Electronic Research Archive

https://www.aimspress.com/journal/era

ERA, 33(1): 255–262.
DOI: 10.3934/era.2025013
Received: 13 October 2024
Revised: 27 December 2024
Accepted: 09 January 2025
Published: 21 January 2025

Comment

Comment on: "Solving the conformable Huxley equation using the complex method" [Electron. Res. Arch., 31 (2023), 1303–1322]

Feng Ye[1], **Xiaoting Zhang**[1,*], **Chunling Jiang**[1] **and Bo Zeng**[2]

[1] School of Basic Courses and General Education, Software Engineering Institute of Guangzhou, Guangzhou 510990, China

[2] Academy of General Education, Guangzhou College of Technology and Business, Guangzhou 510850, China

* **Correspondence:** Email: zhangxiaoting1985@163.com.

Abstract: Using the complex method, Guoqiang Dang and Qiyou Liu [Guoqiang Dang, Qiyou Liu, Electron. Res. Arch., 31 (2023), 1303–1322] have found some exact solutions of the conformable Huxley equation. In this comment, we first demonstrate that the elliptic function solutions and rational function solutions do not satisfy the complex conformable Huxley equation. Finally, all exact solutions of the conformable Huxley equation are given by us.

Keywords: differential equation; exact solution; meromorphic function; complex method; conformable Huxley equation

1. Main mistakes and conclusions

In [1] , Dang and Liu used the complex method [2–8] to search for exact solutions of the conformable Huxley equation [9]

$$\frac{\partial^\alpha}{\partial t^\alpha}u(x,t) - \frac{\partial^2}{\partial x^2}u(x,t) = \beta u(x,t)(1-u(x,t))(u(x,t)-\gamma), \tag{1.1}$$

where $\alpha \in (0,1]$, β is a non-zero constant, and $\gamma \in (0,1)$.

Using the transformation $u(x,t) = u(z),\ z = Kx - \frac{\lambda t^\alpha}{\alpha}$ to Eq (1.1) [1; Eq (3.1)], where $\alpha \in (0,1]$, K and λ are non-zero constants, it follows that

$$K^2u'' + \lambda u' - \beta\gamma u + \beta(1+\gamma)u^2 - \beta u^3 = 0, \tag{1.2}$$

where β is a non-zero constant, and $\gamma \in (0,1)$.

We rewrite Eq (1.2) [1; Eq (3.3)] into the following form [1; Eq (3.23)]:

$$u'' + \frac{\lambda}{K^2}u' - \frac{\beta}{K^2}u(u-1)(u-\gamma) = 0, \tag{1.3}$$

where K, λ, and β are non-zero constants, and $\gamma \in (0, 1)$.

Dang and Liu [1] obtained main results as below.

Conclusion 1. Eq (1.2) [1; Eq (3.3)] has the solutions (3.10) and (3.11).

Remark 1.1 The constraints on Eq (1.2) [1; Eq (3.3)] on page 1306 are β be a non-zero constant, and $\gamma \in (0, 1)$. The proof provided in lines 4–8 on page 1307 and lines 12–18 on page 1308 does not satisfy the above limitations. So, the solutions (3.10) and (3.11) also do not meet these constraints.

Conclusion 2. All meromorphic solutions of Eq (1.2) [1; Eq (3.3)] belong to the class W. The author discusses on the second line of page 1308 in the article: Eq (1.2) [1; Eq (3.3)] has two integer Fuchs indexes, $-1, 4$. From Eq (3.6) [1], the coefficient c_3 is an arbitrary constant, and the other coefficients $c_4, c_5, \cdots$ can be represented using c_3. Then, Eq (1.2) [1; Eq (3.3)] satisfies the $\langle p, q\rangle$ condition, and Eq (1.2) [1; Eq (3.3)] is integrable. Therefore, all meromorphic solutions of Eq (1.2) [1; Eq (3.3)] belong to the class W.

Remark 1.2 We know that the coefficient c_3 is an arbitrary constant, indicating that there exists infinite Laurent expansions, which means that p is infinite. Then, Eq (1.2) [1; Eq (3.3)] does not satisfy the $\langle p, q\rangle$ condition. Therefore, it does not follow that all meromorphic solutions of Eq (1.2) [1; Eq (3.3)] belong to the class W. In fact, in Section 2 of this comment, we will give some meromorphic solutions that do not belong to the class W for Eq (1.2) [1; Eq (3.3)].

