



Research article

Experimental evaluation of PSO-optimized LQR control for a Quanser 3-DOF gyroscope: A multi-criteria comparative study under practical constraints

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Abstract: This paper presents a rigorous experimental evaluation of a particle swarm optimization (PSO)-optimized linear quadratic regulator (LQR) controller deployed on a physical Quanser 3-degree-of-freedom (3-DOF) gyroscope platform. To address the limitations of manual trial-and-error parameter tuning under real-world constraints, a multi-criteria optimization framework was established using three distinct performance indices: integral of absolute error (IAE), integral of time-weighted absolute error (ITAE), and integral of squared error (ISE). Real-time hardware experiments under multi-frequent square-wave and continuous sinusoidal trajectories, alongside impulse disturbance scenarios, demonstrated that the proposed PSO-LQR configured via the IAE criterion achieves the most superior balance between dynamic tracking agility and actuator energy efficiency. Specifically, the proposed PSO-IAE framework yields a settling time of 1.68 s, zero overshoot, and a minimal steady-state error of 0.10 °. Crucially, quantitative energy analysis revealed that the proposed framework effectively circumvents motor voltage saturation, operating at an optimal control torque root mean square (RMS) value of 0.0962 Nm, which significantly outperforms the ISE configuration (0.3184 Nm) and the ITAE variant (0.1446 Nm). These empirical findings provide valuable practical benchmarks for constraint-enforced automation in high-level mechatronic mechanisms.

Keywords: 3-DOF gyroscope; actuator saturation; LQR; PSO; real-time control

1. Introduction

Gyroscope mechanisms play a pivotal role in numerous modern mechatronic and aerospace systems, ranging from satellite stabilization and missile guidance to complex robotic platforms [1–3]. Designing an accurate trajectory-tracking system for these mechanisms remains a formidable challenge due to their highly nonlinear dynamics, underactuated nature, and intricate cross-coupling between rotational axes [4,5]. In modern control engineering, the LQR is widely used owing to its ability to optimize state-feedback systems, providing an effective trade-off between tracking error and control energy [6,7].

Nevertheless, the practical feasibility and performance of the LQR strictly depend on the selection of the weighting matrices, Q and R . The conventional trial-and-error approach to tuning these matrices is not only time-consuming but also heuristic, and it struggles to guarantee global optimality when the system operates under stringent physical constraints [8,9]. To overcome this parameter-selection dilemma, metaheuristic optimization algorithms have been widely integrated into the control design process. While the genetic algorithm (GA) has demonstrated efficacy in enhancing LQR performance on the identical Quanser model through the authors' previous experimental evaluations [10], particle swarm optimization emerges as a potentially more robust alternative due to its simpler mathematical structure and faster convergence rate. PSO has been successfully deployed across various complex nonlinear systems [11–13]. Furthermore, to tackle the inherent uncertainties and severe nonlinearities of gyroscopic mechanisms, several advanced methodologies—such as applied type-3 fuzzy logic configurations and observer-based intelligent control strategies—have been developed recently to maximize tracking precision under structural perturbations [14,15].

However, most current studies are confined to simulation-based evaluations [16,17]. In practice, mechatronic systems are inevitably subjected to unmodeled dynamic factors such as nonlinear friction, sensor noise, and time delays [18,19]. Notably, phenomena such as backlash or actuator dead zones can severely degrade system stability [20,21]. Consequently, the transition from theoretical formulation to practical deployment necessitates rigorous experimental validation [22,23].

Furthermore, the quest for optimal and robust trajectory tracking control has significantly expanded across various high-level nonlinear and constrained platforms in recent literature. To ensure physical safety and operational efficiency, advanced control paradigms have been extensively explored, including finite-time learning-based optimal elliptical encircling control [24] and enhanced fixed-time collision-free elliptical circumnavigation coordination for unmanned aerial vehicles (UAVs) [25]. To handle complex vehicle configurations without the inherent vulnerabilities of conventional backstepping, adaptive critic attitude learning control for hypersonic morphing vehicles without backstepping [26] and concurrent-learning-based adaptive critic formation for multi-robots under safety constraints [27] have established powerful theoretical benchmarks. Additionally, modern intelligent methodologies—such as efficient path-following for urban logistics, a fuzzy control strategy for consumer UAVs under disturbance constraints [28], safety-certified optimal formation control for nonlinear multiagents via high-order control barrier function [29], and enhanced state-constrained adaptive fuzzy exact tracking control for nonlinear strict-feedback systems [30]—underscore the paradigm shift toward constraint-enforced automation. Integrating these cutting-edge state-constraint frameworks further highlights the critical necessity of validating practical metaheuristic-driven LQR structures on real-time hardware mechanisms to bridge the gap between complex theoretical formulations and mechatronic constraints.

