Electronic Research Archive

https://www.aimspress.com/journal/era

ERA, 33(10): 6514–6532.
DOI: 10.3934/era.2025287
Received: 17 August 2025
Revised: 27 September 2025
Accepted: 09 October 2025
Published: 30 October 2025

Research article

Stability analysis of delayed neural networks via improved negative definite conditions

Xiao-Gui Liu[1,2], Xiang-Jie Zhou[1,2,*], Min-Hai Zhang[1,2] and Xin Zhou[3]

[1] Hunan High Speed Railway Operation Safety Assurance Engineering Technology Research Center, Zhuzhou 412006, China

[2] School of Railway Power Supply and Electrical Engineering, Hunan Vocational College of Railway Technology, Zhuzhou 412006, China

[3] College of Science, Hunan University of Technology, Zhuzhou 412007, China

* **Correspondence:** Email: xiangjiez2024@163.com.

Abstract: This article discusses the issue of delay-dependent stability in generalized neural networks (GNNs). First, a suitable Lyapunov-Krasovskii functional (LKF), including more state information on time delays, was constructed, effectively reducing the system's conservatism. Second, a novel stability condition was established by utilizing the suitable LKF and the matrix-valued cubic polynomials to determine the negative definite conditions. Finally, the experimental simulation results were validated using three typical numerical examples. The experimental results demonstrated the efficiency and merits of our proposed approach compared with the existing methods.

Keywords: stability condition; neural networks; linear matrix inequality (LMI); polynomial inequalities; negative definite conditions

---

1. Introduction

In recent years, with the development of artificial intelligence technology and the continuous update of neural network algorithms, the application of neural networks in signal processing, speech recognition, image decryption, and other natural sciences fields has gradually emerged [1, 2]. As a computational model simulating the human brain, neural networks can process a large amount of data quickly by simulating the signal transmission and learning mode between neurons. Recently, data show that it has surpassed human performance in various fields and will become the focus of scientific research in the future. Neural networks can be broadly classified into local field networks and static neural networks. Initially, researchers conducted separate studies on the two types of neural networks. To analyze the two types of neural networks simultaneously, the generalized neural networks (GNNs) model was proposed in [3] by combining the above two types of neural networks. Reference [4] points out that in the practical application of GNNs, a time-delay phenomenon occurs, primarily caused by the inherent communication delay of neurons and the limited switching speed of amplifiers. From this perspective, the time delay phenomenon is inevitable and will affect the stability of the GNNs. Therefore, analyzing the stability of GNNs with a time delay is of great practical significance.

Recently, there has been significant interest in developing less conservative stability conditions for GNNs with time-varying delays, which has emerged as a prominent research area [5–11]. In general, the system has two kinds of delay stability conditions: delay-dependent and delay-independent. The delay-dependent stability condition is less conservative compared with the delay-independent stability condition. Up to now, much research has been devoted and some results have been achieved in deriving less conservative delay-dependent stability conditions for GNNs with a time-varying delay.

Based on the Lyapunov-Krasovskii functional (LKF) analysis method, there are two main ways to reduce the conservatism of the time-delay system: one is to construct an appropriate LKF [12–16], and the other is to estimate the integral term in the time derivative of the constructed LKF via efficient techniques [17–21]. In [22], a stability criterion of GNNs with less conservatism was derived by constructing an LKF that considers more augmented information. In [23], the authors partitioned the delay interval into N equal subintervals, resulting in a stability condition for GNNs. In [24], the LKF with triple integral terms was constructed. Novel LKFs incorporating cross-terms of state and activation function were developed in [25]. The stability problem of GNNs in [26, 27] was discussed via constructing LKFs with more delay state information. The approaches mentioned above are mainly meant to make the LKFs include more delay state information to reduce the GNNs' conservatism.

Estimating the upper bound value of the LKF's derivative is another essential way to reduce the conservatism of GNNs. The integral inequality approach, since the proposal of the Wirtinger-based integral inequality [28], has played a predominant role in estimating integral terms. Inspired by the Wirtinger-based integral inequality, researchers have done a lot of research work and proposed many integral inequalities, such as the Bessel-Legendre integral inequality [29, 30], auxiliary functions-based integral inequality [31], and free-matrix-based integral inequality [32–35]. The extra reciprocally convex matrix inequalities are less conservative in bounding quadratic integral terms, except for free-matrix-based integral inequalities. Still, when dealing with time-varying delays, reciprocally convex problems need to be transformed into linear matrix inequality (LMI) conditions [36, 37]. Considering the superiority of the free-matrix-based integral inequality method, this paper applies the technique to bound the integral term in the LKF's derivative.

Due to the appearance of higher-order polynomials with time-varying delay functions, it is essential to establish the negative definite conditions for derivatives in the last step of deriving the stability criterion. References [31, 32] point out that convex analysis is the most effective method to deal with linear polynomials. However, the perfect solution to form negative definite conditions has yet to be found for processing higher-order polynomials with time-varying delays. In [38], although the negative definite condition contains only one LMI, it requires the introduction of an additional high-dimensional matrix, significantly increasing computational complexity. Reference [39] presents negative definite conditions composed of several LMIs with appropriate parameters; however, the resulting stability criterion exhibits some degree of conservatism. In [40], the negative definite conditions of quadratic polynomials were extended to infinite series and included the advantages mentioned in [38, 39, 41]. Although the negative definite conditions with infinite series do not introduce additional matrices, the increased number of LMIs leads to computational complexity. Recently, a class of negative definite conditions involving higher-order polynomials was proposed in [42], which can establish LMI conditions. However, the negative definite conditions still require the introduction of additional matrices. From the above discussion, there is still room for improvement in reducing the computational complexity.

For GNNs with time-varying delay, many stability criteria have been derived. In [43, 44], a less conservative stability condition was obtained by introducing slack variables and constructing an augmented LKF with triple integral terms. In [9], an LKF with double integral terms was constructed, and a greatly improved stability condition for GNNs was presented. Many additional state variables were introduced into the Lyapunov functional to form quadratic polynomials and linear conditions concerning time-varying delays. However, our research finds that if no additional state variables were introduced, a cubic polynomial with a time-varying delay will appear in the derivative of the LKF. At the same time, we use the free-matrix-based integral inequality to accurately bound the integral term in the derivative of the LKF and transform the nonlinear term into a polynomial. Thus, to obtain the negative definite condition of the stability criterion, we apply cubic polynomial inequalities to deal with the nonlinear problem.

Based on the above discussion, this paper mainly studies the delay-dependent stability of GNNs with time-varying delay. The main contributions are summarized as follows:

1) An augmented LKF considering more delay state information is constructed, which improves the direct relationship between each delay state variable and effectively reduces the conservatism of the system.

2) In deriving the stability criterion of GNNs with time-varying delay, the integral term in the derivative of the augmented LKF is bounded precisely by the method of the free-matrix-based integral inequality without introducing additional state variables.

3) The positive definite constraint of the LKF is relaxed appropriately, which reduces the conservativeness of the system from another aspect.

4) A new negative definite condition is derived using cubic polynomials. This innovative approach decreases the matrix's dimension and significantly reduces the computational complexity when deriving LMIs. This reduction in complexity provides a sense of relief and reassurance about the practicality of our research.

Through the rigorous examination of three typical numerical examples, we verify the new criterion's effectiveness and demonstrate its clear superiority over the existing results. This robust validation instills confidence and trust in the reliability of our findings.

Notation: Throughout the paper, $\mathbb{Z}^n$ stands for the n-dimensional Euclidean space, and $\mathbb{Z}^{n\times m}$ is the $n \times m$ real matrix; $P > 0$ means P is positive definite and symmetric; the superscripts -1 and T represent the inverse and transpose of a matrix, respectively; 0 and I are the zero matrix and identity matrix of appropriate dimensions, respectively; col$\{\cdots\}$ and diag$\{\cdots\}$ stand for a block-column vector and a block diagonal matrix, respectively; $*$ is the symmetric term in a symmetric matrix, and $\mathrm{Sym}\{H\} = H + H^{\mathrm{T}}$.

2. Preliminaries

Consider the following GNNs.

$$\dot{\chi}(t) = -A\chi(t) + W_0\lambda(W_2\chi(t)) + W_1\lambda(W_2\chi(t-k(t))), \tag{2.1}$$

where $\chi(t) \in \mathbb{Z}^n$ is the neuron state variable; n is the number of neurons; $A = \text{diag}\{a_1, a_2, \cdots, a_n\}$ is a positive diagonal matrix; and $W_i \in \mathbb{Z}^{n\times n}, i = 0, 1, 2$, are the real matrices. For the time-delay signal, $k(t)$ is a continuous function satisfying

$$0 \le k(t) \le k, \dot{k}(t) \le \mu, \tag{2.2}$$

where k and μ are constants, $k > 0$ and $\mu > 0$, $\lambda(\cdot) = \text{col}\{\lambda_1(\cdot), \lambda_2(\cdot), \cdots, \lambda_{\text{n}}(\cdot)\} \in \mathbb{Z}^{\text{n}}$ is the activation function of neurons, and the functions $\lambda_{\text{n}}(\cdot)$ are continuous, bounded, and satisfy

$$j_i^- \le \frac{\lambda_i(a) - \lambda_i(b)}{a-b} \le j_i^+, i = 1, 2, \cdots n, \forall a \neq b, \tag{2.3}$$

where j_i^- and j_i^+ are constant scalars, and we denote $J_1 = \text{diag}\{j_1^-, j_2^-, \cdots j_n^-\}$ and $J_2 = \text{diag}\{j_1^+, j_2^+, \cdots j_n^+\}$.

The focus of this article is to derive delay-dependent stability conditions with less conservatism for GNNs (2.1). In order to obtain the main result, Lemmas 1–5 are crucial.