Conclusion 3. Eq (1.2) [1; Eq (3.3)] has the rational function solution and elliptic function solution. In Case 1 on page 1039 of the article, the authors provide rational function solution (3.21) for Eq (1.2) [1; Eq (3.3)], in the following form:

$$w(z) = -\frac{\sqrt{2K^2}}{\sqrt{\beta}} \cdot \frac{1}{z-z_0} + \frac{\lambda}{\sqrt{2\beta K^2}}, \tag{1.4}$$

where $\beta(1+\gamma) = \frac{\lambda\sqrt{2\beta}}{K^2}, \beta\gamma = \frac{\lambda^2}{2K^2}$.

They provide the elliptic function solution (3.22) for Eq (1.2) [1; Eq (3.3)] on page 1310 of the article, in the following:

$$W(z) = -\frac{1}{\sqrt{-2D}}\frac{\wp'(z-z_0, g_2, g_3) + B_1}{\wp(z-z_0, g_2, g_3) - A_1} - \frac{\lambda}{\sqrt{2\beta K^2}}, \tag{1.5}$$

where $\beta(1+\gamma) = -\frac{\lambda\sqrt{2\beta}}{K^2}, A_1 = \frac{\lambda^2}{12K^4} - \frac{\beta\gamma}{6K^2}, B_1 = 0, g_2 = \frac{(2K^2\beta\gamma-\lambda^2)^2}{12K^8}, g_3 = \frac{(2K^2\beta\gamma-\lambda^2)^3}{216K^{12}}$, and z_0 is arbitrary.

Remark 1.3 Eq (1.2) [1; Eq (3.3)] does not have elliptic function and rational function solutions. For detailed proofs, please refer to Remarks 2.2 and 2.3 in Section 2.

Conclusion 4. Eq (1.2) [1; Eq (3.3)] has new exact solutions. In this paper, a great deal of space is devoted to finding new exact solutions to Eq (1.2) [1; Eq (3.3)], and all the new solutions are given in Subsection 4.1 on page 1320.

Remark 1.4 We can notice that some solutions do not satisfy Eq (1.2) [1; Eq (3.3)], such as solutions (3.12), (3.13), (3.21), and (3.22). Some solutions are merely different in their representation, for example, (3.51) and (3.52), (3.55) and (3.56), (3.59) and (3.60), (3.63) and (3.64), (3.67) and (3.68), (3.71) and (3.72), (3.80) and (3.81), (3.84) and (3.85), (3.88) and (3.89), (3.92) and (3.93), (3.96) and (3.97), and (3.100) and (3.101). Some solutions are identical, such as (3.88) and (3.96) and (3.92) and (3.100), and some solutions differ by a constant, such as (3.51) and (3.84), (3.55) and (3.80), (3.59) and (3.92), (3.63) and (3.88), (3.67) and (3.100), and (3.71) and (3.96).

2. All exact solutions of the complex conformable Huxley equation

In [10], Conte et al. used the Loewy factorizable method to look for meromorphic solutions for the nonlinear second-order algebraic ordinary differential equation

$$w'' + cw' - \frac{2}{\mu^2}(w - q_1)(w - q_2)(w - q_3) = 0, \tag{2.1}$$

where $\mu(\neq 0), c, q_1, q_2,$ and q_3 are complex constants.

They proved Theorem A as follows below.

Theorem A. Eq (2.1) has nonconstant meromorphic solutions if and only if c satisfies

$$c\prod(c\mu + q_i + q_j - 2q_k)(-c\mu + q_i + q_j - 2q_k) = 0, \tag{2.2}$$

where $(i\ j\ k)$ is any permutation of $(1\ 2\ 3)$ and, for $c \neq 0$ satisfying Eq (2.2), Eq (2.1) has two class nonconstant meromorphic solutions. The first class solution is

$$w_1(z) = q_k - \frac{q_i - q_k}{2} e^{-\frac{q_i - q_k}{\mu}z} \frac{\wp\prime(e^{-\frac{q_i-q_k}{\mu}z} - \zeta_0; g_2, 0)}{\wp(e^{-\frac{q_i-q_k}{\mu}z} - \zeta_0; g_2, 0)}, \tag{2.3}$$

where ζ_0, g_2 are arbitrary, if $c = \frac{2q_i - q_j - q_k}{\mu} = \frac{-q_i + 2q_j - q_k}{-\mu}$. The other class solution is

$$w_2(z) = \frac{q_j e^{\frac{q_j(z-z_0)}{\pm\mu}} - q_k e^{\frac{q_k(z-z_0)}{\pm\mu}}}{e^{\frac{q_j(z-z_0)}{\pm\mu}} - e^{\frac{q_k(z-z_0)}{\pm\mu}}}, \tag{2.4}$$

where z_0 is arbitrary, if $c = \frac{2q_i - q_j - q_k}{\pm\mu}$. For $q_j = q_k$, solution (2.4) degenerates to

$$w_3(z) = \frac{\pm\mu}{z - z_0} + q_j, \tag{2.5}$$

where z_0 is arbitrary.

