



Research article

The parameter-Newton iteration for the second-order cone linear complementarity problem

Peng Zhou^{1,2} and Teng Wang^{1,*}

¹ School of Mathematics and Computer Sciences, Nanchang University, China

² Jiangxi Institute of Economic Administrators, China

* **Correspondence:** Email: spring0920@163.com.

Abstract: In this paper, we propose the parameter-Newton (PN) method to solve the second-order linear complementarity problem (SOCLCP). The key idea of PN method is that we transfer the SOCLCP into a system of nonlinear equations by bringing in a parameter. Then we solve the system of nonlinear equations by Newton method. At last, we prove that the PN method has quadratic convergence. Compared with the bisection-Newton (BN) method, the PN method has less CPU time and higher accuracy in numerical tests.

Keywords: second-order cone linear complementarity problem; Newton iteration; parameter; higher accuracy; quadratic convergence

1. Introduction

The second-order cone linear complementarity problem (SOCLCP) is to find a vector $\mathbf{x} \in \mathbf{R}^n$, such that

$$\mathbf{x} \in \mathcal{K}^n, \mathbf{z} = \mathbf{Ax} + \mathbf{q} \in \mathcal{K}^n, \mathbf{x}^\top \mathbf{z} = 0, \tag{1.1}$$

where $\mathbf{A} \in \mathbf{R}^{n \times n}$, $\mathbf{q} \in \mathbf{R}^n$ and $\mathcal{K}^n \subset \mathbf{R}^n$. \mathcal{K}^n (second-order cone) is defined as following:

$$\mathcal{K}^n := \{\mathbf{x} \mid \mathbf{x} = [x_1, \tilde{\mathbf{x}}^\top]^\top \in \mathbf{R}^n : x_1 \in \mathbf{R}_+, \tilde{\mathbf{x}} \in \mathbf{R}^{n-1} \text{ and } \|\tilde{\mathbf{x}}\|_2 \leq x_1\}, \tag{1.2}$$

where $\|\cdot\|_2$ denotes the Euclidean norm. Specially, \mathcal{K}^1 is equal to the set of nonnegative reals \mathbf{R}_+ . The SOCLCP is a kind of extension of linear complimentary problems(LCP). If you are interested in the LCP, please see [1–3]. The SOCLCP also could reduce to the nonlinear complementarity problem (NCP), please see [4–6].

The SOCLCP plays a very important role in many research fields. For example: the problem of support vector machine with noisy data and missing data [7]; combination optimization problem [8];

engineering design [9, 10]; convex network flow [11]; image processing [12, 13]; array antenna design [14].

What properties does the solution of the SOCLCP have? How to get the solution? These questions are very important. First of all, the problem about the unique of the solution should be solved. According to [15–18], we get the answer. If the matrix \mathbf{A} has the globally uniquely solvable (GUS) property, the solution of the SOCLCP is unique. Hence, many researches about how to get the unique solution arised in recent years. Here, we mainly introduce some methods in numerical algebra and optimization. For example: Hayashi et al. give the combined smoothing and regularization method [19] in 2005; Xiang et al. give the smoothing method [20] in 2009; Wang et al. give the interior-point method [21–25] in 2010; Fang et al. give the one-step Newton method [26] in 2011; Tang et al. give the smoothing Newton method [27] and Zhang et al. give the bisection-Newton method [28] in 2013; Hao et al. give the power penalty method [29] in 2015.

In this paper, we propose the parameter-Newton (PN) method to solve the SOCLCP. The key idea of PN method is that we transfer the SOCLCP into a system of nonlinear equations by bringing in a parameter (see Eq (3.1)). Then we solve the system of nonlinear equations by Newton method. The paper is organized as following. In Section 2, we introduce some notations and lemmas which will be used in the other sections. In Section 3, we give the details of PN method. In Section 4, we prove that the PN method has quadratic convergence. In Section 5, the numerical tests are presented. Finally, we state the conclusions and some future work in Section 6.

2. Preliminaries

In this section, we introduce some basic notations and lemmas which will be used in the other sections. First, we introduce the definition of Fréchet differentiability.

