\left[\frac{x}{2}\right]^4 Tanh\left[\frac{x}{2}\right]+\frac{1}{2}\left(2Sech\left[\frac{x}{2}\right]^4 Tanh\left[\frac{x}{2}\right]-Sech\left[\frac{x}{2}\right]^2 Tanh\left[\frac{x}{2}\right]^3\right)}{s}\right)\right]$$

$$= -\frac{t^{\alpha}\left(-\frac{3}{2}\mathrm{Sech}\left[\frac{x}{2}\right]^4\mathrm{Tanh}\left[\frac{x}{2}\right]+\frac{1}{2}\left(2\mathrm{Sech}\left[\frac{x}{2}\right]^4\mathrm{Tanh}\left[\frac{x}{2}\right]-\mathrm{Sech}\left[\frac{x}{2}\right]^2\mathrm{Tanh}\left[\frac{x}{2}\right]^3\right)\right)}{\Gamma(1+\alpha)}. \tag{5.51}$$

Similarly, we get u_2, u_3 ... as follows:

$$u_2 = -\frac{t^{2\alpha}Sech\left[\frac{x}{2}\right]^8}{4\Gamma(1+2\alpha)} - \frac{3t^{3\alpha}\Gamma(1+2\alpha)Sech\left[\frac{x}{2}\right]^{10}Tanh\left[\frac{x}{2}\right]}{4\Gamma(1+\alpha)^2\Gamma(1+3\alpha)} + \frac{3t^{2\alpha}Sech\left[\frac{x}{2}\right]^4 Tanh\left[\frac{x}{2}\right]^4}{4\Gamma(1+2\alpha)} + \frac{9t^{3\alpha}\Gamma[1+2\alpha]Sech\left[\frac{x}{2}\right]^6 Tanh\left[\frac{x}{2}\right]^5}{4\Gamma(1+\alpha)^2\Gamma(1+3\alpha)} + \frac{t^{2\alpha}Sech\left[\frac{x}{2}\right]^2 Tanh\left[\frac{x}{2}\right]^6}{2\Gamma(1+2\alpha)} + \frac{3t^{3\alpha}\Gamma(1+2\alpha)Sech\left[\frac{x}{2}\right]^4 Tanh\left[\frac{x}{2}\right]^7}{2\Gamma(1+\alpha)^2\Gamma(1+3\alpha)}. \tag{5.52}$$

$$\vdots$$

Consequently, the approximate series solution is provided as

$$u(x,t) = \frac{1}{2}Sech\left[\frac{x}{2}\right]^2 - \frac{t^{\alpha}\left(-\frac{3}{2}\mathrm{Sech}\left[\frac{x}{2}\right]^4\mathrm{Tanh}\left[\frac{x}{2}\right]+\frac{1}{2}\left(2\mathrm{Sech}\left[\frac{x}{2}\right]^4\mathrm{Tanh}\left[\frac{x}{2}\right]-\mathrm{Sech}\left[\frac{x}{2}\right]^2\mathrm{Tanh}\left[\frac{x}{2}\right]^3\right)\right)}{\Gamma(1+\alpha)} - \frac{t^{2\alpha}Sech\left[\frac{x}{2}\right]^8}{4\Gamma(1+2\alpha)} - \frac{3t^{3\alpha}\Gamma(1+2\alpha)Sech\left[\frac{x}{2}\right]^{10}Tanh\left[\frac{x}{2}\right]}{4\Gamma(1+\alpha)^2\Gamma(1+3\alpha)} + \frac{3t^{2\alpha}Sech\left[\frac{x}{2}\right]^4 Tanh\left[\frac{x}{2}\right]^4}{4\Gamma(1+2\alpha)} + \frac{9t^{3\alpha}\Gamma[1+2\alpha]Sech\left[\frac{x}{2}\right]^6 Tanh\left[\frac{x}{2}\right]^5}{4\Gamma(1+\alpha)^2\Gamma(1+3\alpha)} + \frac{t^{2\alpha}Sech\left[\frac{x}{2}\right]^2 Tanh\left[\frac{x}{2}\right]^6}{2\Gamma(1+2\alpha)} + \frac{3t^{3\alpha}\Gamma(1+2\alpha)Sech\left[\frac{x}{2}\right]^4 Tanh\left[\frac{x}{2}\right]^7}{2\Gamma(1+\alpha)^2\Gamma(1+3\alpha)} + \cdots. \tag{5.53}$$

Following computation at $\alpha = 1$ and three iterations, the approximate solution takes the following form:

$$u(x,t) \approx \frac{1}{1720320}\text{Sech}\left[\frac{x}{2}\right]^{15}\Big(-840(-429+132t^2+834t^4+208t^6)\text{Cosh}\left[\frac{x}{2}\right]+315(858-231t^2+552t^4+368t^6)\text{Cosh}\left[\frac{3x}{2}\right]+150150\text{Cosh}\left[\frac{5x}{2}\right]-28875t^2\text{Cosh}\left[\frac{5x}{2}\right]+349650t^4\text{Cosh}\left[\frac{5x}{2}\right]+58800t^6\text{Cosh}\left[\frac{5x}{2}\right]+60060\text{Cosh}\left[\frac{7x}{2}\right]-4620t^2\text{Cosh}\left[\frac{7x}{2}\right]+23730t^4\text{Cosh}\left[\frac{7x}{2}\right]-29400t^6\text{Cosh}\left[\frac{7x}{2}\right]+16380\text{Cosh}\left[\frac{9x}{2}\right]+1260t^2\text{Cosh}\left[\frac{9x}{2}\right]-37170t^4\text{Cosh}\left[\frac{9x}{2}\right]+2520t^6\text{Cosh}\left[\frac{9x}{2}\right]+2730\text{Cosh}\left[\frac{11x}{2}\right]+735t^2\text{Cosh}\left[\frac{11x}{2}\right]+2310t^4\text{Cosh}\left[\frac{11x}{2}\right]+210\text{Cosh}\left[\frac{13x}{2}\right]+105t^2\text{Cosh}\left[\frac{13x}{2}\right]+27720t\text{Sinh}\left[\frac{x}{2}\right]-13020t^3\text{Sinh}\left[\frac{x}{2}\right]+184212t^5\text{Sinh}\left[\frac{x}{2}\right]-1854720t^7\text{Sinh}\left[\frac{x}{2}\right]+62370t\text{Sinh}\left[\frac{3x}{2}\right]-27405t^3\text{Sinh}\left[\frac{3x}{2}\right]+198828t^5\text{Sinh}\left[\frac{3x}{2}\right]+864000t^7\text{Sinh}\left[\frac{3x}{2}\right]+57750t\text{Sinh}\left[\frac{5x}{2}\right]-21875t^3\text{Sinh}\left[\frac{5x}{2}\right]-30870t^5\text{Sinh}\left[\frac{5x}{2}\right]-172800t^7\text{Sinh}\left[\frac{5x}{2}\right]+32340t\text{Sinh}\left[\frac{7x}{2}\right]-9310t^3\text{Sinh}\left[\frac{7x}{2}\right]-40698t^5\text{Sinh}\left[\frac{7x}{2}\right]+11520t^7\text{Sinh}\left[\frac{7x}{2}\right]+11340t\text{Sinh}\left[\frac{9x}{2}\right]-1890t^3\text{Sinh}\left[\frac{9x}{2}\right]+4914t^5\text{Sinh}\left[\frac{9x}{2}\right]+2310t\text{Sinh}\left[\frac{11x}{2}\right]-35t^3\text{Sinh}\left[\frac{11x}{2}\right]+126t^5\text{Sinh}\left[\frac{11x}{2}\right]+210t\text{Sinh}\left[\frac{13x}{2}\right]+35t^3\text{Sinh}\left[\frac{13x}{2}\right]\Big). \tag{5.54}$$

It is necessary to observe that in this study, the proposed method provides higher accuracy, faster convergence, and lower computational complexity than the method presented by Khirsariya et al. [39]. There, authors used semi-analytic solution for KdV equations using the fractional residual power series method. The AADM requires fewer iterations to achieve a given accuracy level, making it computationally efficient. On the other hand, the FRPSM demands a larger number of series terms for precise solutions.

Tables 6–9 show that the absolute errors in LAADM are almost the same as those in FRPSM, especially for $\alpha = 1$, where LAADM closely approximates the exact solution. Additionally, LAADM achieves comparable accuracy with fewer terms, resulting in less computational effort. The method remains stable and efficient for different fractional values of α, whereas the FRPSM requires extra corrections and expansions to improve its results.