Despite these advanced theoretical frameworks, a critical research gap remains unaddressed in

the literature, since most existing studies are strictly confined to numerical simulations under idealized, unconstrained assumptions. Consequently, there is a distinct lack of comprehensive experimental research investigating how mathematically diverse optimization criteria, specifically IAE, ITAE, and ISE, interact with real-world mechatronic boundaries such as nonlinear joint friction and actuator voltage saturation on physical multi-DOF gyroscope platforms. To bridge this literature gap and build upon the foundation of the previous GA-based optimization study [10], this manuscript aims to establish a rigorous, practical multi-criteria PSO-LQR integration framework deployed directly onto a physical 3-DOF Quanser gyroscope hardware setup. Furthermore, this study systematically evaluates and compares the real-time tracking agility under both multi-frequent square-wave and continuous time-varying sinusoidal trajectories, while quantitatively analyzing the physical trade-off between tracking precision and control energy expenditure via root mean square analysis under physical actuator constraints and manual impulse disturbances. Accordingly, the primary contributions of this work lie in the successful real-time hardware implementation of the PSO-LQR architecture under physical saturation boundaries, alongside an in-depth comparative analysis detailing the physical implications of distinct objective functions on control effort, trajectory tracking fidelity, and overall system robustness.

2. Materials and methods

2.1. Mathematical modeling of the 3-DOF gyroscope

The synthesis of a practical optimal control algorithm fundamentally relies on the formulation of a precise mathematical model—one that comprehensively captures the gyroscope's inherent nonlinear dynamics and intricate cross-coupling effects [1,2]. In this study, the system's dynamic model is established by applying D'Alembert's principle in conjunction with multibody rigid mechanics [4]. Structurally, the physical architecture of the 3-DOF gyroscope, as illustrated in Figure 1, consists of three primary mechanical components arranged in a nested rotational configuration: the outer gimbal (o), the inner gimbal (g), and the rotor (r).

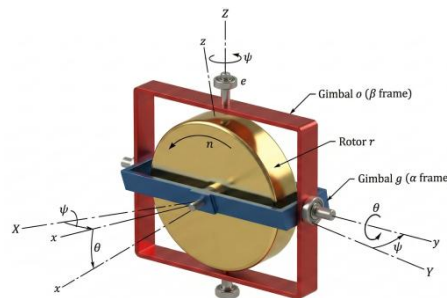


Figure 1. Spatial configuration and coordinate frame definitions of the gyroscope.

By employing Eulerian coordinate transformations, the absolute angular velocity components of these reference frames are explicitly determined as follows:

$$\Omega^\beta = \mathbf{1}_z \dot{\psi} \quad (1)$$

$$\Omega^\alpha = \Omega^{\alpha/\beta} + \Omega^\beta = \mathbf{1}_y \dot{\theta} + \mathbf{1}_z \dot{\psi} = -\mathbf{1}_x \dot{\psi} \sin \theta + \mathbf{1}_y \dot{\theta} + \mathbf{1}_z \dot{\psi} \cos \theta \quad (2)$$

$$\Omega^r = \Omega^{r/\alpha} + \Omega^\alpha = \mathbf{1}_x \dot{n} + \mathbf{1}_y \dot{\theta} + \mathbf{1}_z \dot{\psi} = \mathbf{1}_x (n - \dot{\psi} \sin \theta) + \mathbf{1}_y \dot{\theta} + \mathbf{1}_z \dot{\psi} \cos \theta. \quad (3)$$

Where θ and ψ denote the angular displacements of the inner and outer gimbals, respectively, and n represents the nominal spin rate of the rotor relative to the inner gimbal. Invoking Euler's equations of rigid body dynamics, the applied external torque is equated to the time derivative of the angular momentum $\mathbf{H}$. Accordingly, the moment balance equations governing the dynamics of the rotor (r), inner gimbal (g), and outer gimbal (o) are formulated as follows:

$$-\mathbf{M}_i^r = \dot{\mathbf{H}}^r = \mathbf{H}^r + \boldsymbol{\Omega}^\alpha \times \mathbf{H}^r \quad (4)$$

$$-\mathbf{M}_i^g = \dot{\mathbf{H}}^g = \mathbf{H}^g + \boldsymbol{\Omega}^\alpha \times \mathbf{H}^g \quad (5)$$

$$-\mathbf{M}_i^0 = \dot{\mathbf{H}}^0 = \mathbf{H}^0 + \boldsymbol{\Omega}^\beta \times \mathbf{H}^0 \quad (6)$$

To derive the equations of motion for each degree of freedom, a systematic analysis of the isolated free-body diagrams, as illustrated in Figure 2, is conducted.