Definition 2.1. Let \mathbb{V} and \mathbb{W} be normed linear spaces, and \mathbb{U} be an open subset of \mathbb{V} . A function $\mathcal{F} : \mathbb{U} \rightarrow \mathbb{W}$ is called Fréchet differentiable at $\mathbf{x} \in \mathbb{U}$, if there exists a bounded linear operator $\mathbf{A} : \mathbb{V} \rightarrow \mathbb{W}$ such that

$$\lim_{\|\mathbf{h}\| \rightarrow 0} \frac{\|\mathcal{F}(\mathbf{x} + \mathbf{h}) - \mathcal{F}(\mathbf{x}) - \mathbf{A}\mathbf{h}\|}{\|\mathbf{h}\|} = 0. \quad (2.1)$$

Next, we give some lemmas about the norm of matrix.

Lemma 2.1. Let $\|\cdot\|_F$ denote the Euclidean norm of the matrix. Assume that matrices \mathbf{A} , $\mathbf{B} \in \mathbf{R}^{n \times n}$ and \mathbf{A} is nonsingular. If $\|\mathbf{A}^{-1}\|_F \leq \alpha$, $\|\mathbf{A} - \mathbf{B}\|_F \leq \beta$ and $\alpha\beta < 1$, we yield that the matrix \mathbf{B} is also nonsingular and $\|\mathbf{B}^{-1}\|_F \leq \frac{\alpha}{1-\alpha\beta}$.

Proof. Let the matrix $\mathbf{C} = \mathbf{I} - \mathbf{A}^{-1}\mathbf{B}$. Then we have

$$\begin{aligned} \|\mathbf{C}\|_F &= \|\mathbf{I} - \mathbf{A}^{-1}\mathbf{B}\|_F \\ &= \|\mathbf{A}^{-1}(\mathbf{A} - \mathbf{B})\|_F \\ &\leq \|\mathbf{A}^{-1}\|_F \cdot \|\mathbf{A} - \mathbf{B}\|_F \\ &\leq \alpha\beta < 1. \end{aligned}$$

It is clear that the matrix $\mathbf{A}^{-1}\mathbf{B} = \mathbf{I} - \mathbf{C}$ is nonsingular which means the matrix \mathbf{B} being also nonsingular.

Furthermore, we yield the following inequality as

$$\begin{aligned}
 \|\mathbf{B}^{-1}\|_F &= \|(\mathbf{I} - \mathbf{C})^{-1}\mathbf{A}^{-1}\|_F \\
 &\leq \|(\mathbf{I} - \mathbf{C})^{-1}\|_F \cdot \|\mathbf{A}^{-1}\|_F \\
 &= \alpha \left\| \lim_{n \rightarrow \infty} (\mathbf{I} + \mathbf{C} + \cdots + \mathbf{C}^n) \right\|_F \\
 &\leq \alpha \lim_{n \rightarrow \infty} (1 + \|\mathbf{C}\|_F + \cdots + \|\mathbf{C}\|_F^n) \\
 &= \frac{\alpha}{1 - \|\mathbf{C}\|_F} \\
 &\leq \frac{\alpha}{1 - \alpha\beta}.
 \end{aligned}$$

Here, we introduce some other notations which will be used in the following part of the paper. The $bd(\mathcal{K}^n)$ and the $int(\mathcal{K}^n)$ stand for the boundary and the interior of \mathcal{K}^n . The matrix $\mathbf{J}_n = \text{diag}\{1, -1, \dots, -1\} = \text{diag}\{1, -\mathbf{I}_{n-1}\}$.

Lemma 2.2. [18] For the SOCLCP, the matrix \mathbf{A} has the GUS property if and only if it satisfies the following assumptions:

(i) The matrix $\mathbf{A}\mathbf{J}_n$ has nonnegative eigenvalues and there exists $\tau > 0$ such that all nonnegative eigenvalues of $\mathbf{A}\mathbf{J}_n$ are equal to τ . Moreover, $\text{rank}(\mathbf{A}\mathbf{J}_n - \tau\mathbf{I}_n) = n - 1$. There exists $\mathbf{w} \in int(\mathcal{K}^n)$ such that \mathbf{w} is the eigenvector of $\mathbf{A}\mathbf{J}_n$ associated with τ ;

(ii) There exists $\mathbf{v} \in int(\mathcal{K}^n)$ such that \mathbf{v} is the eigenvector of $\mathbf{A}^\top\mathbf{J}_n$ associated with τ ;

(iii)

$$\mathbf{a}^\top \mathbf{A} \mathbf{a} \geq 0, \quad \forall \mathbf{a} \in bd(\mathcal{K}^n);$$

(iv)

$$\mathbf{a}^\top \mathbf{A}^{-1} \mathbf{a} \geq 0, \quad \forall \mathbf{a} \in bd(\mathcal{K}^n).$$

According to the above lemma, the positive definite (not necessarily symmetric) matrix \mathbf{A} has the GUS property. Next, we introduce two important conclusions of the solution to the SOCLCP when the matrix \mathbf{A} has the GUS property.