Table 6. Absolute errors and experimental rates of convergence produced by applying Laplace transform combined with accelerated Adomian method and FRPS method, when $x = 0$ and $\alpha = 1$, for Example 5.2.

	LAADM		FRPSM [39]	
T	Absolute error	Exp. Rate	Absolute error	Exp. Rate
0.1	1.30335×10^{-5}	–	2.90000×10^{-9}	–
0.2	2.09145×10^{-4}	4.0042	1.87933×10^{-7}	6.0180
0.3	1.06395×10^{-3}	4.0120	2.12660×10^{-6}	5.9837
0.4	3.38549×10^{-3}	4.0236	1.18419×10^{-5}	5.9688
0.5	8.3375×10^{-3}	4.0389	4.46589×10^{-5}	5.9487

Table 7. When $x = 10$ and $\alpha = 1$.

	LAADM		FRPSM [39]	
T	Absolute error	Exp. Rate	Absolute error	Exp. Rate
0.1	1.81866×10^{-5}	–	1.59158×10^{-5}	–
0.2	3.65496×10^{-5}	1.0070	3.64406×10^{-5}	1.1951
0.3	5.52581×10^{-5}	1.0194	5.49109×10^{-5}	1.0113
0.4	7.44741×10^{-5}	1.0374	7.36995×10^{-5}	1.0229
0.5	9.43525×10^{-5}	1.0602	9.29355×10^{-5}	1.0393

Table 8. When $x = 20$ and $\alpha = 1$.

	LAADM		FRPSM [39]	
T	Absolute error	Exp. Rate	Absolute error	Exp. Rate
0.1	8.25819×10^{-10}	–	8.25166×10^{-10}	–
0.2	1.65965×10^{-9}	1.0070	1.65471×10^{-9}	1.0038
0.3	2.50917×10^{-9}	1.0194	2.49341×10^{-9}	1.0112
0.4	3.38172×10^{-9}	1.0374	3.34659×10^{-9}	1.0230
0.5	4.28432×10^{-9}	1.0602	4.22009×10^{-9}	1.0393

Table 9. Approximate solutions produced by applying Laplace transform combined with accelerated Adomian with several fractional order α, when $x = 2$.

t	$\alpha = 0.7$	$\alpha = 0.8$	$\alpha = 0.9$	$\alpha = 1$	Exact solution
0.1	0.2470916340	0.2384175270	0.2316141660	0.2263576410	0.1943894100
0.2	0.2708642920	0.2604975130	0.2512577320	0.2434130590	0.1796006580
0.3	0.2897549870	0.2802322700	0.2702789150	0.2609825520	0.1656391340
0.4	0.3041699500	0.2976294390	0.2885119120	0.2788099040	0.1525099980
0.5	0.3138573830	0.3122700770	0.3055477070	0.2965538430	0.1402074330

Table 10. Absolute errors produced by applying Laplace transform combined with accelerated Adomian, using four iterations with higher precision (e.g., 34 digits) with different fractional order α, when $x = 2$.

	Absolute errors of LAADM ($\alpha = 1$) for four iterations $u(x,t) = u_0 + u_1 + u_2 + u_3 + u_4$		
t	$x = 0$	$x = 10$	$x = 20$
0.1	$2.772686078511555\times10^{-7}$	$1.818692724094007\times10^{-5}$	$8.25835882430243\times10^{-10}$
0.2	$1.766453096052611\times10^{-5}$	$3.655556479286437\times10^{-5}$	$1.659926339969509\times10^{-9}$
0.3	$1.996952831082588\times10^{-4}$	$5.528856713422186\times10^{-5}$	$2.510564566177375\times10^{-9}$
0.4	$1.110299155892447\times10^{-3}$	$7.457041264682962\times10^{-5}$	$3.386117916828298\times10^{-9}$
0.5	$4.179256116472962\times10^{-3}$	$9.45882130174891\times10^{-5}$	$4.295060081629869\times10^{-9}$

As shown in Table 10, the error reduced notably at $x = 0$, while it remains stable at $x = 10$ and $x = 20$. This reflects the robustness of proposed method and demonstrates that its accuracy and convergence properties are enhanced with the increase in the number of iterations.

Table 11. Approximate solutions produced by applying Laplace transform combined with accelerated Adomian, using four iterations with higher precision (e.g., 34 digits) with different fractional order α, when $x = 2$.

	Approximate solutions of LAADM for four iterations $u(x,t) = u_0 + u_1 + u_2 + u_3 + u_4$				
t	$\alpha = 0.7$	$\alpha = 0.8$	$\alpha = 0.9$	$\alpha = 1$	Exact solution
0.1	0.2476890098689968	0.2385348715766247	0.2316368951564358	0.226362007714672	0.1943894096012508
0.2	0.2765651886651225	0.261979176056213	0.2516323484027358	0.2435058642687789	0.1796006580801374
0.3	0.3115798207761698	0.286968275831019	0.2722893606683328	0.2615664386183427	0.1656391339814823
0.4	0.3613475365570887	0.3176107586250896	0.2952462693360546	0.2810102440934397	0.1525099981037045
0.5	0.4354382544452342	0.3590689505447365	0.3229033386862492	0.3027816124281348	0.1402074330902163

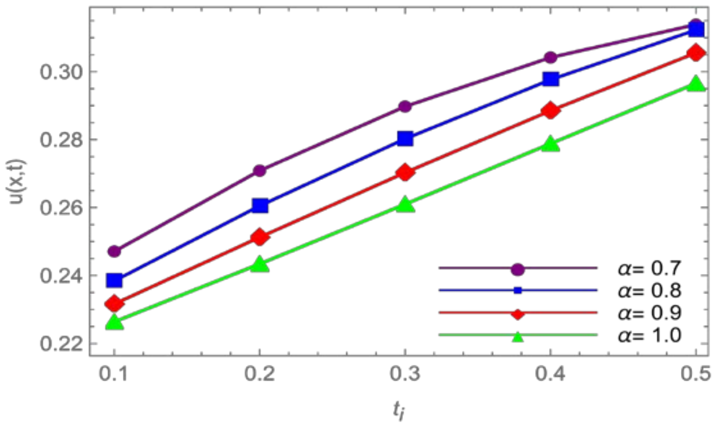

Figure 2. Comparison of approximate solutions produced for various fractional values of α, employing Laplace transform coupled with accelerated Adomian for Example 5.2.

Table 11 clearly illustrates that the approximate solution converges toward the exact solution as the fractional order α tends to 1. This behavior confirms the consistency of the suggest method with the classical case and validates its reliability for fractional values of α.

Figure 2 shows a set of curves resulting from gradually changing the parameter α from 0.7 to 1.0 (0.7, 0.8, 0.9, and finally 1). It is noticeable that the curves are very close to each other, indicating the stability and consistency of the numeric solution with varying α. This closeness reflects the superior accuracy and effectiveness of the applied approximation technique, as the results are not significantly affected by small changes in α, which enhances the reliability of the employed model.

Example 5.3: [40] Examine the partial differential equation for nonlinear time-fractional advection

$$D_t^{\alpha}u(x,t) + u(x,t)u_x(x,t) + u(x,t)u_{xxx}(x,t) = 0, \text{ where } t > 0, 0 < \alpha \le 1, \quad (5.55)$$

with initial condition $u(x,0) = x$, and exact solution for $\alpha = 1$ is

$$u(x,t) = \frac{x}{1+t}. \quad (5.56)$$

This topic has been studied by several academics. The most recent were Ahmed et al. [40] who presented a homotopy perturbation approach combined with Yang transform to solve KdV and Burger equation for nonlinear fractional order with exponential-decay kernel.

For the above, we utilize the Laplace transform, with respect to the time t, with the proposed accelerated Adomian for comparing the two accuracy of the approaches.