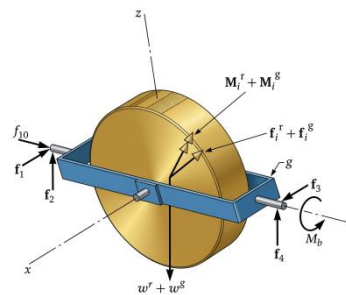


Figure 2. Free-body diagram of the inner gimbal-rotor assembly.

By evaluating the moment equilibrium of the combined inner gimbal and rotor assembly (g+r) about the y-axis, based on the free-body diagram illustrated in Figure 3, the dynamic equation—formulated to decouple and eliminate the internal reaction forces at the bearings—is derived as follows:

$$(\Sigma M^*)_y = (M_i^r + M_i^g)_y + M_b = 0 \quad (7)$$

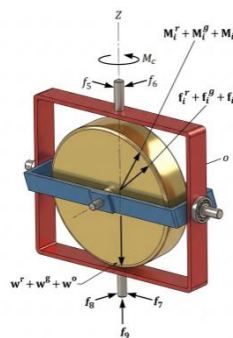


Figure 3. Free-body diagram of the complete gyroscope assembly (g+r+o).

Subsequently, by enforcing the moment equilibrium condition on the complete assembly (g + r + o) about the spatial Z-axis, the following dynamic relationship is obtained:

$$(\Sigma M^*)_z = (M_i^r + M_i^g + M_i^o)_z + M_c = 0 \quad (8)$$

Transforming Equation (8) into the body-fixed reference frame of the gimbal assembly yields the following general form:

$$(M_i^r + M_i^g)_z \cos \theta - (M_i^r + M_i^g)_x \sin \theta + (M_i^o)_z + M_c = 0 \quad (9)$$

Substituting the angular velocity components from Eqs (1)–(3) into the angular momentum equations yields:

$$\mathbf{H}^r = \mathbf{1}_x [J_x^r (n - \dot{\psi} \sin \theta)] + \mathbf{1}_y [J_y^r \dot{\theta}] + \mathbf{1}_z [J_z^r \dot{\psi} \cos \theta] \quad (10)$$

$$\mathbf{H}^g = \mathbf{1}_x [J_x^g (-\dot{\psi} \sin \theta)] + \mathbf{1}_y [J_y^g \dot{\theta}] + \mathbf{1}_z [J_z^g \dot{\psi} \cos \theta] \quad (11)$$

$$\mathbf{H}^o = \mathbf{1}_z [J_z^o \dot{\psi}] \quad (12)$$

To simplify the mathematical framework, the equivalent moments of inertia for the entire system are redefined in accordance with the technical specifications [10]:

$$h \triangleq J_x^r n, J_x \triangleq J_x^r + J_x^g, J_y \triangleq J_y^r + J_y^g, J_z \triangleq J_z^r + J_z^g \quad (13)$$

Where h represents the angular momentum of the rotor. By substituting Eq (13) into Eqs (10)–(12) and evaluating the vector cross products, the full nonlinear differential equations—characterizing the inertial forces and complex gyroscopic coupling terms—are obtained as follows:

$$-(M_i^r + M_i^g)_y = J_y \ddot{\theta} + h \dot{\psi} \cos \theta + (J_z - J_x) \dot{\psi}^2 \sin \theta \cos \theta \quad (14)$$

$$\begin{aligned} -(M_i^r + M_i^g)_z \cos \theta \\ = J_z (\ddot{\psi} \cos^2 \theta - \dot{\psi} \dot{\theta} \sin \theta \cos \theta) - h \dot{\theta} \cos \theta + (J_x - J_y) \dot{\psi} \dot{\theta} \sin \theta \cos \theta \end{aligned} \quad (15)$$

$$-(M_i^r + M_i^g)_x \sin \theta = -J_z (\ddot{\psi} \sin^2 \theta + \dot{\psi} \dot{\theta} \cos \theta \sin \theta) + (J_z - J_y) \dot{\psi} \dot{\theta} \cos \theta \sin \theta \quad (16)$$