Lemma 2.3. [28] Suppose that \mathbf{A} has the GUS property and $\tau > 0$ is the positive eigenvalue of $\mathbf{A}^\top\mathbf{J}_n$ with the unit eigenvector $\mathbf{v} \in int(\mathcal{K}^n)$. Then the solution \mathbf{x} to the SOCLCP is locally a continuously differentiable function of \mathbf{q} .

Lemma 2.4. [28] Suppose that \mathbf{A} has the GUS property and τ is the unique positive eigenvalue of $\mathbf{A}\mathbf{J}_n$. Then for any $0 < \rho \neq \tau$ and for all nonzero vector $\mathbf{a} \in bd(\mathcal{K}^n)$, we have

$$\mathbf{a}^\top (\mathbf{A} - \rho\mathbf{J}_n)^{-1} \mathbf{a} \begin{cases} > 0, & \text{if } 0 < \rho < \tau, \\ < 0, & \text{if } \rho > \tau. \end{cases} \quad (2.2)$$

For a given \mathbf{A} with the GUS property and a vector $\mathbf{q} \in \mathbf{R}^n$, our main interest in this paper is to find the unique solution for the SOCLCP. There are two special cases that can be handled very easily: $\mathbf{q} \in \mathcal{K}^n$ and: $\mathbf{q} \in -\mathbf{A}\mathcal{K}^n$. The former gives the solution $\mathbf{x} = \mathbf{0}$ whereas the latter leads to the solution $\mathbf{x} = -\mathbf{A}^{-1}\mathbf{q}$. The case $\mathbf{q} \notin -\mathbf{A}\mathcal{K}^n \cup \mathcal{K}^n$ is the main topic which will be discussed in next section.

3. The parameter-Newton method

In [18], Yang et al. prove that when $\mathbf{q} \notin -\mathbf{A}\mathcal{K}^n \cup \mathcal{K}^n$, the exact solution \mathbf{x}_* and $\mathbf{z}_* = \mathbf{A}\mathbf{x}_* + \mathbf{q}$ are belong to the $bd(\mathcal{K}^n)$. Adding $\mathbf{x}_*^\top \mathbf{z}_* = 0$, we yield that $\mathbf{z}_* = \lambda_* \mathbf{J}_n \mathbf{x}_*$ where λ_* is a positive parameter. The problem (1.1) could be transformed into the following system of nonlinear equations:

$$\mathcal{F}(\mathbf{x}, \lambda) := \begin{cases} (\mathbf{A} - \lambda \mathbf{J}_n) \mathbf{x} + \mathbf{q} \\ \frac{1}{2} \mathbf{x}^\top \mathbf{J}_n \mathbf{x} \end{cases} = \mathbf{0} \quad (3.1)$$

Then we use Newton iteration to solve (3.1). It is easy to get the iteration equation:

$$\begin{pmatrix} \mathbf{A} - \lambda_k \mathbf{J}_n & -\mathbf{J}_n \mathbf{x}_k \\ \mathbf{x}_k^\top \mathbf{J}_n & 0 \end{pmatrix} \begin{pmatrix} \mathbf{x}_{k+1} \\ \lambda_{k+1} \end{pmatrix} = \begin{pmatrix} -\lambda_k \mathbf{J}_n \mathbf{x}_k - \mathbf{q} \\ \frac{1}{2} \mathbf{x}_k^\top \mathbf{J}_n \mathbf{x}_k \end{pmatrix}. \quad (3.2)$$

We combine our described techniques and present the complete PN algorithm in Algorithm 1.

Algorithm 1 The parameter-Newton method (The PN method)

INPUT: The matrix $\mathbf{A} \in \mathbf{R}^{n \times n}$ with GUS property, a vector $\mathbf{q} \in \mathbf{R}^n$, the initial vector $\mathbf{x}_0 = (1, 0, \dots, 0)^\top$, $\lambda_0 = 1$ and a relative tolerance ϵ .