5.4. Analytical solution via Laplace transform combined with accelerated Adomian decomposition method

Utilizing the Laplace transform, with respect to the time t, and its fractional derivatives characteristics for Eq (5.55), we obtain the following:

$$s^{\alpha}U(x,s) - s^{\alpha-1}u(x,0) = -\mathcal{L}[u(x,t)u_x(x,t) + u(x,t)u_{xxx}(x,t)], \quad (5.57)$$

$$U(x,s) = \frac{x}{s} - \frac{1}{s^{\alpha}}\mathcal{L}[u(x,t)u_x(x,t) + u(x,t)u_{xxx}(x,t)]. \quad (5.58)$$

Then, by applying the inverse Laplace transform to each side of Eq (5.58), we have:

$$u(x,t) = x - \mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\mathcal{L}[u(x,t)u_x(x,t) + u(x,t)u_{xxx}(x,t)]\right]. \quad (5.59)$$

We can formulate the solution as an infinite series, and the nonlinear term in the form is as described below:

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t), uu_x = \sum_{n=0}^{\infty} \bar{A}_n(u), uu_{xxx} = \sum_{n=0}^{\infty} \bar{B}_n(u), \quad (5.60)$$

where $\bar{A}_n$ and $\bar{B}_n$ are the accelerated Adomian polynomials, which can be obtained using the following formulas:

$$\bar{A}_n = N(s_n) - \sum_{i=0}^{n-1} \bar{A}_i, \text{ where } s_n = u_0u_{0x} + u_1u_{1x} + \cdots + u_nu_{nx}. \quad (5.61)$$

$$\bar{B}_n = N(s_n) - \sum_{i=0}^{n-1} \bar{B}_i, \text{ where } s_n = u_0u_{0xxx} + u_1u_{1xxx} + \cdots + u_nu_{nxxx}. \quad (5.62)$$

By substituting Eq (5.60) in Eq (5.59), then we have:

$$\sum_{n=0}^{\infty} u_n(x,t) = x - \mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\mathcal{L}\left[\sum_{n=0}^{\infty} \bar{A}_n(u) + \sum_{n=0}^{\infty} \bar{B}_n(u)\right]\right], \quad (5.63)$$

Then, from Eq (5.63), we get:

$$u_0 = x, \quad (5.64)$$

$$u_1 = -\mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\mathcal{L}[\bar{A}_0 + \bar{B}_0]\right], \quad (5.65)$$

$$u_2 = -\mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\mathcal{L}[\bar{A}_1 + \bar{B}_1]\right], \quad (5.66)$$

$$\vdots$$

$$u_n(x,t) = -\mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\mathcal{L}[\bar{A}_{n-1}(u) + \bar{B}_{n-1}(u)]\right], n = 1, 2, 3, \ldots . \quad (5.67)$$

For $\bar{A}_n$ and $\bar{B}_n$, the first components are provided by

$$\left.\begin{aligned} \bar{A}_0 &= u_0u_{0x}, \\ \bar{A}_1 &= u_0u_{1x} + u_1u_{0x} + u_1u_{1x}, \\ \bar{A}_2 &= u_0u_{2x} + u_1u_{2x} + u_2u_{0x} + u_2u_{1x} + u_2u_{2x}, \\ &\vdots \end{aligned}\right\} \quad (5.68)$$

$$\left.\begin{aligned}\bar{B}_0 &= u_0 u_{0xxx},\\ \bar{B}_1 &= u_0 u_{1xxx} + u_1 u_{0xxx} + u_1 u_{1xxx},\\ \bar{B}_2 &= u_0 u_{2xxx} + u_1 u_{2xxx} + u_2 u_{0xxx} + u_2 u_{1xxx} + u_2 u_{2xxx},\\ &\vdots\end{aligned}\right\} \tag{5.69}$$

From Eq (5.65), Eqs (5.68), and (5.69), we may represent the following

$$u_1 = -\mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\ \mathcal{L}[\bar{A}_0 + \bar{B}_0]\right] = -\mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\ \mathcal{L}[x]\right] = -\mathcal{L}^{-1}\left[\frac{x}{s^{\alpha+1}}\ \right] = -\frac{t^{\alpha}x}{\Gamma(1+\alpha)}. \tag{5.70}$$

Similarly, we get u_2, u_3 ... as follows:

$$u_2 = \frac{2t^{2\alpha}x}{\Gamma(1+2\alpha)} - \frac{t^{3\alpha}x\Gamma(1+2\alpha)}{\Gamma(1+\alpha)^2\Gamma(1+3\alpha)}, \tag{5.71}$$

$$\vdots$$

Consequently, the approximate series solution is provided as

$$u(x,t) = x - \frac{t^{\alpha}x}{\Gamma(1+\alpha)} + \frac{2t^{2\alpha}x}{\Gamma(1+2\alpha)} - \frac{t^{3\alpha}x\Gamma(1+2\alpha)}{\Gamma(1+\alpha)^2\Gamma(1+3\alpha)} + \cdots \tag{5.72}$$

Hence, the approximate solution with two iterations and calculation at $\alpha = 1$ is as follows:

$$u(x,t) \approx\ x - \frac{1}{3}t\ \ (3 + (-3 + t)t)x. \tag{5.73}$$

It necessary to observe that in this study, the proposed method provides higher accuracy, faster convergence, and lower computational complexity than the method presented in Ahmed et al. [40], in which authors investigated the Yang transform homotopy perturbation approach for solving the nonlinear Burger equation with exponential-decay kernel and KdV of fractional order.

Table 12. Absolute errors and experimental rates of convergence between approximate and exact solutions produced by the Laplace transform, with respect to the time t, merged with accelerated Adomian decomposition and the homotopy perturbation combined with Yang transform using two iterations for Example 5.3 when $t = 0.5$ and $\alpha = 1$.

	Laplace accelerated Adomian method using two iterations				Yang homotopy perturbation method using two iterations [40]		
x	Exact sol.	Approximate sol.	Absolute error	Exp. Rate	Approximate sol.	Absolute error	Exp. Rate
0.1	0.0667	0.0708	0.0042	–	0.0750	0.0083	–
0.2	0.1330	0.1420	0.0083	0.9827	0.1500	0.0167	1.0087
0.3	0.2000	0.2130	0.0125	1.0099	0.2250	0.0250	0.9951
0.4	0.2670	0.2830	0.0167	1.0069	0.3000	0.0333	0.9965
0.5	0.3330	0.3540	0.0208	0.9839	0.3750	0.0417	1.0081
0.6	0.4000	0.4250	0.0250	1.0088	0.4500	0.0500	0.9956
0.7	0.4670	0.4960	0.0292	1.0074	0.5250	0.0583	0.9963
0.8	0.5330	0.5670	0.0333	0.9840	0.6000	0.0667	1.008
0.9	0.6000	0.6380	0.0375	1.0085	0.6750	0.0750	0.9958
1.0	0.6670	0.7080	0.0417	1.0076	0.7500	0.08333	0.9996

Table 13. Absolute error between the two approximate solutions derived from applying the Laplace transform combined with the accelerated Adomian decomposition method at $\alpha = 1$ and $\alpha = 0.98$, compared to those obtained with the homotopy perturbation method utilizing the Yang transform for Example 5.3, $\alpha = 1$, and $\alpha = 0.98$.

	Approximate solutions by LAADM using two iterations			Approximate solutions by YTHPM using two iterations [40]		
x	$\alpha = 1$	$\alpha = 0.98$	$\lvert U_{\alpha=1}(x,t) - U_{\alpha=0.98}(x,t)\rvert$	$\alpha = 1$	$\alpha = 0.98$	$\lvert U_{\alpha=1}(x,t) - U_{\alpha=0.98}(x,t)\rvert$
0.1	0.0708	0.0710	0.0002	0.0750	0.0770	0.0020
0.2	0.1420	0.1420	0.0000	0.1500	0.1540	0.0040
0.3	0.2130	0.2130	0.0000	0.2250	0.2310	0.0060
0.4	0.2830	0.2840	0.0010	0.3000	0.3080	0.0080
0.5	0.3540	0.3550	0.0010	0.3750	0.3851	0.0101
0.6	0.4250	0.4260	0.0010	0.4500	0.4621	0.0121
0.7	0.4960	0.4970	0.0010	0.5250	0.5391	0.0141
0.8	0.5670	0.5680	0.0010	0.6000	0.6161	0.0161
0.9	0.6380	0.6390	0.0010	0.6750	0.6931	0.0181
1.0	0.7080	0.7100	0.0020	0.7500	0.7701	0.0201

As shown in Table 13, the approximate solution at $\alpha = 0.98$ obtained by our proposed method converges to the solution at $\alpha = 1$ more quickly and accurately than the Yang transform homotopy perturbation method.

Tables 12 and 13 show that the proposed method of combining Laplace transform with accelerated Adomian yields higher accuracy and lower absolute errors than the Yang transform homotopy perturbation approach, showing the ability of the proposed method to accelerate convergence while ensuring minimal approximation error.

It is evident from Table 14 that the absolute error improves significantly after three iterations. This observation indicates that the suggest approach becomes more accurate as the number of iterations increases, highlighting its efficiency and convergence behavior.