For $c \neq 0$, all the meromorphic solutions of Eq (2.1) are given by (2.3)–(2.5) and the solution (2.3) is the general solution.

According to Theorem A, it can be inferred that:

Remark 2.1 When $c = 0$, Conte et al. [10] and Yuan et al. [11] obtained all nonconstant meromorphic solutions of Eq (2.1).

By comparing Eqs (1.3) and (2.1), we can set $c = \frac{\lambda}{K^2} \neq 0$ and $\mu^2 = \frac{2K^2}{\beta}$. Here $q_1 = 0, q_2 = 1$, $q_3 = \gamma \in (0, 1)$. By Theorem A, we obtain main results as below:

Theorem 1. Let $\gamma \in (0,1)$, β, K and λ be non-zero constants.

1) If and only if $K^2 = \frac{8\lambda^2}{9\beta}$ and $\gamma = \frac{1}{2}$, Eq (1.3) has the genaral meromorphic solutions

$$u_m(z) = \frac{1}{2} \pm \frac{1}{4} e^{-\frac{3\beta}{8\lambda}z} \frac{\wp\prime(e^{-\frac{3\beta}{8\lambda}z} - \zeta_0; g_2, 0)}{\wp(e^{-\frac{3\beta}{8\lambda}z} - \zeta_0; g_2, 0)}, \tag{2.6}$$

where ζ_0, g_2 are arbitrary, $\beta\lambda \neq 0$, β, $\lambda \in \mathbb{R}$.

2) All simply periodic solutions of Eq (1.3) are the following three forms:

(i) If $\lambda^2 = \frac{(1+\gamma)^2}{2} K^2 \beta$, then

$$u_{s1}(z) = \frac{e^{-\frac{(1+\gamma)\beta}{2\lambda}(z-z_0)} - \gamma e^{-\frac{\gamma(1+\gamma)\beta}{2\lambda}(z-z_0)}}{e^{-\frac{(1+\gamma)\beta}{2\lambda}(z-z_0)} - e^{-\frac{\gamma(1+\gamma)\beta}{2\lambda}(z-z_0)}}, \tag{2.7}$$

where z_0 is arbitrary.

(ii) If $\lambda^2 = \frac{(2-\gamma)^2}{2} K^2 \beta$, then

$$u_{s2}(z) = \frac{\gamma e^{\frac{\gamma(2-\gamma)\beta}{2\lambda}(z-z_0)}}{e^{\frac{\gamma(2-\gamma)\beta}{2\lambda}(z-z_0)} - 1}, \tag{2.8}$$

where z_0 is arbitrary.

(iii) If $\lambda^2 = \frac{(2\gamma-1)^2}{2} K^2 \beta$, then

$$u_{s3}(z) = \frac{e^{\frac{(2\gamma-1)\beta}{2\lambda}(z-z_0)}}{e^{\frac{(2\gamma-1)\beta}{2\lambda}(z-z_0)} - 1}, \tag{2.9}$$

where z_0 is arbitrary.

Remark 2.2 It is easy to know that $\wp$ in $u_m(z)$ is the Weierstrass elliptic function, and the growth order of $\wp$ is $\rho(\wp) = 2$. Thus, $\rho(u_m(z)) = +\infty$. Therefore, Eq (1.3) has no elliptic function solutions and $u_m(z) \notin W$

Remark 2.3 Since $q_1 = 0, q_2 = 1$, and $q_3 = \gamma \in (0,1)$ are not equal to each other, it is known by Theorem A that Eq (1.3) does not have rational solutions.

Proof of Theorem 1.

Let $\gamma \in (0,1)$, β, K and λ be non-zero constants. For Eq (1.3) we discuss its solutions in the following two scenarios:

1) By comparing the coefficients of Eqs (1.3) and (2.1) and combining the conditions from (2.3), we have $\frac{\lambda}{K^2} = \frac{2q_i - q_j - q_k}{\mu} = \frac{-q_i + 2q_j - q_k}{-\mu}$ and $\frac{2}{\mu^2} = \frac{\beta}{K^2}$, which leads to

$$q_k = \frac{q_i + q_j}{2}, \lambda^2 = \frac{(2q_i - q_j - q_k)^2}{2} \cdot \beta K^2, \tag{2.10}$$

where $(i\ j\ k)$ is any permutation of $(1\ 2\ 3)$.