Lemma 1. *[32] For a differentiable and integrable function $\chi : [d \;\; e] \to \mathbb{Z}^n$, matrix $\bar{H} > 0$, and $\bar{M}_j$, $j = 1, 2, 3$, the inequality* (2.4) *holds:*

$$-\int_d^e \dot{\chi}^{\text{T}}(\theta)\bar{\text{H}}\dot{\chi}(\theta)d\theta \le \sum_{i=1}^{3}(\frac{e-d}{2i-1}\varsigma^{\text{T}}\bar{M}_i\bar{\text{H}}^{-1}\bar{M}_i^{\text{T}}\varsigma + \text{Sym}\{\varsigma^{\text{T}}\bar{M}_i\delta_i\}), \tag{2.4}$$

where

$$\delta_1 = -\chi(d) + \chi(e), \delta_2 = \chi(d) + \chi(e) - \tfrac{2}{e-d}\int_d^e \chi(\theta)d\theta,$$
$$\delta_3 = \chi(e) - \chi(d) + \tfrac{6}{e-d}\int_d^e \chi(\theta)d\theta - \tfrac{12}{(e-d)^2}\int_d^e\int_\theta^e \chi(u)dud\theta.$$

Lemma 2. *[28] We are given a symmetric and positive definite matrix $\bar{H}$. Then, for all integrable functions $\chi : [d \;\; e] \to \mathbb{Z}^n$, inequality* (2.5) *holds:*

$$\int_d^e \chi^{\text{T}}(\theta)\bar{H}\chi(\theta)d\theta \ge \frac{1}{e-d}\int_d^e \chi^{\text{T}}(\theta)d\theta\bar{H}\int_d^e \chi(\theta)d\theta + \frac{3}{e-d}\bar{\delta}^{\text{T}}\bar{H}\bar{\delta}, \tag{2.5}$$

where

$$\bar{\delta} = \int_d^e \chi(\theta)d\theta - \tfrac{2}{e-d}\int_d^e\int_d^\theta \chi(u)dud\theta.$$

Lemma 3. *[45] For any given matrix $\bar{M} > 0$, a scalar ℓ in the interval* (0, 1)*, two matrices $\bar{N}_1$ and $\bar{N}_2$ in $\mathbb{Z}^{n\times m}$, we define, for all vectors ζ in $\mathbb{Z}^m$, the function $\bar{\vartheta}(\ell, \bar{M})$ given by:*

$$\bar{\vartheta}(\ell, \bar{M}) = \frac{1}{\ell}\zeta^{\text{T}}\bar{N}_1^{\text{T}}\bar{M}\bar{N}_1\zeta + \frac{1}{1-\ell}\zeta^{\text{T}}\bar{N}_2^{\text{T}}\bar{M}\bar{N}_2\bar{\zeta}.$$

Then, if there exists a matrix $\bar{X}$ in $\mathbb{Z}^{n\times n}$ such that $\begin{bmatrix} \bar{M} & \bar{X} \\ * & \bar{M} \end{bmatrix} > 0$*, then the following inequality holds:*

$$\min_{\ell\in(0,1)} \bar{\vartheta}(\ell, \bar{M}) \geqslant \begin{bmatrix} \bar{N}_1\zeta \\ \bar{N}_2\zeta \end{bmatrix}^{\text{T}} \begin{bmatrix} \bar{M} & \bar{X} \\ * & \bar{M} \end{bmatrix} \begin{bmatrix} \bar{N}_1\zeta \\ \bar{N}_2\zeta \end{bmatrix}.$$

To obtain a negative definite condition for the stability criterion, the following inequalities are necessary.

Lemma 4. *For the polynomial function* $g(h) = \bar{a}h^3 + \bar{b}h^2 + \bar{c}h + \bar{d}$ *with* $h \in [0 \;\; k]$*, and* $\bar{a}, \bar{b}, \bar{c}, \bar{d} \in \mathbb{Z}^n$*,* $g(h) < 0$ *if there exists matrix* $\bar{N} \in \mathbb{Z}^{2n\times n}$*, such that inequalities* (2.6) *hold:*

$$\bar{\omega}(h) + \mathrm{Sym}\{\bar{N}\beta(h)\} = \begin{bmatrix} \Theta_{11}(h) & \Theta_{12}(h) \\ * & \Theta_{22}(h) \end{bmatrix} < 0; \; h = 0, k, \tag{2.6}$$

where

$$\bar{\omega}(h) = \begin{bmatrix} \bar{d} + h\bar{c} & \frac{h}{2}\bar{b} \\ * & h\bar{a} \end{bmatrix}, \; \beta(h) = [hI_n \;\; -I_n].$$

Proof: Given $\gamma^{\mathrm{T}}(h) = [I_n \;\; hI_n]$, we can get $\beta(h)\gamma(h) = 0$ and $g(h) = \gamma^{\mathrm{T}}(h)\bar{\omega}(h)\gamma(h)$. Then, for any matrix $\bar{N} \in \mathbb{Z}^{2n\times n}$, define

$$\bar{\psi}(h) = \gamma^{\mathrm{T}}(h)[\bar{\omega}(h) + \mathrm{Sym}\{\bar{N}\beta(h)\}]\gamma(h).$$

Performing the calculation for $\bar{\psi}(h)$ yields $\bar{\psi}(h) = g(h) + \gamma^{\mathrm{T}}(h)\bar{N}\beta(h)\gamma(h) + [\beta(h)\gamma(h)]^{\mathrm{T}}\bar{N}^{\mathrm{T}}\gamma(h)$, while $\beta(h)\gamma(h) = 0$. Thus, for $h \in [0 \;\; k]$, $g(h) < 0$ can be guaranteed by $\bar{\psi}(h) < 0$ ($h = 0, k$), This ends the proof.

Remark 1. *Since a cubic polynomial with a time-varying delay appears in the LKF's derivative, it is necessary to form the negative definite condition of the cubic polynomial when establishing the stability criterion. The cubic polynomials in Lemma 4 can not only deal with the nonlinear term in the integral quadratic term effectively, but also reduce the dimension of LMIs.*

Lemma 5. *[31] For an integrable function* $\chi : [d \;\; e] \to \mathbb{Z}^n$*, given a symmetric and positive definite matrix* $\bar{H}$*, inequalities* (2.7) *hold:*

$$\begin{aligned} &\int_d^e \int_\theta^e \dot{\chi}^{\mathrm{T}}(u)\bar{H}\dot{\chi}(u)dud\theta \geqslant 2\rho_1^{\mathrm{T}}\bar{H}\rho_1 + 4\rho_2^{\mathrm{T}}\bar{H}\rho_2, \\ &\int_d^e \int_\theta^e \chi^{\mathrm{T}}(u)\bar{H}\chi(u)dud\theta \geqslant \frac{2}{(e-d)^2} \int_d^e \int_\theta^e \chi^{\mathrm{T}}(u)dud\theta\bar{H} \int_d^e \int_\theta^e \chi(u)dud\theta, \end{aligned} \tag{2.7}$$

where

$$\rho_1 = \chi(e) - \frac{1}{e-d}\int_d^e \chi(u)du, \;\; \rho_2 = \chi(e) + \frac{2}{e-d}\int_d^e \chi(u)du - \frac{6}{(e-d)^2}\int_d^e \int_\theta^e \chi(u)dud\theta.$$

3. Main results

In this part, some new stability conditions with less conservatism are provided for GNNs (2.1) based on a suitable augmented LKF, an integral inequality proposed in Lemmas 1 and 2, and negative definite conditions obtained in Lemma 4. To simplify the description, the mathematical notations we need are introduced as follows:

$$\eta_1(t) = \left[\begin{array}{ccccc} \chi^{\mathrm{T}}(t) & \int_{t-k}^{t}\chi^{\mathrm{T}}(u)du & \int_{t-k(t)}^{t}\chi^{\mathrm{T}}(u)du & \int_{t-k(t)}^{t}\int_{\theta}^{t}\chi^{\mathrm{T}}(u)dud\theta & \int_{t-k}^{t}\int_{\theta}^{t}\chi^{\mathrm{T}}(u)dud\theta \end{array}\right]^{\mathrm{T}},$$
$$\eta_2(t,\theta) = \left[\begin{array}{ccccc} \chi^{\mathrm{T}}(t) & \chi^{\mathrm{T}}(\theta) & \int_{\theta}^{t}\chi^{\mathrm{T}}(\alpha)d\alpha & \int_{t-k}^{\theta}\chi^{\mathrm{T}}(u)du & \int_{t-k(t)}^{\theta}\chi^{\mathrm{T}}(\theta)d\theta \end{array}\right]^{\mathrm{T}},$$
$$\eta_3(t,\theta) = \left[\begin{array}{cccc} \chi^{\mathrm{T}}(t) & \chi^{\mathrm{T}}(\theta) & \int_{\theta}^{t}\chi^{\mathrm{T}}(\alpha)d\alpha & \int_{t-k}^{\theta}\chi^{\mathrm{T}}(\alpha)d\alpha \end{array}\right]^{\mathrm{T}},\ \eta_4(t) = \left[\begin{array}{cc} \chi^{\mathrm{T}}(t) & \lambda(W_2\chi^{\mathrm{T}}(t)) \end{array}\right]^{\mathrm{T}},$$
$$\eta_5(t) = \left[\begin{array}{cc} \dot{\chi}^{\mathrm{T}}(t) & \chi^{\mathrm{T}}(t) \end{array}\right]^{\mathrm{T}},\ \upsilon_1(t) = \left[\begin{array}{cc} \int_{t-k(t)}^{t}\eta_5^{\mathrm{T}}(u)du & \int_{t-k}^{t-k(t)}\eta_5^{\mathrm{T}}(u)du \end{array}\right]^{\mathrm{T}},$$
$$\upsilon_2(t) = \frac{2}{k(t)}\int_{t-k(t)}^{t}\int_{\theta}^{t}\eta_5(u)dud\theta - \int_{t-k(t)}^{t}\eta_5(u)du,$$
$$\upsilon_3(t) = \frac{2}{k-k(t)}\int_{t-k}^{t-k(t)}\int_{\theta}^{t-k(t)}\eta_5(u)dud\theta - \int_{t-k}^{t-k(t)}\eta_5(u)du,$$
$$\bar{\zeta}_{\mathrm{a}}(t) = \left[\begin{array}{ccccc} \chi^{\mathrm{T}}(t) & \chi^{\mathrm{T}}(t-k(t)) & \chi^{\mathrm{T}}(t-k) & \frac{1}{k(t)}\int_{t-k(t)}^{t}\chi^{\mathrm{T}}(\alpha)d\alpha & \frac{1}{k-k(t)}\int_{t-k}^{t-k(t)}\chi^{\mathrm{T}}(\alpha)d\alpha \end{array}\right]^{\mathrm{T}},$$
$$\bar{\zeta}_{\mathrm{b}}(t) = \left[\begin{array}{ccc} \frac{1}{k^2(t)}\int_{t-k(t)}^{t}\int_{\theta}^{t}\chi^{\mathrm{T}}(u)dud\theta & \frac{1}{(k-k(t))^2}\int_{t-k}^{t-k(t)}\int_{\theta}^{t-k(t)}\chi^{\mathrm{T}}(u)dud\theta & \lambda(W_2\chi^{\mathrm{T}}(t)) \end{array}\right]^{\mathrm{T}},$$
$$\bar{\zeta}_{\mathrm{c}}(t) = \left[\begin{array}{cc} \lambda(W_2\chi^{\mathrm{T}}(t-k(t))) & \lambda(W_2\chi^{\mathrm{T}}(t-k)) \end{array}\right]^{\mathrm{T}},\ \bar{\zeta}(t) = \left[\begin{array}{ccc} \bar{\zeta}_{\mathrm{a}}^{\mathrm{T}}(t) & \bar{\zeta}_{\mathrm{b}}^{\mathrm{T}}(t) & \bar{\zeta}_{\mathrm{c}}^{\mathrm{T}}(t) \end{array}\right]^{\mathrm{T}},$$
$$e_{\mathrm{i}} = \left[\begin{array}{ccc} 0_{n\times(\mathrm{i}-1)n} & I_n & 0_{n\times(10-\mathrm{i})n} \end{array}\right], \mathrm{i} = 1,2,\cdots,10.$$