Table 14. Approximate solutions for $\alpha = 1$ and $\alpha = 0.98$ and the absolute errors between the approximate and exact solutions produced by our proposed method using three iterations with higher precision (e.g., 34 digits) for Example 5.3 when $t = 0.5$ and $\alpha = 1$.

	Laplace accelerated Adomian method using three iterations			
x	Exact sol.	Approximate sol. ($\alpha = 0.98$)	Approximate sol. ($\alpha = 1$)	Absolute error ($\alpha = 1$)
0.1	$6.666666666666666\times10^{-2}$	$6.536245643411278\times10^{-2}$	$6.578621031746033\times10^{-2}$	$8.80456349206338\times10^{-4}$
0.2	$1.333333333333333\times10^{-1}$	$1.307249128682256\times10^{-1}$	$1.315724206349207\times10^{-1}$	$1.760912698412676\times10^{-3}$
0.3	$2.\times10^{-1}$	$1.960873693023384\times10^{-1}$	$1.97358630952381\times10^{-1}$	$2.641369047619014\times10^{-3}$
0.4	$2.666666666666667\times10^{-1}$	$2.614498257364511\times10^{-1}$	$2.631448412698413\times10^{-1}$	$3.521825396825351\times10^{-3}$
0.5	$3.333333333333333\times10^{-1}$	$3.268122821705639\times10^{-1}$	$3.289310515873016\times10^{-1}$	$4.402281746031688\times10^{-3}$
0.6	$4.\times10^{-1}$	$3.921747386046767\times10^{-1}$	$3.947172619047619\times10^{-1}$	$5.282738095238027\times10^{-3}$
0.7	$4.666666666666667\times10^{-1}$	$4.575371950387895\times10^{-1}$	$4.605034722222223\times10^{-1}$	$6.163194444444365\times10^{-3}$
0.8	$5.333333333333333\times10^{-1}$	$5.228996514729022\times10^{-1}$	$5.262896825396827\times10^{-1}$	$7.043650793650702\times10^{-3}$
0.9	$6.\times10^{-1}$	$5.882621079070151\times10^{-1}$	$5.920758928571429\times10^{-1}$	$7.924107142857039\times10^{-3}$
1.0	$6.666666666666666\times10^{-1}$	$6.536245643411278\times10^{-1}$	$6.578621031746033\times10^{-1}$	$8.80456349206338\times10^{-3}$

Figure 3 demonstrates that the approximate solution derived through the accelerated Adomian decomposition method exhibits a higher accuracy and closer agreement with the exact solution than that achieved by the Yang transform homotopy perturbation method. This indicates the higher convergence of the accelerated Adomian approach.

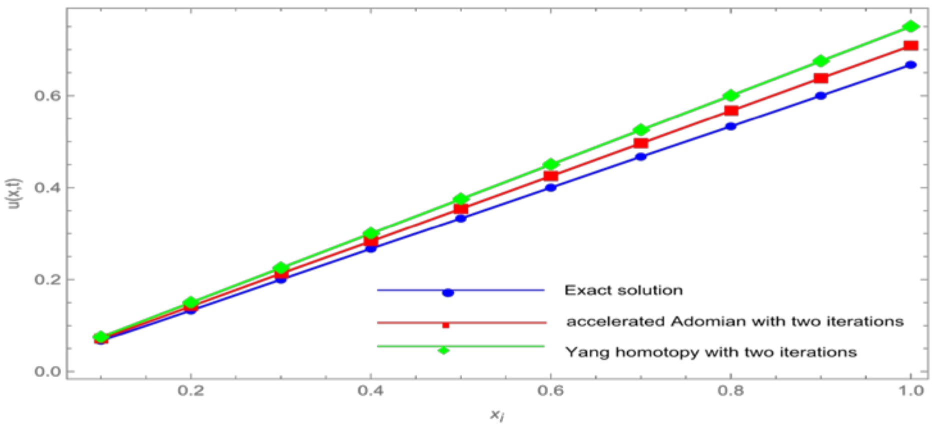

Figure 3. Comparison of the approximate and the exact solution for fractional order $\alpha = 1$, obtained through two iterations using Laplace transform combined with the accelerated Adomian decomposition method and Yang transform homotopy perturbation technique, for Example 5.3.

Example 5.4: [41] Consider the nonlinear partial differential equation for time-fractional advection

$$D_t^{\alpha} u(x,t) + u(x,t)u_x(x,t) - u_{xxx}(x,t) = 0, \text{ where } t > 0, 0 < \alpha \leq 1, \tag{5.74}$$

with the initial condition $u(x,0) = 2x$, and the exact solution for $\alpha = 1$ is given by

$$u(x,t) = \frac{2x}{2t+1}. \tag{5.75}$$

Several researchers have examined this example. The most recent were Noor et al. [41], who used the Aboodh transformation to compare analytically several linear and nonlinear time-fractional partial differential equations.

Here, we use the Laplace transform, with respect to the time t, with the proposed accelerated Adomian to compare the accuracy of the two approaches.

5.5. Analytical solution via Laplace transform combined with accelerated Adomian decomposition method

By employing Laplace transform, with respect to the time t, and its fractional derivative characteristics for Eq (5.74), we obtain the following

$$s^{\alpha}U(x,s) - s^{\alpha-1}u(x,0) = \mathcal{L}[u_{xxx}(x,t) - u(x,t)u_x(x,t)], \quad (5.76)$$

$$U(x,s) = \frac{2x}{s} + \frac{1}{s^{\alpha}}\mathcal{L}[u_{xxx}(x,t) - u(x,t)u_x(x,t)]. \quad (5.77)$$

For Eq (5.77), by applying the inverse Laplace transform to each side, we find:

$$u(x,t) = 2x + \mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\mathcal{L}[u_{xxx}(x,t) - u(x,t)u_x(x,t)]\right]. \quad (5.78)$$

We can formulate the solution as an infinite series, and the nonlinear term in the form is as described below:

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t), \;\; uu_x(x,t) = \sum_{n=0}^{\infty} \bar{A}_n(u), \;\; u_{xxx}(x,t) = \sum_{n=0}^{\infty} u_{nxxx}(x,t). \quad (5.79)$$

The accelerated Adomian polynomials, denoted by $\bar{A}_n$, can be obtained by applying the following formula

$$\bar{A}_n = N(s_n) - \sum_{i=0}^{n-1} \bar{A}_i, \text{ where } s_n = u_0u_{0x} + u_1u_{1x} + \cdots + u_nu_{nx}. \quad (5.80)$$

By substituting Eq (5.79) in Eq (5.78), then we find:

$$\sum_{n=0}^{\infty} u_n(x,t) = 2x + \mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\mathcal{L}[\sum_{n=0}^{\infty} u_{nxxx}(x,t) - \sum_{n=0}^{\infty} \bar{A}_n(u)]\right], \quad (5.81)$$

Then, from Eq (5.81), we get:

$$u_0 = 2x, \quad (5.82)$$

$$u_1 = \mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\mathcal{L}[u_{0xxx} - \bar{A}_0]\right], \quad (5.83)$$

$$u_2 = \mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\mathcal{L}[u_{1xxx} - \bar{A}_1]\right], \quad (5.84)$$

$$u_3 = \mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\mathcal{L}[u_{2xxx} - \bar{A}_2]\right], \quad (5.85)$$

$$\vdots$$

$$u_n(x,t) = \mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\mathcal{L}[u_{(n-1)xxx}(x,t) - \bar{A}_{n-1}(u)]\right], \; n = 1,2,3, \; \ldots \; . \quad (5.86)$$

For $\bar{A}_n$, the first components are provided by

$$\left.\begin{aligned} \bar{A}_0 &= u_0u_{0x}, \\ \bar{A}_1 &= u_0u_{1x} + u_1u_{0x} + u_1u_{1x}, \\ \bar{A}_2 &= u_0u_{2x} + u_1u_{2x} + u_2u_{0x} + u_2u_{1x} + u_2u_{2x}, \\ &\vdots \end{aligned}\right\} \quad (5.87)$$

from Eq (5.83), Eq (5.86), and Eq (5.87), we have that

$$u_1 = \mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\mathcal{L}[u_{0xxx} - \bar{A}_0]\right] = -\mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\mathcal{L}\left[\frac{4x}{s}\right]\right] = -\mathcal{L}^{-1}\left[\frac{4x}{s^{\alpha+1}}\right] = -\frac{4t^{\alpha}x}{\Gamma(1+\alpha)}. \quad (5.88)$$