Considering the different values of q_i, q_j, and q_k, we will discuss the following cases.

Case 1. When $q_i = 0, q_j = 1$, and $q_k = \gamma$, from (2.10) we can obtain $\gamma = \frac{1}{2} \in (0,1), K^2 = \frac{8\lambda^2}{9\beta}$. By (2.3), Eq (1.3) has the genaral meromorphic solution

$$u_{m1}(z) = \frac{1}{2} + \frac{1}{4} e^{-\frac{3\beta}{8\lambda}z} \frac{\wp\prime(e^{-\frac{3\beta}{8\lambda}z} - \zeta_0; g_2, 0)}{\wp(e^{-\frac{3\beta}{8\lambda}z} - \zeta_0; g_2, 0)}, \tag{2.11}$$

where ζ_0, g_2 are arbitrary, $\beta\lambda \neq 0$, and $\beta,\ \lambda \in \mathbb{R}$.

Case 2. When $q_i = 0, q_j = \gamma, q_k = 1$, or $q_i = \gamma, q_j = 0, q_k = 1$, from (2.10) we can obtain $\gamma = 2 \notin (0,1)$. The requirements for the coefficients of Eq (1.3) are not met, so Eq (1.3) has no solution in this case.

Case 3. When $q_i = 1, q_j = 0$, and $q_k = \gamma$, from (2.10) we can obtain $\gamma = \frac{1}{2} \in (0,1), K^2 = \frac{8\lambda^2}{9\beta}$. By (2.3), Eq (1.3) has the genaral meromorphic solution

$$u_{m2}(z) = \frac{1}{2} - \frac{1}{4} e^{-\frac{3\beta}{8\lambda}z} \frac{\wp\prime(e^{-\frac{3\beta}{8\lambda}z} - \zeta_0; g_2, 0)}{\wp(e^{-\frac{3\beta}{8\lambda}z} - \zeta_0; g_2, 0)}, \tag{2.12}$$

where ζ_0, g_2 are arbitrary, $\beta\lambda \neq 0,\ \beta,\ \lambda \in \mathbb{R}$.

Case 4. When $q_i = 1, q_j = \gamma, q_k = 0$, or $q_i = \gamma, q_j = 1, q_k = 0$, from (2.10) we have $\gamma = -1 \notin (0,1)$, so its result is the same as Case 2.

Therefore, if and only if $K^2 = \frac{8\lambda^2}{9\beta}$ and $\gamma = \frac{1}{2}$, Eq (1.3) has the genaral meromorphic solutions

$$u_m(z) = \frac{1}{2} \pm \frac{1}{4} e^{-\frac{3\beta}{8\lambda}z} \frac{\wp\prime(e^{-\frac{3\beta}{8\lambda}z} - \zeta_0; g_2, 0)}{\wp(e^{-\frac{3\beta}{8\lambda}z} - \zeta_0; g_2, 0)}, \tag{2.13}$$

where ζ_0, g_2 are arbitrary, $\beta\lambda \neq 0$, and $\beta,\ \lambda \in \mathbb{R}$.

2) By comparing the coefficients of Eqs (1.3) and (2.1) and combining the conditions from (2.4), we have $\frac{\lambda}{K^2} = \frac{2q_i - q_j - q_k}{\pm\mu}$ and $\frac{2}{\mu^2} = \frac{\beta}{K^2}$, which deduces

$$\lambda^2 = \frac{(2q_i - q_j - q_k)^2}{2} \cdot \beta K^2, \tag{2.14}$$

where $(i\ j\ k)$ is any permutation of $(1\ 2\ 3)$.

Considering the different values of q_i, q_j, and q_k, we will discuss the following cases.

Case 1. When $q_i = 0, q_j = 1, q_k = \gamma$, or $q_i = 0, q_j = \gamma, q_k = 1$, from (2.10) we can obtain $\lambda^2 = \frac{(1+\gamma)^2}{2} \cdot \beta K^2$.