Theorem 1. *For given constants $k > 0$ and $\mu > 0$, if there exist symmetric and positive definite matrices $P \in \mathbb{Z}^{5n\times5n}$, $Q_1 \in \mathbb{Z}^{5n\times5n}$, $Q_2 \in \mathbb{Z}^{4n\times4n}$, $Z_1 \in \mathbb{Z}^{2n\times2n}$, $Z_2 \in \mathbb{Z}^{2n\times2n}$, $R_1 \in \mathbb{Z}^{n\times n}$, $R_2 \in \mathbb{Z}^{2n\times2n}$, symmetric matrices $P_1 \in \mathbb{Z}^{n\times n}$, $P_2 \in \mathbb{Z}^{n\times n}$, any matrices $N \in \mathbb{Z}^{2n\times n}$, $S_1 \in \mathbb{Z}^{2n\times2n}$, $S_2 \in \mathbb{Z}^{2n\times2n}$, and $Y_j \in \mathbb{Z}^{10n\times3n}$, $j = 1,2$, diagonal matrices $D_l \in \mathbb{Z}^{n\times n}, G_l \in \mathbb{Z}^{n\times n}, T_l \in \mathbb{Z}^{n\times n}, U_l \in \mathbb{Z}^{n\times n}, l = 1,2,3$, and $\Lambda_i \in \mathbb{Z}^{n\times n}, i = 1,2$, the following inequalities* (3.1) *and* (3.2) *hold with positive definite matrices $R_{21} \in \mathbb{Z}^{2n\times2n}$, $R_{22} \in \mathbb{Z}^{2n\times2n}$. Then GNNs* (2.1) *are asymptotically stable.*

$$\left[\begin{array}{ccc} \Theta_{11}(0) & \Theta_{12}(0) & \sqrt{k}Y_2 \\ * & \Theta_{22}(0) & 0 \\ * & * & -\bar{R}_1 \end{array}\right] < 0, \tag{3.1}$$

$$\left[\begin{array}{ccc} \Theta_{11}(k) & \Theta_{12}(k) & \sqrt{k}Y_1 \\ * & \Theta_{22}(k) & 0 \\ * & * & -\bar{R}_1 \end{array}\right] < 0, \tag{3.2}$$

where

$$\left[\begin{array}{cc} d+tc & \frac{t}{2}b \\ * & ta \end{array}\right] + \mathrm{Sym}\{N\alpha(t)\} = \left[\begin{array}{cc} \Theta_{11}(t) & \Theta_{12}(t) \\ * & \Theta_{22}(t) \end{array}\right],\ \alpha(t) = [tI_n \quad -I_n],$$
$$\bar{R}_1 = \mathrm{diag}\{R_1, 3R_1, 5R_1\}, R_{21} = R_2 + \left[\begin{array}{cc} 0 & P_1 \\ P_1 & 0 \end{array}\right],\ R_{22} = R_2 + \left[\begin{array}{cc} 0 & P_2 \\ P_2 & 0 \end{array}\right],$$

$$d = \mathrm{Sym}\{\lambda_{1a}^{T}P\lambda_{2a} + (H_{11}^{T}\Lambda_1 + H_{21}^{T}\Lambda_2)W_2\Gamma + k\Sigma_1\} + \lambda_7^{T}Q_2\lambda_{12a} + Y_1E_1 + Y_2E_2 + \lambda_{3a}^{T}Q_1\lambda_{3a} + \lambda_{4a}^{T}Q_2\lambda_{4a} - (1-\mu)\lambda_{5a}^{T}Q_1\lambda_{5a} - \lambda_{10a}^{T}Q_2\lambda_{10a} + \lambda_{13}^{T}(Z_1 + Z_2)\lambda_{13} - (1-\mu)\lambda_{14}^{T}Z_1\lambda_{14} - \lambda_{15}^{T}Z_2\lambda_{15} + k\Gamma^{T}R_1\Gamma + k^2\lambda_{16}^{T}R_2\lambda_{16} - E_{3a}^{T}\bar{R}_{21}E_{3a} - 3E_{4a}^{T}\bar{R}_{22}E_{4a} + k(e_1^{T}P_1e_1 - e_2^{T}P_1e_2 + e_2^{T}P_2e_2 - e_3^{T}P_2e_3),$$

$$c = \mathrm{Sym}\{\lambda_{1a}^{T}P\lambda_{2b} + \lambda_{1b}^{T}P\lambda_{2a} + \lambda_{3a}^{T}Q_1\lambda_{3b} + \lambda_{4a}^{T}Q_2\lambda_{4b} - (1-\mu)\lambda_{5a}^{T}Q_1\lambda_{5b} - \lambda_{10a}^{T}Q_2\lambda_{10b} + \lambda_6^{T}Q_1\lambda_{8b} + \lambda_7^{T}Q_2\lambda_{12b} - \Sigma_1 + \Sigma_2 - E_{3b}^{T}\bar{R}_{21}E_{3a} - 3E_{4a}^{T}\bar{R}_{22}E_{4b}\},$$

$$b = \mathrm{Sym}\{\lambda_{1a}^{T}P\lambda_{2c} + \lambda_{1b}^{T}P\lambda_{2b} + \lambda_{3b}^{T}Q_1\lambda_{3b} + \lambda_{4b}^{T}Q_2\lambda_{4b} - (1-\mu)\lambda_{5b}^{T}Q_1\lambda_{5b} - \lambda_{10b}^{T}Q_2\lambda_{10b} + \lambda_6^{T}Q_1\lambda_{8c} + \lambda_7^{T}Q_2\lambda_{12c}\} - E_{3b}^{T}\bar{R}_{21}E_{3b} - 3E_{4b}^{T}\bar{R}_{22}E_{4b},$$

$a = \mathrm{Sym}\{\lambda_{1b}^{T}P\lambda_{2c}\}, \lambda_{1a} = [\Gamma^{T}\ e_1^{T} - e_3^{T}\ \ e_1^{T} - e_2^{T} + \dot{k}(t)e_2^{T}\ \ 0\ \ k(e_1^{T} - e_5^{T})]^{T}$,

$\lambda_{1b} = \begin{bmatrix} 0 & 0 & 0 & e_1^{T} - (1-\dot{k}(t))e_4^{T} & e_5^{T} - e_4^{T} \end{bmatrix}^{T}, \lambda_{2a} = \begin{bmatrix} e_1^{T} & ke_5^{T} & 0 & 0 & k^2e_7^{T} \end{bmatrix}^{T}$,

$\lambda_{2b} = \begin{bmatrix} 0 & e_4^{T} - e_5^{T} & e_4^{T} & 0 & k(e_4^{T} - 2e_7^{T}) \end{bmatrix}^{T}, \lambda_{2c} = \begin{bmatrix} 0 & 0 & 0 & e_6^{T} & e_6^{T} + e_7^{T} - e_4^{T} \end{bmatrix}^{T}$,

$\lambda_{3a} = \begin{bmatrix} e_1^{T} & e_1^{T} & 0 & ke_5^{T} & 0 \end{bmatrix}^{T}, \lambda_{3b} = \begin{bmatrix} 0 & 0 & 0 & e_4^{T} - e_5^{T} & e_4^{T} \end{bmatrix}^{T}$,

$\lambda_{4a} = \begin{bmatrix} e_1^{T} & e_1^{T} & 0 & ke_5^{T} \end{bmatrix}^{T}, \lambda_{4b} = \begin{bmatrix} 0 & 0 & 0 & e_4^{T} - e_5^{T} \end{bmatrix}^{T}, \lambda_{5a} = \begin{bmatrix} e_1^{T} & e_2^{T} & 0 & ke_5^{T} & 0 \end{bmatrix}^{T}$,

$\lambda_{5b} = \begin{bmatrix} 0 & 0 & e_4^{T} & -e_5^{T} & 0 \end{bmatrix}^{T}, \lambda_6 = \begin{bmatrix} \Gamma^{T} & 0 & e_1^{T} & -e_3^{T} & (\dot{k}(t) - 1))e_2^{T} \end{bmatrix}^{T}$,

$\lambda_7 = \begin{bmatrix} \Gamma^{T} & 0 & e_1^{T} & -e_3^{T} \end{bmatrix}^{T}, \lambda_{8b} = \begin{bmatrix} e_1^{T} & e_4^{T} & 0 & ke_5^{T} & 0 \end{bmatrix}^{T}$,

$\lambda_{8c} = \begin{bmatrix} 0 & 0 & e_6^{T} & e_4^{T} - e_5^{T} - e_6^{T} & e_4^{T} - e_6^{T} \end{bmatrix}^{T}, \lambda_{10a} = \begin{bmatrix} e_1^{T} & e_3^{T} & ke_5^{T} & 0 \end{bmatrix}^{T}$,

$\lambda_{10b} = \begin{bmatrix} 0 & 0 & e_4^{T} - e_5^{T} & 0 \end{bmatrix}^{T}, \lambda_{12a} = \begin{bmatrix} ke_1^{T} & ke_5^{T} & k^2e_7^{T} & k^2(e_5^{T} - e_7^{T}) \end{bmatrix}^{T}$,

$\lambda_{12b} = \begin{bmatrix} 0 & e_4^{T} - e_5^{T} & k(e_4^{T} - 2e_7^{T}) & k(2e_7^{T} - e_5^{T}) \end{bmatrix}^{T}, \lambda_{13} = \begin{bmatrix} e_1^{T} & e_8^{T} \end{bmatrix}^{T}$,