Similarly, we get u_2, u_3 ... as follows:

$$u_2 = \frac{16t^{2\alpha}x}{\Gamma(1+2\alpha)} - \frac{16t^{3\alpha}x\Gamma(1+2\alpha)}{\Gamma(1+\alpha)^2\Gamma(1+3\alpha)}, \tag{5.89}$$

$$u_3 = -\frac{64t^{3\alpha}x}{\Gamma(1+3\alpha)} + \frac{64t^{4\alpha}x\Gamma(1+2\alpha)}{\Gamma(1+\alpha)^2\Gamma(1+4\alpha)} + \frac{128t^{4\alpha}x\Gamma(1+3\alpha)}{\Gamma(1+\alpha)\Gamma(1+2\alpha)\Gamma(1+4\alpha)} - \frac{256t^{5\alpha}x\Gamma(1+4\alpha)}{\Gamma(1+2\alpha)^2\Gamma(1+5\alpha)} - \frac{128t^{5\alpha}x\Gamma(1+2\alpha)\Gamma(1+4\alpha)}{\Gamma(1+\alpha)^3\Gamma(1+3\alpha)\Gamma(1+5\alpha)} + \frac{512t^{6\alpha}x\Gamma(1+5\alpha)}{\Gamma(1+\alpha)^2\Gamma(1+3\alpha)\Gamma(1+6\alpha)} - \frac{256t^{7\alpha}x\Gamma(1+2\alpha)^2\Gamma(1+6\alpha)}{\Gamma(1+\alpha)^4\Gamma(1+3\alpha)^2\Gamma(1+7\alpha)}, \tag{5.90}$$

$$\vdots$$

Hence, the approximate series solution is provided by

$$u(x,t) = 2x - \frac{4t^{\alpha}x}{\Gamma(1+\alpha)} + \frac{16t^{2\alpha}x}{\Gamma(1+2\alpha)} - \frac{16t^{3\alpha}x\Gamma(1+2\alpha)}{\Gamma(1+\alpha)^2\Gamma(1+3\alpha)} - \frac{64t^{3\alpha}x}{\Gamma(1+3\alpha)} + \frac{64t^{4\alpha}x\Gamma(1+2\alpha)}{\Gamma(1+\alpha)^2\Gamma(1+4\alpha)} + \frac{128t^{4\alpha}x\Gamma(1+3\alpha)}{\Gamma(1+\alpha)\Gamma(1+2\alpha)\Gamma(1+4\alpha)} - \frac{256t^{5\alpha}x\Gamma(1+4\alpha)}{\Gamma(1+2\alpha)^2\Gamma(1+5\alpha)} - \frac{128t^{5\alpha}x\Gamma(1+2\alpha)\Gamma(1+4\alpha)}{\Gamma(1+\alpha)^3\Gamma(1+3\alpha)\Gamma(1+5\alpha)} + \frac{512t^{6\alpha}x\Gamma(1+5\alpha)}{\Gamma(1+\alpha)^2\Gamma(1+3\alpha)\Gamma(1+6\alpha)} - \frac{256t^{7\alpha}x\Gamma(1+2\alpha)^2\Gamma(1+6\alpha)}{\Gamma(1+\alpha)^4\Gamma(1+3\alpha)^2\Gamma(1+7\alpha)}. \tag{5.91}$$

Consequently, the approximate solution with three iterations and calculation at $\alpha = 1$ is as follows

$$u(x,t) \approx 2x - \frac{4}{63}t\left(63 + 2t\left(-63 + 2t\left(63 + 4t\left(-21 + t\big(21 + 2t(-7 + 2t)\big)\right)\right)\right)\right)x. \tag{5.92}$$

It necessary to observe that in this study, the proposed method provides higher accuracy, faster convergence, and lower computational complexity than the method presented in Noor et al. [41]. There, authors introduced an analytical comparison of sometime-fractional partial differential equations using the Aboodh transformation as a framework.

Table 15. Approximate solution using three iterations and applying the Laplace transform combined with the accelerated Adomian decomposition method (LAADM) of Example 5.4, with various fractional order α at $t = 0.01$.

	Approximate solution for three iterations $u(x,t) = u_0 + u_1 + u_2 + u_3$				
X	$\alpha = 0.5$	$\alpha = 0.7$	$\alpha = 1$	Exact solution	Absolute error ($\alpha = 1$)
0.2	0.331350	0.368560	0.392160	0.392160	2.05023×10^{-8}
0.4	0.662700	0.737120	0.784310	0.784310	4.10045×10^{-8}
0.6	0.994040	1.105700	1.176500	1.176500	6.15068×10^{-8}
0.8	1.325400	1.474200	1.568600	1.568600	8.2009×10^{-8}
1.0	1.656700	1.842800	1.960800	1.960800	1.02511×10^{-7}
1.2	1.988100	2.211400	2.352900	2.352900	1.23014×10^{-7}
1.4	2.319400	2.579900	2.745100	2.745100	1.43516×10^{-7}
1.6	2.650800	2.948500	3.137300	3.137300	1.64018×10^{-7}
1.8	2.982100	3.317100	3.529400	3.529400	1.8452×10^{-7}
2.0	3.313500	3.685600	3.921600	3.921600	2.05023×10^{-7}

Table 16. Absolute errors and experimental rates of convergence of the approximate and exact solution derived from applying the Laplace transform combined with the accelerated Adomian decomposition method (LAADM) compared to that obtained using the Aboodh residual power series method (ARPSM) for Example 5.4, when $\alpha = 1$ and $t = 0.01$.

		LAADM			ARPSM [41]		
X	Exact solution	Approx. solution	Absolute error	Exp. rate	Approx. solution	Absolute error	Exp. rate
0.2	0.392160	0.392160	2.05023×10^{-8}	–	0.392157	6.274509×10^{-8}	–
0.4	0.784310	0.784310	4.10045×10^{-8}	0.999996	0.784313	1.254901×10^{-7}	0.9999991
0.6	1.176500	1.176500	6.15068×10^{-8}	1.000002	1.17647	1.882352×10^{-7}	1.0000007
0.8	1.568600	1.568600	8.2009×10^{-8}	0.999997	1.56863	2.509803×10^{-7}	1.0000005
1.0	1.960800	1.960800	1.02511×10^{-7}	0.999989	1.96078	3.137254×10^{-7}	1.0000004
1.2	2.352900	2.352900	1.23014×10^{-7}	1.000036	2.35294	3.764705×10^{-7}	1.0000003
1.4	2.745100	2.745100	1.43516×10^{-7}	0.999985	2.7451	4.392156×10^{-7}	1.0000002
1.6	3.137300	3.137300	1.64018×10^{-7}	0.999987	3.13725	5.019607×10^{-7}	1.0000002
1.8	3.529400	3.529400	1.8452×10^{-7}	0.999988	3.52941	5.647058×10^{-7}	1.0000002
2.0	3.921600	3.921600	2.05023×10^{-7}	1.000036	3.92157	6.274509×10^{-7}	1.0000002

As shown in Table 15, the comparison results make it evident that the approximate solutions acquired using the suggested technique are more accurate than those obtained by the Aboodh residual power series method.

Tables 15 and 16 demonstrate that the proposed approach of applying Laplace transform combined with the accelerated Adomian decomposition method yields higher accuracy and lower absolute errors than the residual power series technique of Aboodh. This shows the ability of the proposed approach to accelerate convergence while ensuring minimal approximation error.

Table 17. Approximate solution after applying the Laplace transform combined with the accelerated Adomian decomposition method (LAADM) and using four iterations with higher precision (e.g., 34 digits) of Example 5.4, with various fractional order α at $t = 0.01$.

	Approximate solution for four iterations $u(x,t) = u_0 + u_1 + u_2 + u_3 + u_4$			
X	$\alpha = 0.5$	$\alpha = 0.7$	$\alpha = 1$	Exact solution
0.2	0.3330440278346288	0.3685846746820659	0.3921568629075005	0.392156862745098
0.4	0.6660880556692575	0.7371693493641319	0.784313725815001	0.784313725490196
0.6	0.999132083503887	1.105754024046198	1.176470588722502	1.176470588235294
0.8	1.332176111338515	1.474338698728264	1.568627451630002	1.568627450980392
1.0	1.665220139173144	1.842923373410329	1.960784314537502	1.96078431372549
1.2	1.998264167007773	2.211508048092395	2.352941177445003	2.352941176470588
1.4	2.331308194842402	2.580092722774461	2.745098040352504	2.745098039215686
1.6	2.66435222267703	2.948677397456527	3.137254903260004	3.137254901960784
1.8	2.997396250511659	3.317262072138593	3.529411766167505	3.529411764705882
2.0	3.330440278346288	3.685846746820659	3.921568629075005	3.92156862745098

Table 18. Absolute errors of the approximate and exact solution derived from applying the LAADM after using four iterations with higher precision (e.g., 34 digits) for Example 5.4, when $\alpha = 1$ and $t = 0.01$.