OUTPUT: The solution \mathbf{x}_* .

Case 1: If $\mathbf{q} \in \mathcal{K}^n$ then

$$\mathbf{x}_* = \mathbf{0};$$

Case 2: If $-\mathbf{A}^{-1} \mathbf{q} \in \mathcal{K}^n$ then

$$\mathbf{x}_* = -\mathbf{A}^{-1} \mathbf{q};$$

Case 3: If $\mathbf{q} \notin -\mathbf{A}\mathcal{K}^n \cup \mathcal{K}^n$ then

while ($\frac{\|\mathbf{x}_{k+1} - \mathbf{x}_k\|_2}{\|\mathbf{x}_k\|_2} \geq \epsilon$)

Solve equation (3.2);

If the first element of \mathbf{x}_{k+1} is negative, then let it equal $\|\bar{\mathbf{x}}_{k+1}\|_2$;

If $\lambda_{k+1} < 0$, then let $\lambda_{k+1} = -\lambda_{k+1}$;

end while

% We use GMRES in Case 3 to solve the Eq (3.2).

4. The convergence of the PN method

In this section, we prove that the convergence of the PN method has quadratic convergence.

Theorem 4.1. *Suppose that the sequence $\{\mathbf{x}_k\}$ is produced by the PN method, then the sequence has quadratic convergence.*

Proof. First of all, we need to prove the following conclusion.

When \mathbf{A} has the GUS property, then for any $\mathbf{q} \notin -\mathbf{A}\mathcal{K}^n \cup \mathcal{K}^n$, the coefficient matrix of Eq (3.2) is nonsingular at the solution pair $(\mathbf{x}_*, \lambda_*)$.

Suppose there exists a vector $(\mathbf{a}^\top, b)^\top \in \mathbf{R}^{n+1}$ satisfying

$$\begin{pmatrix} \mathbf{A} - \lambda_* \mathbf{J}_n & -\mathbf{J}_n \mathbf{x}_* \\ \mathbf{x}_*^\top \mathbf{J}_n & 0 \end{pmatrix} \begin{pmatrix} \mathbf{a} \\ b \end{pmatrix} = \mathbf{0}.$$

In case $\lambda_* = \tau$, Lemma 2.3 indicates $(\mathbf{a}^\top, b)^\top = \mathbf{0}$. If $\lambda_* \neq \tau$, then it follows that $\mathbf{A} - \lambda_* \mathbf{J}_n$ is nonsingular and thus one has

$$\mathbf{a} = b(\mathbf{A} - \lambda_* \mathbf{J}_n)^{-1}(\mathbf{J}_n \mathbf{x}_*), \tag{4.1}$$

and

$$0 = b(\mathbf{J}_n \mathbf{x}_*)^\top (\mathbf{A} - \lambda_* \mathbf{J}_n)^{-1}(\mathbf{J}_n \mathbf{x}_*). \tag{4.2}$$

This relation together with Lemma 2.4 implies $b = 0$ which leads to $\mathbf{a} = \mathbf{0}$. Therefore, the conclusion follows.

Furthermore, at the neighborhood of the solution pair $(\mathbf{x}_*, \lambda_*)$, the matrix

$$\mathcal{F}'(\mathbf{x}, \lambda) := \begin{pmatrix} \mathbf{A} - \lambda \mathbf{J}_n & -\mathbf{J}_n \mathbf{x} \\ \mathbf{x}^\top \mathbf{J}_n & 0 \end{pmatrix}$$

is nonsingular.

Next, we prove the following inequality

$$\|\mathcal{F}'(\mathbf{x}_k, \lambda_k) - \mathcal{F}'(\mathbf{x}_*, \lambda_*)\|_F \leq \sqrt{n}(\|\mathbf{x}_k - \mathbf{x}_*\|_F^2 + (\lambda_k - \lambda_*)^2)^{\frac{1}{2}}. \tag{4.3}$$