$\lambda_{12c} = \begin{bmatrix} 0 & 0 & e_6^{T} + e_7^{T} - e_4^{T} & e_4^{T} - e_6^{T} - e_7^{T} \end{bmatrix}^{T}, \lambda_{14} = \begin{bmatrix} e_2^{T} & e_9^{T} \end{bmatrix}^{T}, \lambda_{15} = \begin{bmatrix} e_3^{T} & e_{10}^{T} \end{bmatrix}^{T}$,

$\lambda_{16} = \begin{bmatrix} \Gamma^{T} & e_1^{T} \end{bmatrix}^{T}, \Gamma = -Ae_1 + W_0e_8 + W_1e_9, E_1 = [e_1^{T} - e_2^{T}\ \ e_1^{T} + e_2^{T} - 2e_4^{T}\ \ e_1^{T} - e_2^{T} + 6e_4^{T} - 12e_6^{T}]^{T}$,

$E_2 = [e_2^{T} - e_3^{T}\ \ e_2^{T} + e_3^{T} - 2e_5^{T}\ \ e_2^{T} - e_3^{T} + 6e_5^{T} - 12e_7^{T}]^{T}, E_{3a} = \begin{bmatrix} e_1^{T} - e_2^{T} & 0 & e_2^{T} - e_3^{T} & ke_5^{T} \end{bmatrix}^{T}$,

$E_{3b} = \begin{bmatrix} 0 & e_4^{T} & 0 & -e_5^{T} \end{bmatrix}^{T}, E_{4a} = [e_1^{T} + e_2^{T} - 2e_4^{T}\ \ 0\ \ e_2^{T} + e_3^{T} - 2e_5^{T}\ \ 2ke_7^{T} - ke_5^{T}]^{T}$,

$E_{4b} = \begin{bmatrix} 0 & 2e_6^{T} - e_4^{T} & 0 & -2e_7^{T} + e_5^{T} \end{bmatrix}^{T}, \Sigma_1 = \sum_{i=1}^{3} H_{1i}^{T}D_iH_{2i} + \sum_{i=1}^{3} \Delta_{1i}^{T}T_i\Delta_{2i}$,

$\Sigma_2 = \sum_{i=1}^{3} H_{1i}^{T}G_iH_{2i} + \sum_{i=1}^{3} \Delta_{1i}^{T}U_i\Delta_{2i}, H_{1i} = -e_{i+7} + J_2W_2e_i,\ H_{2i} = e_{i+7} - J_1W_2e_i$,

$\Delta_{1i} = J_2W_2e_i - J_2W_2e_{i+1} + e_{i+8} - e_{i+7}, i = 1, 2,\ \Delta_{2i} = e_{i+7} - e_{i+8} - J_1W_2e_i + J_1W_2e_{i+1}, i = 1, 2$,

$\Delta_{13} = J_2W_2e_1 - J_2W_2e_3 - e_8 + e_{10},\ \Delta_{23} = e_8 - e_{10} - J_1W_2e_1 + J_1W_2e_3$.

Proof: Construct the following augmented LKF candidate:

$$\vartheta(t) = \sum_{i=1}^{3} \vartheta_i(t), \tag{3.3}$$

$$\vartheta_1(t) = \eta_1^{\mathrm{T}}(t)P\eta_1(t) + \int_{t-k(t)}^{t} \eta_2^{\mathrm{T}}(t,\theta)Q_1\eta_2(t,\theta)d\theta + \int_{t-k}^{t} \eta_3^{\mathrm{T}}(t,\theta)Q_2\eta_3(t,\theta)d\theta$$
$$+ \int_{t-k(t)}^{t} \eta_4^{\mathrm{T}}(u)Z_1\eta_4(u)du + \int_{t-k}^{t} \eta_4^{\mathrm{T}}(u)Z_2\eta_4(u)du,$$
$$\vartheta_2(t) = 2\sum_{\mathrm{i}=1}^{n} \int_0^{W_{2\mathrm{i}}\chi(t)} \gamma_{1\mathrm{i}}(\lambda_{\mathrm{i}}(u) - j_{\mathrm{i}}^{-}u)du + 2\sum_{\mathrm{i}=1}^{n} \int_0^{W_{2\mathrm{i}}\chi(t)} \gamma_{2\mathrm{i}}(j_{\mathrm{i}}^{+}u - \lambda_{\mathrm{i}}(u))du,$$
$$\vartheta_3(t) = \int_{t-k}^{t}\int_{\theta}^{t} \dot{\chi}^{\mathrm{T}}(u)R_1\dot{\chi}(u)dud\theta + k\int_{t-k}^{t}\int_{\theta}^{t} \eta_5^{\mathrm{T}}(u)R_2\eta_5(u)dud\theta.$$

Then, differentiating $\vartheta_1(t)$ along with the trajectory of system (2.1) yields

$$\begin{aligned}\dot{\vartheta}_1(t) = \bar{\zeta}^{\mathrm{T}}(t)[&2(k(t)\lambda_{1\mathrm{b}}^{\mathrm{T}} + \lambda_{1\mathrm{a}}^{\mathrm{T}})P(k^2(t)\lambda_{2\mathrm{c}} + k(t)\lambda_{2\mathrm{b}} + \lambda_{2\mathrm{a}}) + (k(t)\lambda_{3\mathrm{b}}^{\mathrm{T}} + \lambda_{3\mathrm{a}}^{\mathrm{T}})Q_1(k(t)\lambda_{3\mathrm{b}} + \lambda_{3\mathrm{a}})\\ &+ \lambda_{13}^{\mathrm{T}}(Z_1 + Z_2)\lambda_{13} + (k(t)\lambda_{4\mathrm{b}}^{\mathrm{T}} + \lambda_{4\mathrm{a}}^{\mathrm{T}})Q_2(k(t)\lambda_{4\mathrm{b}} + \lambda_{4\mathrm{a}}) - (1-\mu)\lambda_{14}^{\mathrm{T}}Z_1\lambda_{14} - \lambda_{15}^{\mathrm{T}}Z_2\lambda_{15}\\ &- (k(t)\lambda_{10\mathrm{b}}^{\mathrm{T}} + \lambda_{10\mathrm{a}}^{\mathrm{T}})Q_2(k(t)\lambda_{10\mathrm{b}} + \lambda_{10\mathrm{a}}) - (1-\mu)(k(t)\lambda_{5\mathrm{b}}^{\mathrm{T}} + \lambda_{5\mathrm{a}}^{\mathrm{T}})Q_1(k(t)\lambda_{5\mathrm{b}} + \lambda_{5\mathrm{a}})\\ &+ 2\lambda_7^{\mathrm{T}}Q_2(k^2(t)\lambda_{12\mathrm{c}} + k(t)\lambda_{12\mathrm{b}} + \lambda_{12\mathrm{a}}) + 2\lambda_6^{\mathrm{T}}Q_1(k^2(t)\lambda_{8\mathrm{c}} + k(t)\lambda_{8\mathrm{b}})]\bar{\zeta}(t).\end{aligned} \tag{3.4}$$

Similarly, we get $\vartheta_2(t)$ and $\vartheta_3(t)$:

$$\begin{aligned}\dot{\vartheta}_2(t) &= 2\sum_{\mathrm{i}=1}^{n} W_{2\mathrm{i}}\dot{\chi}(t)\gamma_{1\mathrm{i}}(\lambda_{\mathrm{i}}(W_{2\mathrm{i}}\chi(t)) - j_{\mathrm{i}}^{-}W_{2\mathrm{i}}\chi(t)) + 2\sum_{\mathrm{i}=1}^{n} W_{2\mathrm{i}}\dot{\chi}(t)\gamma_{2\mathrm{i}}(j_{\mathrm{i}}^{+}W_{2\mathrm{i}}\chi(t) - \lambda_{\mathrm{i}}(W_{2\mathrm{i}}\chi(t)))\\ &= \bar{\zeta}^{\mathrm{T}}(t)\left[2(H_{11}^{\mathrm{T}}\Lambda_1 + H_{21}^{\mathrm{T}}\Lambda_2)W_2\Gamma\right]\bar{\zeta}(t),\end{aligned} \tag{3.5}$$

$$\dot{\vartheta}_3(t) = k\dot{\chi}^{\mathrm{T}}(t)R_1\dot{\chi}(t) - \int_{t-k}^{t}\dot{\chi}^{\mathrm{T}}(u)R_1\dot{\chi}(u)du + k^2\eta_5^{\mathrm{T}}(t)R_2\eta_5(t) - k\int_{t-k}^{t}\eta_5^{\mathrm{T}}(u)R_2\eta_5(u)du. \tag{3.6}$$

Applying Lemma 1 to the first integral term of $\dot{\vartheta}_3(t)$, we get

$$-\int_{t-k}^{t}\dot{\chi}^{\mathrm{T}}(u)R_1\dot{\chi}(u)du \leqslant \bar{\zeta}^{\mathrm{T}}(t)[Sym\{Y_1E_1 + Y_2E_2\} + k(t)Y_1\bar{R}_1^{-1}Y_1^{\mathrm{T}} + (k-k(t))Y_2\bar{R}_1^{-1}Y_2^{\mathrm{T}}]\bar{\zeta}(t). \tag{3.7}$$

Inspired by reference [46], for symmetric matrices $P_1 \in \mathbb{Z}^{n\times n}$, $P_2 \in \mathbb{Z}^{n\times n}$, the zero equalities (3.8) and (3.9) are established.

$$k\{\chi^{\mathrm{T}}(t)P_1\chi(t) - \chi^{\mathrm{T}}(t-k(t))P_1\chi(t-k(t)) - 2\int_{t-k(t)}^{t}\chi^{\mathrm{T}}(u)P_1\dot{\chi}(u)du\} = 0, \tag{3.8}$$

$$k\{\chi^{\mathrm{T}}(t-k(t))P_2\chi(t-k(t)) - \chi^{\mathrm{T}}(t-k)P_2\chi(t-k) - 2\int_{t-k}^{t-k(t)}\chi^{\mathrm{T}}(u)P_2\dot{\chi}(u)du\} = 0. \tag{3.9}$$

Let

$$R_{21} = R_2 + \begin{bmatrix} 0 & P_1 \\ P_1 & 0 \end{bmatrix}, R_{22} = R_2 + \begin{bmatrix} 0 & P_2 \\ P_2 & 0 \end{bmatrix}.$$