	Approximate solution for four iterations $u(x,t) = u_0 + u_1 + u_2 + u_3 + u_4$		
X	Exact solution	Approximate solution ($\alpha = 1$)	Absolute error ($\alpha = 1$)
0.2	0.392156862745098	0.3921568629075005	$1.624024914548272\times10^{-10}$
0.4	0.784313725490196	0.784313725815001	$3.248049829096544\times10^{-10}$
0.6	1.176470588235294	1.176470588722502	$4.872074743644817\times10^{-10}$
0.8	1.568627450980392	1.568627451630002	$6.496099658193089\times10^{-10}$
1.0	1.96078431372549	1.960784314537502	$8.12012457274136\times10^{-10}$
1.2	2.352941176470588	2.352941177445003	$9.74414948728963\times10^{-10}$
1.4	2.745098039215686	2.745098040352504	$1.136817440183791\times10^{-9}$
1.6	3.137254901960784	3.137254903260004	$1.299219931638618\times10^{-9}$
1.8	3.529411764705882	3.529411766167505	$1.461622423093445\times10^{-9}$
2.0	3.92156862745098	3.921568629075005	$1.624024914548272\times10^{-9}$

From Tables 17 and 18, it is noteworthy that the use of higher-precision (e.g., 34 digits) arithmetic yields an approximate solution that closely matches the exact solution and, the absolute error improves significantly after four iterations. This observation indicates that the suggest approach becomes more accurate as the number of iterations increases, highlighting its efficiency and convergence behavior and demonstrating the suitability and reliability of the chosen example.

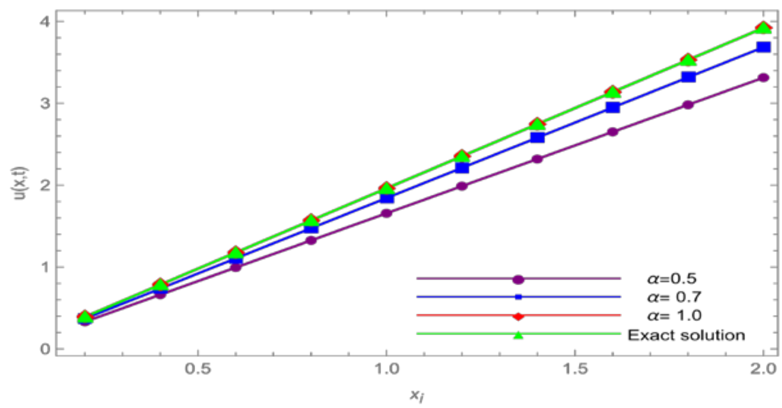

Figure 4. Comparison between approximate solutions produced by several fractional values of α and the exact solution using Laplace transform combined with accelerated Adomian for Example 5.4.

Figure 4 shows the approximate solutions obtained through three iterations of Laplace transform combined with the accelerated Adomian decomposition method (LAADM) for Example 5.4, considering different fractional orders of α ($\alpha = 0.5, 0.7,$ and 1). The curves clearly demonstrate that the solution converges to the exact one, indicating that the methodology is effective and accurate.

Example 5.5: [42,43] Consider the nonlinear time-fractional Fisher partial differential equation.

$$D_t^{\alpha} u(x,t) - u_{xx}(x,t) - 6u(x,t)(1 - u(x,t)) = 0, \text{ where} x \epsilon R,\ t > 0,\ 0 < \alpha \leq 1, \quad (5.93)$$

with initial condition $u(x,0) = \frac{1}{(1+e^x)^2}$, and the exact solution for $\alpha = 1$ is given by

$$u(x,t) = \frac{1}{(1+e^{x-5t})^2}. \quad (5.94)$$

Several researchers have investigated this example. Notably, Merdan [42] employed the modified Riemann-Liouville derivative to address a time-fractional reaction-diffusion equation, while Shwayyea [43] formulated nonlinear partial differential equations for the time-fractional Fisher model.

Here, we apply the Laplace transform with respect to time t, combined with the proposed accelerated Adomian polynomial, to compare the accuracy of the three approaches.

5.6. Analytical solution via Laplace transform combined with accelerated Adomian decomposition method

Employing Laplace transform, with respect to the time t, and its fractional derivative characteristics for Eq (5.93), we obtain the following

$$s^\alpha U(x,s) - s^{\alpha-1}u(x,0) = \mathcal{L}[u_{xx}(x,t)] + 6\mathcal{L}[u(x,t) - u^2(x,t)], \quad (5.95)$$

$$U(x,s) = \frac{1}{s(1+e^x)^2} + \frac{1}{s^\alpha}\mathcal{L}[u_{xx}(x,t)] + \frac{6}{s^\alpha}\mathcal{L}[u(x,t) - u^2(x,t)]. \quad (5.96)$$

For Eq (5.96), by applying the inverse Laplace transform to each side, we find:

$$u(x,t) = \frac{1}{(1+e^x)^2} + \mathcal{L}^{-1}\left[\frac{1}{s^\alpha}\mathcal{L}[u_{xx}(x,t)]\right] + \mathcal{L}^{-1}\left[\frac{6}{s^\alpha}\mathcal{L}[u(x,t) - u^2(x,t)]\right]. \quad (5.97)$$

We can formulate the solution as an infinite series, and the nonlinear term in the form is as described below:

$$(x,t) = \sum_{n=0}^{\infty} u_n(x,t), u_{xx}(x,t) = \sum_{n=0}^{\infty} u_{nxx}(x,t), u^2(x,t) = \sum_{n=0}^{\infty} \bar{A}_n(u). \quad (5.98)$$

The accelerated Adomian polynomials, denoted by $\bar{A}_n$, can be obtained by applying the following formula

$$\bar{A}_n = N(s_n) - \sum_{i=0}^{n-1} \bar{A}_i, \text{ where } s_n = {u_0}^2 + {u_1}^2 + \cdots + {u_n}^2 . \quad (5.99)$$

Substitute Eq (5.98) in Eq (5.97), and then we find

$$\sum_{n=0}^{\infty} u_n(x,t) = \frac{1}{(1+e^x)^2} + \mathcal{L}^{-1}\left[\frac{1}{s^\alpha}\mathcal{L}[\sum_{n=0}^{\infty} u_{nxx}(x,t)]\right] + \mathcal{L}^{-1}\left[\frac{6}{s^\alpha}\mathcal{L}[\sum_{n=0}^{\infty} u_n(x,t) - \sum_{n=0}^{\infty} \bar{A}_n(u)\right]. \quad (5.100)$$

Then, from Eq (5.100), we get:

$$u_0 = \frac{1}{(1+e^x)^2}, \quad (5.101)$$

$$u_1 = \mathcal{L}^{-1}\left[\frac{1}{s^\alpha}\mathcal{L}[u_{0xx}]\right] + \mathcal{L}^{-1}\left[\frac{6}{s^\alpha}\mathcal{L}[u_0 - \bar{A}_0]\right], \quad (5.102)$$

$$u_2 = \mathcal{L}^{-1}\left[\frac{1}{s^\alpha}\mathcal{L}[u_{1xx}]\right] + \mathcal{L}^{-1}\left[\frac{6}{s^\alpha}\mathcal{L}[u_1 - \bar{A}_1]\right], \quad (5.103)$$

$$u_3 = \mathcal{L}^{-1}\left[\frac{1}{s^\alpha}\mathcal{L}[u_{2xx}]\right] + \mathcal{L}^{-1}\left[\frac{6}{s^\alpha}\mathcal{L}[u_2 - \bar{A}_2]\right], \quad (5.104)$$

$$\vdots$$

$$u_n(x,t)=\mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\mathcal{L}[\ u_{2xx}(x,t)]\right]+\mathcal{L}^{-1}\left[\frac{6}{s^{\alpha}}\mathcal{L}[\ u_{(n-1)}(x,t)-\bar{A}_{n-1}(u)]\right], n=1,2,3,\dots\ . \tag{5.105}$$