By incorporating the components from Eqs (14), (15), and (16) back into the fundamental moment balance Eqs (7) and (9), the final nonlinear mathematical model of the gyroscope system is established as follows:

$$J_y \ddot{\theta} + h \dot{\psi} \cos \theta + (J_z - J_x) \dot{\psi}^2 \sin \theta \cos \theta = M_b \quad (17)$$

$$(J_z^o + J_z \cos^2 \theta + J_x \sin^2 \theta) \ddot{\psi} - h \dot{\theta} \cos \theta + 2(J_x - J_z) \dot{\psi} \dot{\theta} \sin \theta \cos \theta = M_c \quad (18)$$

To design the LQR controller, the system must be linearized around the equilibrium point. Given the practical operating assumptions that the oscillation angles are small ($\theta, \psi \approx 0 \Rightarrow \sin \theta \approx \theta, \cos \theta \approx 1$), and the rotor speed n is significantly higher than the gimbal frame rates ($n \gg \dot{\theta}, \dot{\psi}$), the high-order nonlinear terms in Eqs (17) and (18) are neglected. The resulting linearized system of equations is obtained as follows:

$$\begin{cases} J_y \ddot{\theta} + h \dot{\psi} = M_b \end{cases} \quad (19a)$$

$$\begin{cases} J_z \ddot{\psi} - h \dot{\theta} = M_c \end{cases} \quad (19b)$$

In the Quanser 3-DOF experimental setup, the control effort (the torque generated by the DC motor) is applied exclusively to the inner gimbal axis. In contrast, the outer gimbal remains unactuated and continues to rotate freely. Consequently, Equation (19) is simplified into the following form:

$$\begin{cases} J_y \ddot{\theta} + h\dot{\psi} = \tau_y & (20a) \\ J_z \ddot{\psi} - h\dot{\theta} = 0 & (20b) \end{cases}$$

The system of equations (20) serves as the fundamental mathematical basis for constructing the state-space model $\dot{x} = Ax + Bu$, which is directly utilized for the design of the LQR state-feedback controller in the subsequent section. The physical parameters of the Quanser gyroscope system, as detailed in the manufacturer's technical documentation, are summarized in Table 1 [31,32].

Table 1. Physical parameters of the Quanser 3-DOF gyroscope system.

Parameter	Symbol	Value
Moment of inertia about y-axis	J_y	0.0039 Kg.m ²
Moment of inertia about z-axis	J_z	0.0223 Kg.m ²
Rotor angular momentum	h	0.44 Kg.m ² /s

Based on the dynamic parameters shown in Table 1, the state-space characteristics of the system (matrices A and B) are fully quantified. However, a deeper analysis of these physical values reveals that the gyroscope control problem is subject to stringent constraints. Specifically, the significant disparity between the moments of inertia (I) and the viscous friction coefficients at the gimbal axes requires that the actuator provide sufficient, instantaneous kinetic energy to overcome static friction, particularly during trajectory reversal phases. Nevertheless, this capability is strictly limited by the electromechanical characteristics of the drive motor (including armature resistance and torque constant). Designing a controller based solely on an idealized linearized model without energy allocation optimization would easily lead to hardware saturation or significant steady-state errors due to friction. This reality necessitates the application of an LQR control structure integrated with the PSO algorithm to identify the optimal compromise between control effort and dynamic constraints, as detailed in the subsequent sections.

2.2. Design of the LQR controller

The LQR is a robust optimal control algorithm utilized to determine the optimal closed-loop pole locations for a given system. The design procedure requires defining an optimal performance index and solving the algebraic Riccati equation (ARE) to derive the optimal control gain. Specifically, the objective of the LQR framework is to synthesize a state-feedback controller K that minimizes the quadratic performance index J . Within this methodology, the feedback gain matrix K is computed to minimize the cost function, thereby establishing an effective tradeoff between control effort utilization, the relative importance of state deviations, and response speed to ensure overall system stability [33,34]. The designer must meticulously select the weighting matrices Q and R to calculate the optimal gain K , typically in MATLAB. The fundamental architecture of the LQR controller is illustrated in Figure 4.

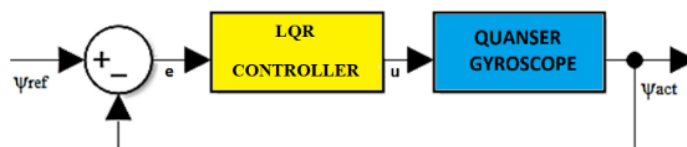


Figure 4. Block diagram of the LQR control architecture.