Then, applying Lemma 2 to the second integral term of $\dot{\vartheta}_3(t)$ and to the integral term in the zero equations (3.8) and (3.9), we have

$$\begin{aligned}&-k\int_{t-k}^{t}\eta_5^{\mathrm{T}}(u)R_2\eta_5(u)du - 2k\int_{t-k(t)}^{t}\chi^{\mathrm{T}}(u)P_1\dot{\chi}(u)du - 2k\int_{t-k}^{t-k(t)}\chi^{\mathrm{T}}(u)P_2\dot{\chi}(u)du\\ &= -k\int_{t-k(t)}^{t}\eta_5^{\mathrm{T}}(u)R_{21}\eta_5(u)du - k\int_{t-k}^{t-k(t)}\eta_5^{\mathrm{T}}(u)R_{22}\eta_5(u)du\\ &\leqslant -\upsilon_1^{\mathrm{T}}(t)\begin{bmatrix} \frac{kR_{21}}{k(t)} & 0 \\ 0 & \frac{kR_{22}}{k-k(t)} \end{bmatrix}\upsilon_1(t) - 3\begin{bmatrix} \upsilon_2^{\mathrm{T}}(t) & \upsilon_3^{\mathrm{T}}(t) \end{bmatrix}\begin{bmatrix} \frac{kR_{21}}{k(t)} & 0 \\ 0 & \frac{kR_{22}}{k-k(t)} \end{bmatrix}\begin{bmatrix} \upsilon_2(t) \\ \upsilon_3(t) \end{bmatrix}.\end{aligned}$$

Next, applying Lemma 3, we get

$$\begin{aligned}&-\upsilon_1^{\mathrm{T}}(t)\begin{bmatrix}\frac{kR_{21}}{k(t)} & 0\\ 0 & \frac{kR_{22}}{k-k(t)}\end{bmatrix}\upsilon_1(t)-3\begin{bmatrix}\upsilon_2^{\mathrm{T}}(t) & \upsilon_3^{\mathrm{T}}(t)\end{bmatrix}\begin{bmatrix}\frac{kR_{21}}{k(t)} & 0\\ 0 & \frac{kR_{22}}{k-k(t)}\end{bmatrix}\begin{bmatrix}\upsilon_2(t)\\ \upsilon_3(t)\end{bmatrix}\\ &\leqslant -\upsilon_1^{\mathrm{T}}(t)\begin{bmatrix}R_{21} & S_1\\ * & R_{22}\end{bmatrix}\upsilon_1(t)-3\begin{bmatrix}\upsilon_2^{\mathrm{T}}(t) & \upsilon_3^{\mathrm{T}}(t)\end{bmatrix}*\begin{bmatrix}R_{21} & S_2\\ * & R_{22}\end{bmatrix}\left[\begin{bmatrix}\upsilon_2(t)\\ \upsilon_3(t)\end{bmatrix}\right]\\ &= -(k(t)E_{3\mathrm{b}}^{\mathrm{T}}+E_{3\mathrm{a}}^{\mathrm{T}})\bar{R}_{21}(k(t)E_{3\mathrm{b}}+E_{3\mathrm{a}})-3(k(t)E_{4\mathrm{b}}^{\mathrm{T}}+E_{4\mathrm{a}}^{\mathrm{T}})\bar{R}_{22}(k(t)E_{4\mathrm{b}}+E_{4\mathrm{a}}).\end{aligned} \tag{3.10}$$

Combined with (3.4)–(3.10), inequality (3.11) can be obtained.

$$\dot{\vartheta}(t) \leqslant \bar{\zeta}^{\mathrm{T}}(t)\bar{\psi}(k(t))\bar{\zeta}(t), \tag{3.11}$$

where

$$\begin{aligned}\bar{\psi}(k(t)) &= 2(k(t)\lambda_{1\mathrm{b}}^{\mathrm{T}}+\lambda_{1\mathrm{a}}^{\mathrm{T}})P(k^2(t)\lambda_{2\mathrm{c}}+k(t)\lambda_{2\mathrm{b}}+\lambda_{2\mathrm{a}})+(k(t)\lambda_{3\mathrm{b}}^{\mathrm{T}}+\lambda_{3\mathrm{a}}^{\mathrm{T}})Q_1(k(t)\lambda_{3\mathrm{b}}+\lambda_{3\mathrm{a}})\\ &+\lambda_{13}^{\mathrm{T}}(Z_1+Z_2)\lambda_{13}+(k(t)\lambda_{4\mathrm{b}}^{\mathrm{T}}+\lambda_{4\mathrm{a}}^{\mathrm{T}})Q_2(k(t)\lambda_{4\mathrm{b}}+\lambda_{4\mathrm{a}})-(1-\mu)\lambda_{14}^{\mathrm{T}}Z_1\lambda_{14}-\lambda_{15}^{\mathrm{T}}Z_2\lambda_{15}\\ &-(k(t)\lambda_{10\mathrm{b}}^{\mathrm{T}}+\lambda_{10\mathrm{a}}^{\mathrm{T}})Q_2(k(t)\lambda_{10\mathrm{b}}+\lambda_{10\mathrm{a}})-(1-\mu)(k(t)\lambda_{5\mathrm{b}}^{\mathrm{T}}+\lambda_{5\mathrm{a}}^{\mathrm{T}})Q_1(k(t)\lambda_{5\mathrm{b}}+\lambda_{5\mathrm{a}})\\ &+2\lambda_7^{\mathrm{T}}Q_2(k^2(t)\lambda_{12\mathrm{c}}+k(t)\lambda_{12\mathrm{b}}+\lambda_{12\mathrm{a}})+2\lambda_6^{\mathrm{T}}Q_1(k^2(t)\lambda_{8\mathrm{c}}+k(t)\lambda_{8\mathrm{b}})+2(H_{11}^{\mathrm{T}}\Lambda_1+H_{21}^{\mathrm{T}}\Lambda_2)W_2\Gamma\\ &+k\Gamma^{\mathrm{T}}R_1\Gamma+k^2\lambda_{16}^{\mathrm{T}}R_2\lambda_{16}+\mathrm{S}ym\{Y_1E_1+Y_2E_2\}+k(t)Y_1\bar{R}_1^{-1}Y_1^{\mathrm{T}}+(k-k(t))Y_2\bar{R}_1^{-1}Y_2^{\mathrm{T}}\\ &+k(e_1^{\mathrm{T}}P_1e_1-e_2^{\mathrm{T}}P_1e_2+e_2^{\mathrm{T}}P_2e_2-e_3^{\mathrm{T}}P_2e_3)-(k(t)E_{3\mathrm{b}}^{\mathrm{T}}+E_{3\mathrm{a}}^{\mathrm{T}})\bar{R}_{21}(k(t)E_{3\mathrm{b}}+E_{3\mathrm{a}})\\ &-3(k(t)E_{4\mathrm{b}}^{\mathrm{T}}+E_{4\mathrm{a}}^{\mathrm{T}})\bar{R}_{22}(k(t)E_{4\mathrm{b}}+E_{4\mathrm{a}}).\end{aligned}$$

Given the diagonal matrices G_{i}, D_{i}, U_{i}, and T_{i}, $\mathrm{i} = 1, 2, 3$, inequalities (3.12) and (3.13) hold.

$$0 \leq 2\bar{\zeta}^{\mathrm{T}}(t)\left[\sum_{\mathrm{i}=1}^{3} H_{1\mathrm{i}}^{\mathrm{T}}(k(t)G_{\mathrm{i}}+(k-k(t))D_{\mathrm{i}})H_{2\mathrm{i}}\right]\bar{\zeta}(t), \tag{3.12}$$

$$0 \leq 2\bar{\zeta}^{\mathrm{T}}(t)\left[\sum_{\mathrm{i}=1}^{3} \Delta_{1\mathrm{i}}^{\mathrm{T}}(k(t)U_{\mathrm{i}}+(k-k(t))T_{\mathrm{i}})\Delta_{2\mathrm{i}}\right]\bar{\zeta}(t). \tag{3.13}$$

Finally, by integrating the inequalities (3.11)–(3.13), inequality (3.14) is established.

$$\dot{\vartheta}(t) \leq \bar{\zeta}^{\mathrm{T}}(t)\Xi(k(t))\bar{\zeta}(t), \tag{3.14}$$

$$\Xi(k(t)) = ak^3(t)+bk^2(t)+ck(t)+d+k(t)Y_1\bar{R}_1^{-1}Y_1^{\mathrm{T}}+(k-k(t))Y_2\bar{R}_1^{-1}Y_2^{\mathrm{T}}.$$

Then, according to the Schur complement and Lemma 4, $\Xi(k(t)) < 0$ if and only if inequalities (3.1) and (3.2) hold, which means GNNs (2.1) are asymptotically stable. This ends the proof.

Remark 2. *From the derivation process of Theorem 1, it can be seen that $\dot{\vartheta}(t)$ is a cubic polynomial function of the time-delay signal $k(t)$. If we directly apply the polynomial inequalities method in [47] to deal with the nonlinear terms in the derivation, it is very difficult. Therefore, Lemma 4 easily solves this problem by changing the positions of a–d. At the same time, the application of Lemma 4 has decreased the LMIs dimension of Theorem 1 effectively.*

Corollary 1. *For given constants $k > 0$ and $\mu > 0$, if there exist symmetric and positive definite matrices $P \in \mathbb{Z}^{5n\times5n}$, $Q_1 \in \mathbb{Z}^{5n\times5n}$, $Z_1 \in \mathbb{Z}^{2n\times2n}$, $Z_2 \in \mathbb{Z}^{2n\times2n}$, symmetric matrices $Q_2 \in \mathbb{Z}^{4n\times4n}$, $R_1 \in \mathbb{Z}^{n\times n}$, $R_2 \in \mathbb{Z}^{2n\times2n}$, $P_1 \in \mathbb{Z}^{n\times n}$, $P_2 \in \mathbb{Z}^{n\times n}$, any matrices $N \in \mathbb{Z}^{2n\times n}$, $S_1 \in \mathbb{Z}^{2n\times2n}$, $S_2 \in \mathbb{Z}^{2n\times2n}$, and $Y_j \in \mathbb{Z}^{10n\times3n}, j = 1, 2,$ positive diagonal matrices $D_l \in \mathbb{Z}^{n\times n}$, $G_l \in \mathbb{Z}^{n\times n}$, $T_l \in \mathbb{Z}^{n\times n}$, $U_l \in \mathbb{Z}^{n\times n}, l = 1, 2, 3,$ and $\Lambda_i \in \mathbb{Z}^{n\times n}, i = 1, 2,$ inequalities* (3.1), (3.2), *and* (3.15) *holds with positive definite matrices $R_{21} \in \mathbb{Z}^{2n\times2n}, R_{22} \in \mathbb{Z}^{2n\times2n}$. Then GNNs* (2.1) *are asymptotically stable.*

$$2\lambda_{18}^{\mathrm{T}}R_1\lambda_{18} + 4\lambda_{19}^{\mathrm{T}}R_1\lambda_{19} + 2\lambda_{20}^{\mathrm{T}}R_2\lambda_{20} + \frac{1}{k}\lambda_{21}^{\mathrm{T}}Q_2\lambda_{21} > 0, \tag{3.15}$$

where

$$\lambda_{18} = \bar{e}_1 - \bar{e}_2,\ \lambda_{19} = \bar{e}_1 + 2\bar{e}_2 - 6\bar{e}_3,\ \lambda_{21} = \begin{bmatrix} k\bar{e}_1^{\mathrm{T}} & k\bar{e}_2^{\mathrm{T}} & k^2\bar{e}_3^{\mathrm{T}} & k^2(\bar{e}_2^{\mathrm{T}} - \bar{e}_3^{\mathrm{T}}) \end{bmatrix}^{\mathrm{T}},$$

$$\lambda_{20} = \begin{bmatrix} \bar{e}_1^{\mathrm{T}} - \bar{e}_2^{\mathrm{T}} & k\bar{e}_3^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}},\ \bar{e}_{\mathrm{i}} = \begin{bmatrix} 0_{n\times(\mathrm{i}-1)n} & I_n & 0_{n\times(3-\mathrm{i})n} \end{bmatrix}, \mathrm{i} = 1, 2, 3.$$