For $\bar{A}_n$, the first components are provided by

$$\left.\begin{aligned}\bar{A}_0&=u_0^2,\\ \bar{A}_1&=2u_0u_1+u_1^2,\\ \bar{A}_2&=2u_0u_2+2u_1u_2+u_2^2,\\ &\vdots\end{aligned}\right\} \tag{5.106}$$

from Eq (5.102), Eq (5.105), and Eq (5.106), we have that

$$\begin{aligned}u_1&=\mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\mathcal{L}[\ u_{0xx}]\right]+\mathcal{L}^{-1}\left[\frac{6}{s^{\alpha}}\ \mathcal{L}[\ u_0-\bar{A}_0]\right]\\ &=\ \mathcal{L}^{-1}\left[\frac{\frac{6e^{2x}}{(1+e^x)^4}-\frac{2e^x}{(1+e^x)^3}}{s^{\alpha+1}}\right]+\mathcal{L}^{-1}\left[6\ \frac{-\frac{1}{(1+e^x)^4}+\frac{1}{(1+e^x)^2}}{s^{\alpha+1}}\right]\\ &=\frac{\left(\frac{6e^{2x}}{(1+e^x)^4}-\frac{2e^x}{(1+e^x)^3}\right)t^{\alpha}}{\Gamma(1+\alpha)}+\frac{6\left(-\frac{1}{(1+e^x)^4}+\frac{1}{(1+e^x)^2}\right)t^{\alpha}}{\Gamma(1+\alpha)}.\end{aligned} \tag{5.107}$$

Similarly, we get u_2, u_3 ... as follows:

$$\begin{aligned}u_2=&-\frac{120e^{2x}t^{2\alpha}}{(1+e^x)^6\Gamma(1+2\alpha)}+\frac{120e^{4x}t^{2\alpha}}{(1+e^x)^6\Gamma(1+2\alpha)}+\frac{24e^{x}t^{2\alpha}}{(1+e^x)^5\Gamma(1+2\alpha)}-\frac{144e^{3x}t^{2\alpha}}{(1+e^x)^5\Gamma(1+2\alpha)}+\\ &\frac{78e^{2x}t^{2\alpha}}{(1+e^x)^4\Gamma(1+2\alpha)}-\frac{14e^{x}t^{2\alpha}}{(1+e^x)^3\Gamma(1+2\alpha)}+6\left(\frac{12t^{2\alpha}}{(1+e^x)^6\Gamma(1+2\alpha)}-\frac{12e^{2x}t^{2\alpha}}{(1+e^x)^6\Gamma(1+2\alpha)}+\frac{4e^{x}t^{2\alpha}}{(1+e^x)^5\Gamma(1+2\alpha)}-\right.\\ &\frac{18t^{2\alpha}}{(1+e^x)^4\Gamma(1+2\alpha)}+\frac{6e^{2x}t^{2\alpha}}{(1+e^x)^4\Gamma(1+2\alpha)}-\frac{2e^{x}t^{2\alpha}}{(1+e^x)^3\Gamma(1+2\alpha)}+\frac{6t^{2\alpha}}{(1+e^x)^2\Gamma(1+2\alpha)}-\\ &\frac{36t^{3\alpha}\Gamma(1+2\alpha)}{(1+e^x)^8\Gamma(1+\alpha)^2\Gamma(1+3\alpha)}+\frac{72e^{2x}t^{3\alpha}\Gamma(1+2\alpha)}{(1+e^x)^8\Gamma(1+\alpha)^2\Gamma(1+3\alpha)}-\frac{36e^{4x}t^{3\alpha}\Gamma(1+2\alpha)}{(1+e^x)^8\Gamma(1+\alpha)^2\Gamma(1+3\alpha)}-\\ &\frac{24e^{x}t^{3\alpha}\Gamma(1+2\alpha)}{(1+e^x)^7\Gamma(1+\alpha)^2\Gamma(1+3\alpha)}+\frac{24e^{3x}t^{3\alpha}\Gamma(1+2\alpha)}{(1+e^x)^7\Gamma(1+\alpha)^2\Gamma(1+3\alpha)}+\frac{72t^{3\alpha}\Gamma(1+2\alpha)}{(1+e^x)^6\Gamma(1+\alpha)^2\Gamma(1+3\alpha)}-\\ &\left.\frac{76e^{2x}t^{3\alpha}\Gamma(1+2\alpha)}{(1+e^x)^6\Gamma(1+\alpha)^2\Gamma(1+3\alpha)}+\frac{24e^{x}t^{3\alpha}\Gamma(1+2\alpha)}{(1+e^x)^5\Gamma(1+\alpha)^2\Gamma(1+3\alpha)}-\frac{36t^{3\alpha}\Gamma(1+2\alpha)}{(1+e^x)^4\Gamma(1+\alpha)^2\Gamma(1+3\alpha)}\right),\end{aligned} \tag{5.108}$$

Hence, the approximate series solution is provided by

$$\begin{aligned}u(x,t)=&\frac{1}{(1+e^x)^2}+\frac{\left(\frac{6e^{2x}}{(1+e^x)^4}-\frac{2e^x}{(1+e^x)^3}\right)t^{\alpha}}{\Gamma[1+\alpha]}+\frac{6\left(-\frac{1}{(1+e^x)^4}+\frac{1}{(1+e^x)^2}\right)t^{\alpha}}{\Gamma[1+\alpha]}-\frac{120e^{2x}t^{2\alpha}}{(1+e^x)^6\Gamma(1+2\alpha)}+\frac{120e^{4x}t^{2\alpha}}{(1+e^x)^6\Gamma(1+2\alpha)}+\\ &\frac{24e^{x}t^{2\alpha}}{(1+e^x)^5\Gamma(1+2\alpha)}-\frac{144e^{3x}t^{2\alpha}}{(1+e^x)^5\Gamma(1+2\alpha)}+\frac{78e^{2x}t^{2\alpha}}{(1+e^x)^4\Gamma(1+2\alpha)}-\frac{14e^{x}t^{2\alpha}}{(1+e^x)^3\Gamma(1+2\alpha)}+6\left(\frac{12t^{2\alpha}}{(1+e^x)^6\Gamma(1+2\alpha)}-\right.\\ &\frac{12e^{2x}t^{2\alpha}}{(1+e^x)^6\Gamma(1+2\alpha)}+\frac{4e^{x}t^{2\alpha}}{(1+e^x)^5\Gamma(1+2\alpha)}-\frac{18t^{2\alpha}}{(1+e^x)^4\Gamma(1+2\alpha)}+\frac{6e^{2x}t^{2\alpha}}{(1+e^x)^4\Gamma(1+2\alpha)}-\frac{2e^{x}t^{2\alpha}}{(1+e^x)^3\Gamma(1+2\alpha)}+\\ &\frac{6t^{2\alpha}}{(1+e^x)^2\Gamma(1+2\alpha)}-\frac{36t^{3\alpha}\Gamma(1+2\alpha)}{(1+e^x)^8\Gamma(1+\alpha)^2\Gamma(1+3\alpha)}+\frac{72e^{2x}t^{3\alpha}\Gamma(1+2\alpha)}{(1+e^x)^8\Gamma(1+\alpha)^2\Gamma(1+3\alpha)}-\frac{36e^{4x}t^{3\alpha}\Gamma(1+2\alpha)}{(1+e^x)^8\Gamma(1+\alpha)^2\Gamma(1+3\alpha)}-\\ &\frac{24e^{x}t^{3\alpha}\Gamma(1+2\alpha)}{(1+e^x)^7\Gamma(1+\alpha)^2\Gamma(1+3\alpha)}+\frac{24e^{3x}t^{3\alpha}\Gamma(1+2\alpha)}{(1+e^x)^7\Gamma(1+\alpha)^2\Gamma(1+3\alpha)}+\frac{72t^{3\alpha}\Gamma(1+2\alpha)}{(1+e^x)^6\Gamma(1+\alpha)^2\Gamma(1+3\alpha)}-\frac{76e^{2x}t^{3\alpha}\Gamma(1+2\alpha)}{(1+e^x)^6\Gamma(1+\alpha)^2\Gamma(1+3\alpha)}+\\ &\left.\frac{24e^{x}t^{3\alpha}\Gamma(1+2\alpha)}{(1+e^x)^5\Gamma(1+\alpha)^2\Gamma(1+3\alpha)}-\frac{36t^{3\alpha}\Gamma(1+2\alpha)}{(1+e^x)^4\Gamma(1+\alpha)^2\Gamma(1+3\alpha)}\right).\end{aligned} \tag{5.109}$$

Consequently, the approximate solution with four iterations and calculation at $\alpha = 1$ is addressed as

$$u(x,t) \approx 0.14253695659655094 + 0.8872345867463686t + 1.9239193892738164t^2 + 0.17498279772388248t^3 - 6.259794369056014t^4 - 8.590285869012614t^5 + 16.657112401737592t^6 + 30.07332711173555t^7 - 6.553026812419212t^8 - 49.66234666289279t^9 - 6.091171959532517t^{10} + 48.918227619216054t^{11} - 9.20475377354154t^{12} - 19.07409607484533t^{13} + 11.03174095751092t^{14} - 1.805485903804283t^{15}. \tag{5.110}$$

It is necessary to observe that in this study, the proposed method provides higher accuracy, faster convergence, and lower computational complexity than the method presented in Merdan [42], who the modified Riemann-Liouville derivative to solve time-fractional reaction-diffusion equation, and in Shwayyea [43], who presented nonlinear partial differential equations for the Fisher time fractional.