By (2.4), Eq (1.3) has the simply periodic solution

$$u_{s1}(z) = \frac{e^{-\frac{(1+\gamma)\beta}{2\lambda}(z-z_0)} - \gamma e^{-\frac{\gamma(1+\gamma)\beta}{2\lambda}(z-z_0)}}{e^{-\frac{(1+\gamma)\beta}{2\lambda}(z-z_0)} - e^{-\frac{\gamma(1+\gamma)\beta}{2\lambda}(z-z_0)}}, \tag{2.15}$$

where z_0 is arbitrary.

Case 2. When $q_i = 1, q_j = 0, q_k = \gamma$, or $q_i = 1, q_j = \gamma, q_k = 0$, from (2.10) we have $\lambda^2 = \frac{(2-\gamma)^2}{2} \cdot \beta K^2$.

By (2.4), Eq (1.3) has the simply periodic solution

$$u_{s2}(z) = \frac{\gamma e^{\frac{\gamma(2-\gamma)\beta}{2\lambda}(z-z_0)}}{e^{\frac{\gamma(2-\gamma)\beta}{2\lambda}(z-z_0)} - 1}, \tag{2.16}$$

where z_0 is arbitrary.

Case 3. When $q_i = \gamma, q_j = 0, q_k = 1$ or $q_i = \gamma, q_j = 1, q_k = 0$, from (2.10) we have $\lambda^2 = \frac{(2\gamma-1)^2}{2} \cdot \beta K^2$.

By (2.4), Eq (1.3) has the simply periodic solution

$$u_{s3}(z) = \frac{e^{\frac{(2\gamma-1)\beta}{2\lambda}(z-z_0)}}{e^{\frac{(2\gamma-1)\beta}{2\lambda}(z-z_0)} - 1}, \tag{2.17}$$

where z_0 is arbitrary.

So far, the proof of Theorem 1 is completed. Substituting $u(x,t) = u(z), z = Kx - \frac{\lambda t^\alpha}{\alpha}$ into all meromorphic solutions $u(z)$ of Eq (1.3), we have obtained all exact solutions for Eq (1.1).

Theorem 2. Let $\alpha \in (0,1]; \gamma \in (0,1);$ *and* β, K, and λ be non-zero constants.

1) If and only if $K^2 = \frac{8\lambda^2}{9\beta}$ and $\gamma = \frac{1}{2}$, Eq (1.1) has the genaral solutions

$$u_m(x,t) = \frac{1}{2} \pm \frac{1}{4} e^{-\frac{3\beta}{8\lambda}(Kx - \frac{\lambda t^\alpha}{\alpha})} \frac{\wp\prime(e^{-\frac{3\beta}{8\lambda}(Kx - \frac{\lambda t^\alpha}{\alpha})} - \zeta_0; g_2, 0)}{\wp(e^{-\frac{3\beta}{8\lambda}(Kx - \frac{\lambda t^\alpha}{\alpha})} - \zeta_0; g_2, 0)}, \tag{2.18}$$

where ζ_0, g_2 is arbitrary, $\beta\lambda \neq 0, K, \beta, \lambda \in \mathbb{R}$.

2) All simply periodic solutions of Eq (1.1) are the following three forms:

(i) If $\lambda^2 = \frac{(1+\gamma)^2}{2} K^2 \beta$, then

$$u_{s1}(x,t) = \frac{e^{-\frac{(1+\gamma)\beta}{2\lambda}(Kx - \frac{\lambda t^\alpha}{\alpha} - Kx_0 + \frac{\lambda t_0^\alpha}{\alpha})} - \gamma e^{-\frac{\gamma(1+\gamma)\beta}{2\lambda}(Kx - \frac{\lambda t^\alpha}{\alpha} - Kx_0 + \frac{\lambda t_0^\alpha}{\alpha}))}}{e^{-\frac{(1+\gamma)\beta}{2\lambda}(Kx - \frac{\lambda t^\alpha}{\alpha} - Kx_0 + \frac{\lambda t_0^\alpha}{\alpha})} - e^{-\frac{\gamma(1+\gamma)\beta}{2\lambda}(Kx - \frac{\lambda t^\alpha}{\alpha} - Kx_0 + \frac{\lambda t_0^\alpha}{\alpha})}}, \tag{2.19}$$

where x_0 and t_0 are real constants.

(ii) If $\lambda^2 = \frac{(2-\gamma)^2}{2} K^2 \beta$, then

$$u_{s2}(x,t) = \frac{\gamma e^{\frac{\gamma(2-\gamma)\beta}{2\lambda}(Kx - \frac{\lambda t^\alpha}{\alpha} - Kx_0 + \frac{\lambda t_0^\alpha}{\alpha})}}{e^{\frac{\gamma(2-\gamma)\beta}{2\lambda}(Kx - \frac{\lambda t^\alpha}{\alpha} - Kx_0 + \frac{\lambda t_0^\alpha}{\alpha})} - 1}, \tag{2.20}$$

where x_0 and t_0 are real constants.