Proof: First, by utilizing Lemma 5, the following two inequalities are obtained.

$$\int_{t-k}^{t}\int_{\theta}^{t} \dot{\chi}^{\mathrm{T}}(u)R_1\dot{\chi}(u)dud\theta \geqslant \bar{\xi}^{\mathrm{T}}(t)(2\lambda_{18}^{\mathrm{T}}R_1\lambda_{18} + 4\lambda_{19}^{\mathrm{T}}R_1\lambda_{19})\bar{\xi}(t),$$
$$\int_{t-k}^{t}\int_{\theta}^{t} \eta_4^{\mathrm{T}}(u)R_2\eta_4(u)dud\theta \geqslant \bar{\xi}^{\mathrm{T}}(t)(2\lambda_{20}^{\mathrm{T}}R_2\lambda_{20})\bar{\xi}(t),$$

where

$$\bar{\xi}(t) = \begin{bmatrix} \chi^{\mathrm{T}}(t) & \frac{1}{k}\int_{t-k}^{t}\chi^{\mathrm{T}}(u)du & \frac{1}{k^2}\int_{t-k}^{t}\int_{\theta}^{t}\chi^{\mathrm{T}}(u)dud\theta \end{bmatrix}^{\mathrm{T}}.$$

Next, according to Lemma 2, the following inequality holds.

$$\int_{t-k}^{t} \eta_3^{\mathrm{T}}(t,\theta)Q_2\eta_3(t,\theta)d\theta \geq \bar{\xi}^{\mathrm{T}}(t)(\frac{1}{k}\lambda_{21}^{\mathrm{T}}Q_2\lambda_{21})\bar{\xi}(t). \tag{3.16}$$

Finally, through the above treatment, the constraint conditions of positive definiteness are relaxed effectively by (3.15), while the positive definiteness of augmented LKF (3.3) still satisfies; this ends the proof.

Remark 3. *In Corollary 1, the terms associated with R_1, R_2, and Q_2 are integrated into the augmented LKF, resulting in a relaxed positive definite constraint (inequality* (3.15)*), which effectively reduces conservatism in the LKF construction. However, integrating related terms becomes increasingly complex with the inclusion of additional augmented state vectors, making it challenging to determine positive conditions.*

4. Numerical examples

In this part, the MAUBs are used as an important evaluation index to measure the system's conservatism. To further demonstrate the validity and superiority of the proposed criteria, we carry out experimental simulations with the help of three numerical examples. *Example 1:* Consider GNNs (2.1) with the following parameters:

$$A = \begin{bmatrix} 1.50 & 0 \\ 0 & 0.7 \end{bmatrix}, W_0 = \begin{bmatrix} 0.0503 & 0.0454 \\ 0.0987 & 0.2075 \end{bmatrix}, W_1 = \begin{bmatrix} 0.2381 & 0.9320 \\ 0.0388 & 0.5062 \end{bmatrix},$$
$$W_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, J_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, J_2 = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.8 \end{bmatrix}.$$

In this example, the comparison results on MAUBs k for various μ are shown in Table 1. It is obvious to find that for all various μ, the calculated results obtained by Theorem 1 and Corollary 1 are better than the results in [5, 8, 44, 48–51]. From Table 1, 28.5178, 19.6106, 16.2501, and 14.3872 obtained by Corollary 1 in this paper are bigger than 24.803, 17.342, 14.446, and 12.670 in [50] when μ = 0.4, 0.45, 0.5, and 0.55, respectively. Moreover, the number of decision variables (NDVs) indicates the computational complexity involved in the stability analysis. To further verify the effectiveness of the calculated results for $\mu = 0.4$ and $k = 28.5178$ in Table 1, the real-time state trajectory of GNNs (2.1) with initial state $\chi(0) = [-3\ \ 2]^{\mathrm{T}}$, time-delay signal $k(t) = 26.9178 + 1.6e^t/(e^t + 1)$, and neuron activation function $\lambda(\chi(t)) = [0.3\tanh(\chi_1(t))\ \ 0.8\tanh(\chi_2(t))]^{\mathrm{T}}$ are exhibited in Figure 1 below. It can be seen from Figure 1 that the real-time state trajectory of GNNs (2.1) is convergent, which fully demonstrates that our proposed method is less conservative.

Table 1. The MAUBs k for given μ in Example 1.

Methods	$\mu = 0.4$	$\mu = 0.45$	$\mu = 0.5$	$\mu = 0.55$	NDVs
[44]	10.4371	9.1910	8.6957	8.3806	$128n^2 + 20n$
[5]	16.8020	11.6745	9.9098	9.0662	$170.5n^2 + 20n$
[48]	17.2697	12.0698	10.2903	9.3879	$319.5n^2 + 35.5n$
[49]	18.8262	12.8039	10.6230	9.5038	$371n^2 + 26n$
Theorem 2 [8]	21.0344	14.4707	11.9667	10.6439	$158n^2 + 17n$
Proposition 3 [50]	24.803	17.342	14.446	12.670	$1047n^2 + 40n$
Theorem 1 [51]	-	17.1680	13.5523	11.7396	$285n^2 + 29n$
Theorem 1	27.5544	18.9676	15.7880	14.0071	$84.5n^2 + 18.5n$
Corollary1	28.5178	19.6106	16.2501	14.3872	$76.5n^2 + 16.5n$

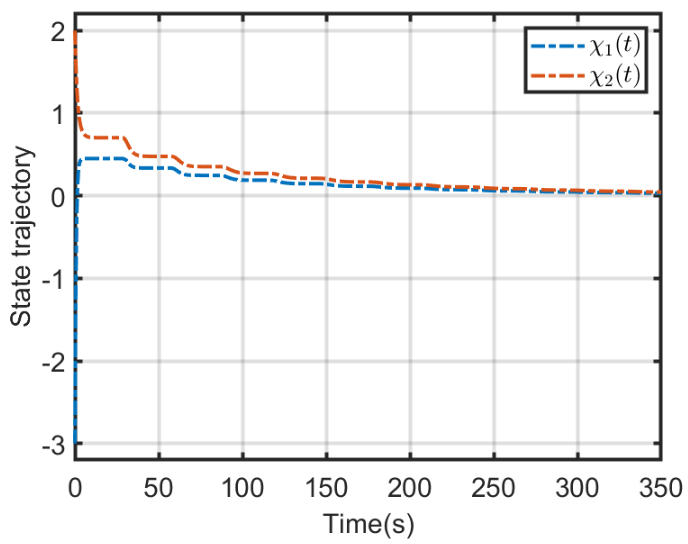

Figure 1. State trajectory of GNNs (2.1) of Example 1.

Example 2: Consider GNNs (2.1) with

$$A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, W_0 = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}, W_1 = \begin{bmatrix} 0.88 & 1 \\ 1 & 1 \end{bmatrix}, W_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, J_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, J_2 = \begin{bmatrix} 0.4 & 0 \\ 0 & 0.8 \end{bmatrix}.$$

For this example, the MAUBs k for given μ is calculated by the stability conditions of Theorem 1 and Corollary 1 as well as some recently reported approaches as shown in Table 2. From Table 2, 8.0549 and 7.8986 obtained by Corollary 1 in this article are bigger than 7.9142, 7.6562, and 7.2033 in [52], [51], [8] when $\mu = 0.8$. When $\mu = 0.9$, we get a similar conclusion. Through the comparison of the above results, it fully illustrates that our proposed conditions are less conservative than in other works.

Table 2. The MAUBs k for given μ in Example 2.

Methods	$\mu = 0.8$	$\mu = 0.9$	NDVs
[5]	6.7001	4.0747	$170.5n^2 + 20n$
[48]	6.7186	3.9623	$319.5n^2 + 35.5n$
Theorem 2 [8]	7.2033	4.0663	$158n^2 + 17n$
Theorem 1 [52]	7.9142	4.4615	$974.5n^2 + 21.5n$
Corollary 1 [51]	7.6562	4.4816	$277n^2 + 27n$
Theorem 1	7.8986	4.4415	$84.5n^2 + 18.5n$
Corollary1	8.0549	4.5250	$76.5n^2 + 16.5n$

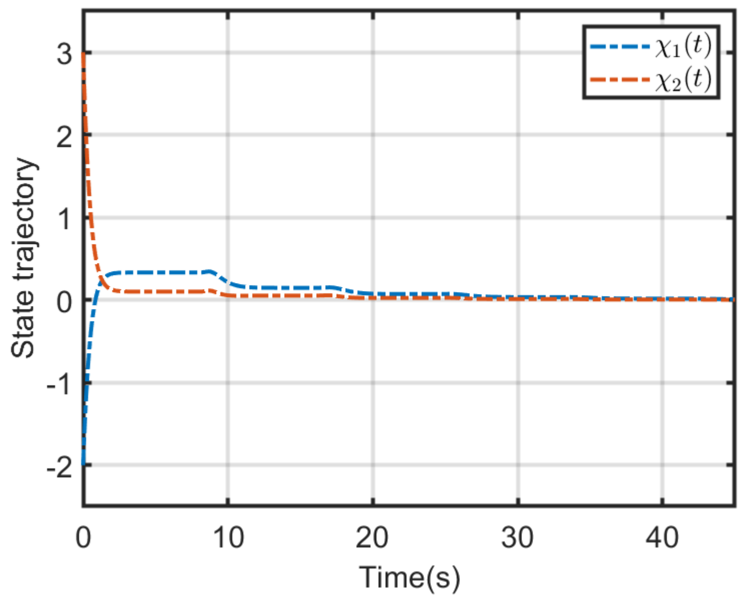

Figure 2. State trajectory of GNNs (2.1) of Example 2.