We yield that

$$\begin{aligned} & \|\mathcal{F}'(\mathbf{x}_k, \lambda_k) - \mathcal{F}'(\mathbf{x}_*, \lambda_*)\|_F \\ &= \left\| \begin{pmatrix} \mathbf{A} - \lambda_k \mathbf{J}_n & -\mathbf{J}_n \mathbf{x}_k \\ \mathbf{x}_k^\top \mathbf{J}_n & 0 \end{pmatrix} - \begin{pmatrix} \mathbf{A} - \lambda_* \mathbf{J}_n & -\mathbf{J}_n \mathbf{x}_* \\ \mathbf{x}_*^\top \mathbf{J}_n & 0 \end{pmatrix} \right\|_F \\ &= \left\| \begin{pmatrix} \lambda_k \mathbf{J}_n - \lambda_* \mathbf{J}_n & \mathbf{J}_n \mathbf{x}_k - \mathbf{J}_n \mathbf{x}_* \\ -\mathbf{x}_k^\top \mathbf{J}_n + \mathbf{x}_*^\top \mathbf{J}_n & 0 \end{pmatrix} \right\|_F. \end{aligned}$$

Let

$$\mathbf{M} = \begin{pmatrix} \lambda_k \mathbf{J}_n - \lambda_* \mathbf{J}_n & \mathbf{J}_n \mathbf{x}_k - \mathbf{J}_n \mathbf{x}_* \\ -\mathbf{x}_k^\top \mathbf{J}_n + \mathbf{x}_*^\top \mathbf{J}_n & 0 \end{pmatrix},$$

we get that

$$\|\mathcal{F}'(\mathbf{x}_k, \lambda_k) - \mathcal{F}'(\mathbf{x}_*, \lambda_*)\|_F^2 = \|\mathbf{M}\|_F^2 = \text{tr}(\mathbf{M}^\top \mathbf{M}).$$

By direct computing, we have that

$$\begin{aligned} \mathbf{M}^\top \mathbf{M} &= \begin{pmatrix} \lambda_k \mathbf{J}_n - \lambda_* \mathbf{J}_n & -\mathbf{J}_n \mathbf{x}_k + \mathbf{J}_n \mathbf{x}_* \\ \mathbf{x}_k^\top \mathbf{J}_n - \mathbf{x}_*^\top \mathbf{J}_n & 0 \end{pmatrix} \begin{pmatrix} \lambda_k \mathbf{J}_n - \lambda_* \mathbf{J}_n & \mathbf{J}_n \mathbf{x}_k - \mathbf{J}_n \mathbf{x}_* \\ -\mathbf{x}_k^\top \mathbf{J}_n + \mathbf{x}_*^\top \mathbf{J}_n & 0 \end{pmatrix} \\ &= \begin{pmatrix} (\lambda_k \mathbf{J}_n - \lambda_* \mathbf{J}_n)^2 + (-\mathbf{J}_n \mathbf{x}_k + \mathbf{J}_n \mathbf{x}_*)(-\mathbf{x}_k^\top \mathbf{J}_n + \mathbf{x}_*^\top \mathbf{J}_n) & (\lambda_k \mathbf{J}_n - \lambda_* \mathbf{J}_n)(\mathbf{J}_n \mathbf{x}_k - \mathbf{J}_n \mathbf{x}_*) \\ (\mathbf{x}_k^\top \mathbf{J}_n - \mathbf{x}_*^\top \mathbf{J}_n)(\lambda_k \mathbf{J}_n - \lambda_* \mathbf{J}_n) & (\mathbf{x}_k^\top \mathbf{J}_n - \mathbf{x}_*^\top \mathbf{J}_n)(\mathbf{J}_n \mathbf{x}_k - \mathbf{J}_n \mathbf{x}_*) \end{pmatrix} \\ &= \begin{pmatrix} (\lambda_k - \lambda_*)^2 \mathbf{I}_n + (-\mathbf{x}_k + \mathbf{x}_*)(-\mathbf{x}_k^\top + \mathbf{x}_*^\top) & (\lambda_k - \lambda_*)(\mathbf{x}_k - \mathbf{x}_*) \\ (\lambda_k - \lambda_*)(\mathbf{x}_k^\top - \mathbf{x}_*^\top) & (\mathbf{x}_k^\top - \mathbf{x}_*^\top)(\mathbf{x}_k - \mathbf{x}_*) \end{pmatrix}. \end{aligned}$$