Table 19 demonstrates that the accelerated Adomian decomposition method combined with the Laplace transform (LAADM) produces a smaller absolute error than the fractional variational iteration method (FVIM) when using the same number of iterations (three iterations). This suggests that the proposed approach delivers more accurate and reliable approximations with improved convergence characteristics.

Table 19. Absolute errors and experimental rates of convergence of the approximate and exact solution derived from applying the Laplace transform combined with the accelerated Adomian decomposition method (LAADM) compared to that obtained using the fractional variational iteration method (FVIM), after utilizing three iterations for Example 5.5, when $\alpha = 1$.

			LAADM			FVIM [42]		
X	t	Exact solution	Approx. solution	Absolute error	Exp. rate	Approx. solution	Absolute error	Exp. rate
0.2	0	0.20264943	0.20264943	0	–	0.20264943	0	–
	0.05	0.26265358	0.26264676	6.8187554×10^{-6}	–	0.26265358	0.00266970	–
	0.1	0.32998421	0.32987329	0.00011091774	4.02384	0.32998420	0.00866211	1.69804
	0.15	0.40212795	0.40152044	0.0006075083	4.19414	0.40212794	0.01471330	1.30663
	0.2	0.47606478	0.47390913	0.0021556584	4.40238	0.47606478	0.01763302	0.62924
T	x	Exact solution	Approx. solution	Absolute error	Exp. rate	Approx. solution	Absolute error	Exp. rate
0.2	0	0.53444665	0.53174702	0.0026996216	–	0.53194780	0.00249884	–
	0.25	0.46128371	0.45932034	0.0019633679	0	0.44013926	0.02125070	0
	0.5	0.38745562	0.38645045	0.0010051646	–0.9659	0.35144129	0.03611834	0.76522
	0.75	0.31604242	0.31547753	0.00056488552	–1.4213	0.27079533	0.04534219	0.56093
	1.0	0.25	0.24909227	0.00090772683	1.64877	0.20157642	0.04850512	0.23440

Table 20 demonstrates that the absolute error and relative error obtained using LAADM is significantly lower than that obtained by the LRPSM with the same number of iterations (4 iterations), confirming the higher accuracy and superior convergence of the suggested approach.

Table 20. Absolute errors and experimental rates of convergence of the approximate and exact solution derived from applying the Laplace transform combined with the accelerated Adomian decomposition method (LAADM) compared to that obtained using the Laplace with residual power series method (LRPSM), after utilizing four iterations for Example 5.5, when $\alpha = 1$ and $x = 0.5$.

		LAADM			
t	Exact solution	Approximate solution	Relative error	Absolute error	Exp. Rate
0.01	0.151602	0.151602	1.27917×10^{-9}	1.9392×10^{-10}	–
0.05	0.191689	0.191689	3.04602×10^{-6}	5.8389×10^{-7}	4.97690
0.1	0.25	0.249982	7.09648×10^{-5}	1.77412×10^{-5}	4.92526
0.15	0.316042	0.315917	3.96564×10^{-4}	1.25331×10^{-4}	4.82179
0.2	0.387456	0.386985	1.21447×10^{-3}	4.70552×10^{-4}	4.59865
		LRPSM [43]			
t	Exact solution	Approximate solution	Relative error	Absolute error	Exp. Rate
0.01	0.151602	0.151602	2.28746×10^{-7}	3.46783×10^{-8}	–
0.05	0.191689	0.191713	1.2251×10^{-4}	2.34838×10^{-5}	4.04983
0.1	0.25	0.250483	1.93108×10^{-3}	4.82771×10^{-4}	4.3616
0.15	0.316042	0.318677	8.33757×10^{-3}	2.63503×10^{-3}	4.18558
0.2	0.387456	0.395971	2.19785×10^{-2}	8.51568×10^{-3}	4.07747

Table 21. Absolute errors of the approximate and exact solution derived from applying the LAADM after using three iterations with higher precision (e.g., 34 digits) for Example 5.5, when $\alpha = 1$.

		Approximate solution for three iterations $u(x,t) = u_0 + u_1 + u_2 + u_3$		
x	t	Exact solution	Approximate solution	Absolute error
0.2	0	0.2026494299756622	0.2026494299756622	0
	0.05	0.2626535814030939	0.2626467626476747	$6.818755419127874\times10^{-6}$
	0.1	0.3299842051209132	0.3298732873850498	0.0001109177358633029
	0.15	0.4021279477866175	0.4015204394844591	0.0006075083021583466
	0.2	0.476064784607318	0.4739091261582335	0.002155658449084566
		Approximate solution for three iterations $u(x,t) = u_0 + u_1 + u_2 + u_3$		
t	x	Exact solution	Approximate solution	Absolute error
0.2	0	0.534446645388523	0.5317470238095239	0.002699621578999278
	0.25	0.4612837054135789	0.4593203374864228	0.001963367927156179
	0.5	0.3874556190002601	0.3864504543765136	0.001005164623746502
	0.75	0.3160424181481997	0.3154775326308731	0.0005648855173265854
	1.0	0.25	0.2490922731679811	0.000907726832018813

Table 22. Absolute errors of the approximate and exact solution derived from applying the LAADM after using four iterations with higher precision (e.g., 34 digits) for Example 5.5, when $\alpha = 1$ and $x = 0.5$.

	Approximate solution for four iterations $u(x,t) = u_0 + u_1 + u_2 + u_3 + u_4$			
T	Exact solution	Approximate solution	Relative error	Absolute error
0.01	0.1516018061396487	0.1516018059457246	$1.279167654362572\times10^{-9}$	$1.939241196823646\times10^{-10}$
0.05	0.1916894163766035	0.191688832486464	$3.046021791961979\times10^{-6}$	$5.838901395707405\times10^{-7}$
0.1	0.25	0.2499822587876142	$7.096484954316473\times10^{-5}$	$1.774121238575996\times10^{-5}$
0.15	0.3160424181481998	0.3159170870943572	$3.965640263636293\times10^{-4}$	$1.25331053842527\times10^{-4}$
0.2	0.3874556190002601	0.3869850668700772	$1.214467172774597\times10^{-3}$	$4.705521301828913\times10^{-4}$

Table 21 shows that after the use of higher precision (e.g., 34 digits) and applying three iterations, the approximate solution closely converges to the exact solution. The superiority in terms of lower absolute error confirms the accuracy and numerical efficiency of our proposed approach.

As illustrated in Table 22 and after using higher precision (e.g., 34 digits) with four iterations, the approximate solution approaches the exact solution; also, the reduced absolute error and relative error highlight the precision and strong convergence of the suggested approach.

6. Conclusions

In conclusion, the proposed method of Laplace transform combined with the accelerated Adomian decomposition method (LAADM) offers a powerful and efficient framework to investigate the solutions of nonlinear partial differential time-fractional equations. The method efficiently combines the Laplace transform, which simplifies the handling of initial conditions and transforms the problem into an algebraic form, with the accelerated Adomian decomposition method, which improves the convergence rate of the solution series. By incorporating fractional derivatives in the sense of Caputo, LAADM is well-suited to handle the memory effects and hereditary properties inherent in fractional-order models. The numerical examples presented in the study confirm that the method not only achieves high accuracy but also reduces the computational burden of traditional methods. These features make LAADM a promising tool for addressing complex problems in various scientific and engineering applications where fractional dynamics are significant, such as viscoelastic materials, diffusion processes, and control systems.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare there is no conflict of interest.

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