In this example, we choose the initial conditions $\chi(0) = [-2\ 3]^{\mathrm{T}}$ and neuron activation function $\lambda(\chi(t)) = [0.4\tanh(\chi_1(t))\ 0.8\tanh(\chi_2(t))]^{\mathrm{T}}$, Then, by setting the time-delay signal

$k(t) = 3.2e^t/(e^t + 1) + 4.8549$ when $\mu = 0.8, k = 8.0549$, the real-time state trajectories of GNNs (2.1) are exhibited in Figure 2 below. From Figure 2, we can see that the real-time state trajectory of GNNs (2.1) are convergent, which proves that our approach is better than the others.

Example 3: Consider GNNs (2.1) with the parameters as follows:

$$A = \begin{bmatrix} 7.3458 & 0 & 0 \\ 0 & 6.9987 & 0 \\ 0 & 0 & 5.5949 \end{bmatrix}, W_0 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, W_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, J_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$J_2 = \begin{bmatrix} 0.3680 & 0 & 0 \\ 0 & 0.1795 & 0 \\ 0 & 0 & 0.2876 \end{bmatrix}, W_2 = \begin{bmatrix} 13.6014 & -2.9616 & -0.6936 \\ 7.4736 & 21.6810 & 3.2100 \\ 0.7920 & -2.6334 & -20.1300 \end{bmatrix}.$$

The MAUBs k for given μ calculated by our proposed conditions of Theorem 1 and Corollary 1 are shown in Table 3. Obviously, 1.1819 and 1.2225 obtained by Theorem 1 and Corollary 1 are much larger than 1.1205 in [53], 1.1365 in [48], 1.1366 in [8], 1.1402 in [51], and 1.1641 in [50], when $\mu = 0.1$. It is obvious that the results obtained by our proposed approach are larger than some existing ones, when $\mu = 0.5, 0.9$. This fully demonstrates that our proposed approach can obtain less conservative results than the others. In this example, by selecting the initial conditions $\chi(0) = [2 \ \ -3 \ \ 5]^{\mathrm{T}}$, time-delay signal $k(t) = 0.4e^t/(e^t + 1) + 0.8225$, and neuron activation function $\lambda(\chi(t)) = [0.368 \tanh(\chi_1(t)) \ \ 0.1795 \tanh(\chi_2(t)) \ \ 0.2876 \tanh(\chi_3(t))]^{\mathrm{T}}$, the real-time state trajectory of GNNs (2.1) is displayed in Figure 3 below. It clearly shows that GNNs (2.1) with a time-varying delay are asymptotically stable at their equilibrium point, which has been shifted to the origin, which fully demonstrates the effectiveness and superiority of our proposed approach.

Table 3. The MAUBs k for given μ in Example 3.

Methods	$\mu = 0.1$	$\mu = 0.5$	$\mu = 0.9$	NDVs
[53]	1.1205	0.4614	0.3963	$122.5n^2 + 22.5n$
[48]	1.1365	0.4678	-	$319.5n^2 + 35.5n$
Theorem 2 [8]	1.1366	0.4887	0.4324	$158n^2 + 17n$
Proposition 3 [50]	1.1641	0.6396	0.5361	$1047n^2 + 40n$
Theorem 1 [51]	1.1402	0.5848	0.4946	$285n^2 + 29n$
Theorem 1	1.1819	0.7035	0.5802	$84.5n^2 + 18.5n$
Corollary1	1.2225	0.7301	0.5951	$76.5n^2 + 16.5n$

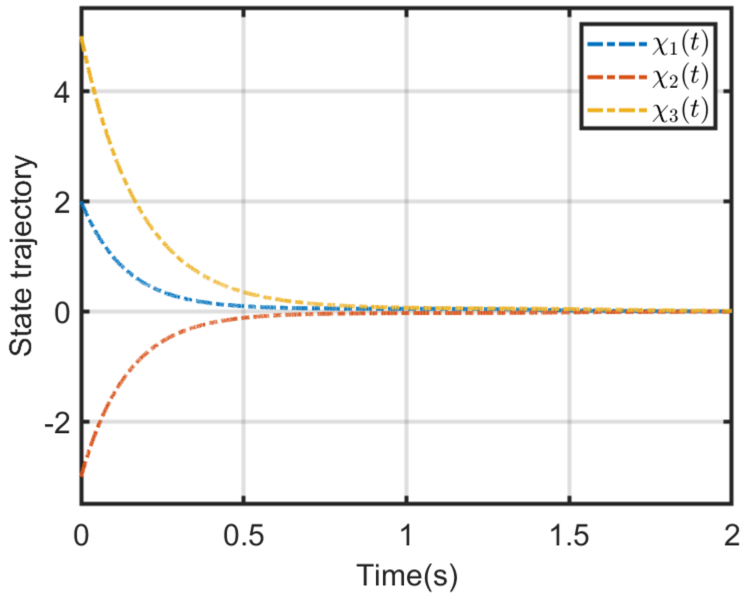

Figure 3. State trajectory of GNNs (2.1) of Example 3.

5. Conclusions

This article investigated the issue of delay-dependent stability in GNNs with a time-varying delay. With a new cubic polynomial and a suitable augmented LKF, some stability conditions emerge as linear matrix inequalities. At the same time, less conservative stability conditions have been obtained via relaxing the augmented LKF's positive definite condition. Finally, three classical numerical examples are devoted to validating the superiority of our proposed method compared to recently reported approaches. Based on the aforementioned research, it is evident that our future research will focus on obtaining stability conditions with less conservatism.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

Acknowledgments

This research work was supported by the Hunan Provincial Natural Science Foundation of China [No. 2023JJ50209], the Science and Technology Innovation Program of Hunan Province [No. 2022RC1097], and the Scientific Research Fund of Hunan Provincial Department of Education [No. 24C1080].

Conflict of interest

The authors declare there is no conflict of interest.

References

1. Y. Q. Yang, J. H. Cai, H. F. Yang, J. F. Zhang, X. J. Zhao, Tad: A trajectory clustering algorithm based on spatial-temporal density analysis, *Expert Syst. Appl.*, **139** (2019), 112846. https://doi.org/10.1016/j.eswa.2019.112846
2. H. C. Lin, J. X. Dong, H. B. Zeng, J. H. Park, Stability analysis of delayed neural networks via a time-varying Lyapunov functional, *IEEE Trans. Syst. Man Cybern.: Syst.*, **54** (2024), 2563–2575. https://doi.org/10.1109/TSMC.2023.3346060
3. X. M. Zhang, Q. L. Han, Global asymptotic stability for a class of generalized neural networks with interval time-varying delays, *IEEE Trans. Neural Networks*, **22** (2011), 1180–1192. https://doi.org/10.1109/TNN.2011.2147331
4. Y. X. Tian, H. C. Yan, H. Zhang, J. Cheng, H. Shen, Asynchronous output feedback control of hidden semi-Markov jump systems with random mode-dependent delays, *IEEE Trans. Autom. Control*, **67** (2021), 4107–4114. https://doi.org/10.1109/TAC.2021.3110006
5. M. J. Park, S. M. Lee, O. M. Kwon, J. H. Ryu, Enhanced stability criteria of neural networks with time-varying delays via a generalized free-weighting matrix integral inequality, *J. Franklin Inst.*, **355** (2018), 6531–6548. https://doi.org/10.1016/j.jfranklin.2018.06.023
6. H. B. Zeng, Z. J. Zhu, S. P. Xiao, X. M. Zhang, A switched system model for exponential stability and dissipativity of delayed neural networks, *IEEE Trans. Neural Networks Learn. Syst.*, (2025). https://doi.org/10.1109/TNNLS.2025.3590251
7. H. C. Lin, J. X. Dong, J. H. Park, Observer-based H_∞ fault-tolerant tracking control of multi-agent systems with nonideal communication links and external disturbances, *IEEE Trans. Autom. Sci. Eng.*, **22** (2025), 14096–14107. https://doi.org/10.1109 /TASE.2025.3557951
8. J. A. Wang, L. Fan, X. Y. Wen, Y. Wang, Enhanced stability results for generalized neural networks with time-varying delay, *J. Franklin Inst.*, **357** (2020), 6932–6950. https://doi.org/10.1016/j.jfranklin.2020.04.049
9. B. Yang, J. J. Cao, M. N. Hao, X. J. Pan, Further stability analysis of generalized neural networks with time-varying delays based on a novel Lyapunov-Krasovskii functional, *IEEE Access*, **7** (2019), 91253–91264. https://doi.org/10.1109/ACCESS.2019.2925912
10. H. B. Zeng, Z. J. Zhu, T. S. Peng, W. Wang, X. M. Zhang, Robust tracking control design for a class of nonlinear networked control systems considering bounded package dropouts and external disturbance, *IEEE Trans. Fuzzy Syst.*, **32** (2024), 3608–3617. https://doi.org/10.1109/TFUZZ.2024.3377799
11. W. Wang, J. M. Liang, H. B. Zeng, X. M. Zhang, Novel looped functionals in designing output feedback controllers for aperiodic sampled-data control systems, *IEEE Trans. Autom. Sci. Eng.*, **22** (2025), 16397–16402. https://doi.org/10.1109/TASE.2025.3573304

12. W. M. Wang, H. B. Zeng, J. M. Liang, S. P. Xiao, Sampled-data-based load frequency control for power systems considering time delays, *J. Franklin Inst.*, **362** (2025), 107477. https://doi.org/10.1016/j.jfranklin.2024.107477

13. H. B. Zeng, Y. J. Chen, Y. He, X. M. Zhang, A delay-derivative-dependent switched system model method for stability analysis of linear systems with time-varying delay, *Automatica*, **175** (2025), 112183. https://doi.org/10.1016/j.automatica.2025.112183

14. H. B. Zeng, W. M. Wang, W. Wang, H. Q. Xiao, Improved looped-functional approach for dwell-time-dependent stability analysis of impulsive systems, *Nonlinear Anal. Hybrid Syst.*, **52** (2024), 101477. https://doi.org/10.1016/j.nahs.2024.101477