Therefore, we yield

$$\begin{aligned} \text{tr}(\mathbf{M}^\top \mathbf{M}) &= \text{tr}[(\lambda_k - \lambda_*)^2 \mathbf{I}_n + (-\mathbf{x}_k + \mathbf{x}_*)(-\mathbf{x}_k^\top + \mathbf{x}_*^\top)] + (\mathbf{x}_k^\top - \mathbf{x}_*^\top)(\mathbf{x}_k - \mathbf{x}_*) \\ &= n(\lambda_k - \lambda_*)^2 + \text{tr}[(-\mathbf{x}_k + \mathbf{x}_*)(-\mathbf{x}_k^\top + \mathbf{x}_*^\top)] + (\mathbf{x}_k^\top - \mathbf{x}_*^\top)(\mathbf{x}_k - \mathbf{x}_*) \\ &= n(\lambda_k - \lambda_*)^2 + 2(\mathbf{x}_k^\top - \mathbf{x}_*^\top)(\mathbf{x}_k - \mathbf{x}_*). \end{aligned} \tag{4.4}$$

Similarly,

$$\|\mathbf{x}_k - \mathbf{x}_*\|_F^2 + (\lambda_k - \lambda_*)^2 = (\mathbf{x}_k - \mathbf{x}_*)^\top (\mathbf{x}_k - \mathbf{x}_*) + (\lambda_k - \lambda_*)^2. \quad (4.5)$$

Getting (4.4) and (4.5) together, we prove the inequality (4.3).

Now, the proof of the theorem is finished.

5. Numerical results

In this section, we will present the numerical experiments of the PN method. It is known that there exist various methods for solving the SOCLCP. In order to evaluate the numerical performance and demonstrate clearly the efficiency of the PN method for solving the SOCLCP, we will also present the numerical results from the bisection-Newton (BN) method [28]. All of our tests are carried out in MATLAB (R2015b) on a PC with Intel(R) Core(TM) m3-7Y30 CPU @ 1.00GHz and 4GB memory.

Example 5.1. *The matrix $\mathbf{N} \in \mathbf{R}^{n \times n}$ is a large sparse random matrix. We set that the dimension n equals 1000, 3000, 5000. The density of matrix \mathbf{N} is 0.5, 1 and 5%. The matrix $\mathbf{A} = \mathbf{N}^\top \mathbf{N}$ has the GUS property. The vector \mathbf{q} is also a random vector.*

We make the initial vector $\mathbf{x}_0 = (1, 0, \dots, 0)^\top$ and parameter $\lambda_0 = 1$. We set the relative tolerance $\epsilon = 10^{-6}$. The results of the numerical test are presented in the Table 1. In this table, we summarize the average numbers of iterations (labelled as Iter) for every method. The corresponding CPU times (labelled as CPU, the unit as second) of each method are also summarized. To check the accuracy (labelled as Err) of the computed solution, we define

$$ac = |\mathbf{x}^\top \mathbf{z}| + |\mathbf{x}_1 - |\tilde{\mathbf{x}}||_2| + |\mathbf{z}_1 - |\tilde{\mathbf{z}}||_2|. \quad (5.1)$$

Table 1. The results of PN method and BN method.

n	density	Iter		CPU(s)		Err	
		PN	BN	PN	BN	PN	BN
1000	0.5%	19	27	0.26	0.17	8.27e-10	3.11e-06
	1%	19	27	0.78	0.10	8.71e-10	7.81e-06
	5%	19	27	0.50	0.06	9.06e-13	7.16e-06
3000	0.5%	19	31	2.23	6.18	1.43e-08	2.14e-06
	1%	20	31	2.04	5.20	7.37e-08	6.37e-06
	5%	19	31	1.85	3.20	5.87e-09	2.60e-06
5000	0.5%	25	33	10.26	21.32	8.94e-08	1.56e-06
	1%	22	32	10.78	25.37	1.79e-08	8.40e-07
	5%	22	33	10.50	21.09	1.21e-07	6.92e-06

6. Conclusions and future work

In this paper, we propose the parameter-Newton method for solving the second-order cone linear complementarity problem. We prove that our algorithm has quadratic convergence at least. Numerical results also show that the PN method is more efficient when \mathbf{A} is a large, sparse and symmetric positive

definite matrix. Since the Eq (3.2) being the saddle point problem, we will use some other numerical methods to solve it in future.

Conflict of interest

The authors declare there is no conflicts of interest.

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