(iii) If $\lambda^2 = \frac{(2\gamma-1)^2}{2} K^2 \beta$, then

$$u_{s3}(x,t) = \frac{e^{\frac{(2\gamma-1)\beta}{2\lambda}(Kx - \frac{\lambda t^\alpha}{\alpha} - Kx_0 + \frac{\lambda t_0^\alpha}{\alpha})}}{e^{\frac{(2\gamma-1)\beta}{2\lambda}(Kx - \frac{\lambda t^\alpha}{\alpha} - Kx_0 + \frac{\lambda t_0^\alpha}{\alpha})} - 1}, \tag{2.21}$$

where x_0 and t_0 are real constants.

3. Conclusions and suggestions for Electron. Res. Arch., 31 (2023), 1303–1322

Starting from raising four questions in this comment, it is clear that Eq (1.3) does not have elliptic function solutions and rational function solutions. In our research, we have obtained the general solutions to Eq (1.3) by using Theorem 1. Thereby all exact solutions of Eq (1.1) are obtained. We hope this comment will be useful to readers.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

Acknowledgments

This work is supported by NSF of Guang dong Province of China (2023A1515011809), Science Research Group Project of SEIG (ST202101) and Science Research Project of SEIG (ky202211).

Conflict of interest

All authors declare no conflicts of interest in this paper.

References

1. G. Dang, Q. Liu, Solving the conformable Huxley equation using the complex method, *Electron. Res. Arch.*, **31** (2023), 1303–1322. https://doi.org/10.3934/era.2023067
2. W. J. Yuan, Y. D. Shang, Y. Huang, H. Wang, The representation of meromorphic solutions of certain ordinary differential equations and its applications, *Sci. Sin. Math.*, **43** (2013), 563–575. https://doi.org/10.1360/012012-159
3. W. J. Yuan, Y. Z. Li, J. M. Lin, Meromorphic solutions of an auxiliary ordinary differential equation using complex method, *Math. Meth. Appl. Sci.*, **36** (2013), 1776–1782. https://doi.org/10.1002/mma.2723
4. W. J. Yuan, Y. Huang, Y. D. Shang, All travelling wave exact solutions of two nonlinear physical models, *Appl. Math. Comput.*, **219** (2013), 6212–6223. https://doi.10.1016/j.amc.2012.12.023
5. W. J. Yuan, Z. Huang, M. Fu, J. Lai, The general solutions of an auxiliary ordinary differential equation using complex method and its applications, *Adv. Differ. Equations*, **2014** (2014), 1–9. https://doi.org/10.1186/1687-1847-2014-147
6. W. J. Yuan, B. Xiao, Y. Wu, J. Qi, The general traveling wave solutions of the Fisher type equations and some related problems, *J. Inequal. Appl.*, **2014** (2014), 1–15. https://doi.org/10.1186/1029-242X-2014-500
7. W. J. Yuan, Y. H. Wu, Q. H. Chen, Y. Huang, All meromorphic solutions for two forms of odd order algebraic differential equations and its applications, *Appl. Math. Comput.*, **240** (2014), 240–251. https://doi.org/10.1016/j.amc.2014.04.099
8. W. J. Yuan, W. L. Xiong, J. M. Lin, Y. H. Wu, All meromorphic solutions of an auxiliary ordinary differential equation using complex method, *Acta. Math. Sci.*, **35** (2015), 1241–1250. https://doi.org/10.1016/S0252-9602(15)30052-7
9. A. C. Cevikel, A. Bekir, E. H. M. Zahran, Novel exact and solitary solutions of conformable Huxley equation with three effective methods, *J. Ocean Eng. Sci.*, **2022** (2022). https://doi.org/10.1016/j.joes.2022.06.010
10. R. Conte, T. W. Ng, C. F. Wu, Hayman's classical conjecture on some nonlinear second-order algebraic ODEs, *Complex Var. Elliptic Equations*, **60** (2015), 1539–1552. https://doi.org/10.1080/17476933.2015.1033414

11. W. J. Yuan, F. N. Meng, J. M. Lin, Y. H. Wu, All meromorphic solutions of an ordinary differential equation and its applications, *Math. Meth. Appl. Sci.*, **39** (2016), 2083–2092. https://doi.10.1002/mma.3625

AIMS Press