15. Y. He, C. K. Zhang, H. B. Zeng, M. Wu, Additional functions of variable-augmented-based free-weighting matrices and application to systems with time-varying delay, *Int. J. Syst. Sci.*, **54** (2022), 991–1003. https://doi.org/10.1080/00207721.2022.2157198

16. Y. Chen, C. Lu, X. M. Zhang, Allowable delay set flexible fragmentation approach to passivity analysis of delayed neural networks, *Neurocomputing*, **629** (2025), 129730. https://doi.org/10.1016/j.neucom.2025.129730

17. H. B. Zeng, Y. He, K. Teo, Monotone-delay-interval-based Lyapunov functionals for stability analysis of systems with a periodically varying delay, *Automatica*, **138** (2022), 110030. https://doi.org/ 10.1016/j.automatica.2021.110030

18. X. F. Jiang, Z. M. Yin, J. J. Wu, Stability analysis of linear systems under time-varying samplings by a non-standard discretization method, *Electronics*, **7** (2018), 278. https://doi.org/10.3390/electronics7110278

19. Y. Chen, Y. Q. Li, G. Chen, New results on stability analysis for a class of generalized delayed neural networks, *Appl. Math. Comput.*, **469** (2024), 128529. https://doi.org/10.1016/j.amc.2024.128529

20. Y. Chen, H. B. Zeng, Y. Li, Stability analysis oflinear delayed systems based on an allowable delay set partitioning approach, *Automatica*, **163** (2024), 111603. https://doi.org/10.1016/j.automatica.2024.111603

21. W. Wang, C. X. Li, A. Q. Luo, H. Q. Xiao, Stability analysis of linear systems with a periodical time-varying delay based on an improved non-continuous piecewise Lyapunov functional, *AIMS Math.*, **10** (2025), 9073–9093. https://doi.org/10.3934/math.2025418

22. O. M. Kwon, J. H. Park, S. M. Lee, E. J. Cha, New augmented Lyapunov-Krasovskii functional approach to stability analysis of neural networks with time-varying delay, *Nonlinear Dyn.*, **76** (2014), 221–236. https://doi.org/10.1007/s11071-013-1122-2

23. C. Ge, C. C. Hua, X. P. Guan, New delay-dependent stability criteria for neural networks with time-varying delay using delay-decomposition approach, *IEEE Trans. Neural Networks Learn. Syst.*, **25** (2014), 1378–1383. https://doi.org/10.1007/s11071-013-1122-2

24. J. A. Wang, X. Y. Wen, B. Y. Hou, Advanced stability criteria for static neural networks with interval time-varying delays via the improved Jensen inequality, *Neurocomputing*, **377** (2020), 49–56. https://doi.org/10.1016/j.neucom.2019.10.034

25. J. Chen, J. H. Park, S. Y. Xu, Stability analysis for neural networks with time-varying delay via improved techniques, *IEEE Trans. Cybern.*, **49** (2019), 4495–4500. https://doi.org/10.1109/TCYB.2018.2868136

26. W. J. Lin, Y. He, C. K. Zhang, M. Wu, Stability analysis of neural networks with time-varying delay: Enhanced stability criteria and conservatism comparisons, *Commun. Nonlinear Sci. Numer. Simul.*, **54** (2018), 118–135. https://doi.org/10.1109/TCYB. 2018.2868136

27. W. J. Lin, Y. He, C. K. Zhang, M. Wu, J. Shen, Extended dissipativity analysis for Markovian jump neural networks with time-varying delay via delay-product-type functionals, *IEEE Trans. Neural Networks Learn. Syst.*, **30** (2019), 2528–2537. https://doi.org/10.1109/ TNNLS.2018.2885115

28. A. Seuret, F. Gouaisbaut, Wirtinger-based integral inequality: Application to time-delay systems, *Automatica*, **49** (2013), 2860–2866. https://doi.org/10.1016/j.automatica.2013.05.030

29. A. Seuret, F. Gouaisbaut, Stability of linear systems with time-varying delays using Bessel-Legendre inequalities, *IEEE Trans. Autom. Control*, **63** (2018), 225–232. https://doi.org/10.1109/TAC.2017.2730485

30. N. Cheng, W. Wang, H. B. Zeng, X. Liu, X. M. Zhang, Novel exponential-weighted integral inequality for exponential stability analysis of time-varying delay systems, *Appl. Math. Lett.*, **172** (2025), 109730. https://doi.org/10.1016/j.aml.2025.109730

31. P. Park, W. I. Lee, S. Y. Lee, Auxiliary function-based integral inequalities for quadratic functions and their applications to time-delay systems, *J. Franklin Inst.*, **352** (2015), 1378–1396. https://doi.org/10.1016/j.jfranklin.2015.01.004

32. H. B. Zeng, X. G. Liu, W. Wang, A generalized free-matrix-based integral inequality for stability analysis of time-varying delay systems, *Appl. Math. Comput.*, **354** (2019), 1–8. https://doi.org/10.1016/j.amc.2019.02.009

33. H. B. Zeng, Z. J. Zhu, W. Wang, X. M. Zhang, Relaxed stability criteria of delayed neural networks using delay-parameters-dependent slack matrices, *Neural Networks*, **180** (2024), 106676. https://doi.org/10.1016/j.neunet.2024.106676

34. W. Wang, J. M. Liang, H. B. Zeng, Sampled-data-based stability and stabilization of Lurie systems, *Appl. Math. Comput.*, **501** (2025), 129455. https://doi.org/10.1016/j.amc.2025.129455

35. T. S. Peng, H. B. Zeng, W. Wang, X. M. Zhang, X. G. Liu, General and less conservative criteria on stability and stabilization of T-S fuzzy systems with time-varying delay, *IEEE Trans. Fuzzy Syst.*, **31** (2023), 1531–1541. https://doi.org/10.1109/TFUZZ.2022.3204899

36. A. Seuret, K. Liu, F. Gouaisbaut, Generalized reciprocally convex combination lemmas and its application to time-delay systems, *Automatica*, **95** (2018), 488–493. https://doi.org/10.1016/j.automatica.2018.06.017

37. H. B. Zeng, H. C. Lin, Y. He, K. L. Teo, W. Wang, Hierarchical stability conditions for time-varying delay systems via an extended reciprocally convex quadratic inequality, *J. Franklin Inst.*, **357** (2020), 9930–9941. https://doi.org/10.1016/j.jfranklin.2020.07.034

38. J. Park, P. Park, Finite-interval quadratic polynomial inequalities and their application to time-delay systems, *J. Franklin Inst.*, **357** (2020), 4316–4327. https://doi.org/10.1016/j.jfranklin.2020.01.022

39. C. K. Zhang, F. Long, Y. He, W. Yao, L. Jiang, M. Wu, A relaxed quadratic function negative-determination lemma and its application to time-delay systems, *Automatica*, **113** (2020), 108764. https://doi.org/10.1016/j.automatica.2019.108764

40. H. B. Zeng, Z. L. Zhai, W. Wang, Hierarchical stability conditions of systems with time-varying delay, *Appl. Math. Comput.*, **404** (2021), 126222. https://doi.org/10.1016/j.amc. 2021.126222

41. S. Xiao, J. Yu, S. X. Yang, Y. Qiu, Stability analysis for time-delay systems via a new negativity condition on quadratic functions, *Mathematics*, **10** (2022), 3096. https://doi.org/10.3390/math10173096

42. X. M. Zhang, Q. L. Han, X. Ge, Suffcient conditions for a class of matrix-valued polynomial inequalities on closed intervals and application to H_∞ filtering for linear systems with time-varying delays, *Automatica*, **125** (2021), 109390. https://doi.org/10.1016/j.automatica.2020.109390

43. B. Wang, J. Yan, J. Cheng, S. Zhong, New criteria of stability analysis for generalized neural networks subject to time-varying delayed signals, *Appl. Math. Comput.*, **314** (2017), 322–333. https://doi.org/10.1016/j.amc.2017.06.031

44. W. Wang, H. B. Zeng, K. Teo, Y. Chen, Relaxed stability criteria of time-varying delay systems via delay-derivative-dependent slack matrices, *J. Franklin Inst.*, **360** (2023), 6099–6109. https://doi.org/10.1016/j.jfranklin.2023.04.019

45. P. Park, J. W. Ko, C. K. Jeong, Reciprocally convex approach to stability of systems with time-varying delays, *Automatica*, **47** (2011), 235–238. https://doi.org/10.1016/j.automatica.2010.10.014

46. S. H. Kim, P. Park, C. Jeong, Robust H_∞ stabilization of networked control systems with packet analyser, *IET Control Theory Appl.*, **4** (2010), 1828–1837. https://doi.org/10.1049/iet-cta.2009.0346

47. X. M. Zhang, Q. L. Han, A. Seuret, F. Gouaisbaut, Y. He, Overview of recent advances in stability of linear systems with time-varying delays, *IET Control Theory Appl.*, **13** (2019), 1–16. https://doi.org/10.1049/iet-cta.2018.5188

48. Z. Y. Feng, H. Y. Shao, L. Shao, Further improved stability results for generalized neural networks with time-varying delays, *Neurocomputing*, **367** (2019), 308–318. https://doi.org/10.1016/j.neucom.2019.07.019

49. M. J. Park, O. M. Kwon, J. H. Ryu, Passivity and stability analysis of neural networks with time-varying delays via extended free-weighting matrices integral inequality, *Neural Networks*, **106** (2018), 67–78. https://doi.org/10.1016/j.neunet.2018.06.010

50. H. C. Lin, H. B. Zeng, X. M. Zhang, Stability analysis for delayed neural networks via a generalized reciprocally convex inequality, *IEEE Trans. Neural Networks Learn. Syst.*, **34** (2023), 7491–7499. https://doi.org/10.1109/TNNLS.2022.3144032

51. F. Long, C. K. Zhang, Y. J. Shen, Q. C. Mei, Q. Chen, Stability analysis of delayed neural networks via novel delay-dependent LKF and integral inequality, *ISA Trans.*, **165** (2025), 222–231. https://doi.org/10.1016/j.isatra.2025.06.027

52. S. Q. Wang, W. C. Ji, Y. L. Jiang, D. R. Liu, Relaxed stability criteria for neural networks with time-varying delay using extended secondary delay partitioning and equivalent reciprocal convex combination techniques, *IEEE Trans. Neural Networks Learn. Syst.*, **31** (2019), 4157–4169. https://doi.org/ 10.1016/j.ins.2017.08.072

53. B. Yang, J. Wang, X. Liu, Improved delay-dependent stability criteria for generalized neural networks with time-varying delays, *Inf. Sci.*, **420** (2017), 299–321. https://doi.org/10.1016/j.ins.2017.08.072