In this configuration, ψ_{ref} denotes the reference input signal, while ψ_{act} represents the actual

measured output of the system. Given the LQR control law defined by $\mathbf{u} = -\mathbf{K}\mathbf{x}$, the weighting matrices $\mathbf{Q}$ and $\mathbf{R}$ are initially selected based on the Quanser technical specifications [31,32].

To establish a seamless consistency between the mathematical state-space representations and the control block diagrams illustrated in Figures 4 and 5, the physical mechanism of gyroscopic cross-coupling must be explicitly integrated into the loop analysis. Although the controlled input voltage or torque τ_y is physically applied directly to the active inner gimbal frame axis (θ), the inherent gyroscopic precession and cross-coupling dynamics automatically link and transfer this operational energy to drive the outer yaw gimbal frame axis (ψ). Consequently, the multi-loop optimization framework targets the final error feedback derived from the actual output heading angle ψ_{act} . This dynamic mapping ensures that the control loop representations in both Figures 4 and 5 are fully synchronized and perfectly aligned with the presented structural state equations.

$$\mathbf{Q} = \text{diag}([2, 16, 0.01, 0.0001]) \text{ and } \mathbf{R} = [5] \quad (21)$$

Determining the optimal gain matrix $\mathbf{K}$ is fundamentally reduced to solving the continuous-time algebraic Riccati equation (CARE). The primary objective is to identify a unique, symmetric, and positive-definite solution $\mathbf{P}$ for Eq (22):

$$\mathbf{A}^T \mathbf{P} + \mathbf{P} \mathbf{A} - \mathbf{P} \mathbf{B} \mathbf{R}^{-1} \mathbf{B}^T \mathbf{P} + \mathbf{Q} = 0 \quad (22)$$

Once the solution $\mathbf{P}$ to the algebraic Riccati equation (22) is determined, the state-feedback gain matrix $\mathbf{K}$ is directly computed using the following expression:

$$\mathbf{K} = -\mathbf{R}^{-1} \mathbf{B}^T \mathbf{P} \quad (23)$$

By applying the aforementioned methodology to the system under study, with predefined weighting matrices $\mathbf{Q}$ and $\mathbf{R}$, Equations (22) and (23) are numerically solved using MATLAB's `lqr` function. The computational results yield the following state-feedback gain matrix: $\mathbf{K} = [0.651, 1.79, 0.12, 0.0045]$.

2.3. Design of the PSO-LQR controller

To address the challenge of optimally selecting the weighting matrices $\mathbf{Q}$ and $\mathbf{R}$ for the LQR controller, this study proposes a hybrid control architecture that integrates the standard LQR framework with the PSO algorithm. This approach is effective in prior research [11,35]. The detailed block diagram of the proposed control structure is illustrated in Figure 5.

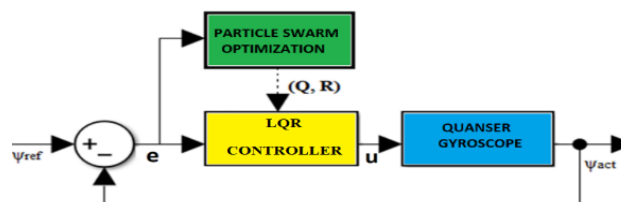


Figure 5. Block diagram of the proposed PSO-LQR control architecture.

In the proposed framework, the controller is structured into two coordinated operational loops. The inner control loop comprises the LQR controller, which directly controls the Quanser gyroscope system based on the tracking error signal. The outer optimization loop serves as an offline design process utilizing the PSO algorithm. The primary objective of the PSO is to autonomously search for the optimal pair of weighting matrices ($\mathbf{Q}$, $\mathbf{R}$) for the LQR, thereby replacing traditional manual tuning and trial-and-error methods. To comprehensively evaluate performance and identify the most suitable design strategy, three independent optimization processes are executed. In each process, the

PSO seeks to minimize one of the three standard performance indices [6,7], which serve as the objective functions:

$$IAE = \int_0^{\infty} |e(t)| dt \quad (24)$$

$$ITAE = \int_0^{\infty} t \cdot |e(t)| dt \quad (25)$$

$$ISE = \int_0^{\infty} [e(t)]^2 dt \quad (26)$$

The primary objective of this architecture is to execute three independent optimization processes. Within each process, the PSO algorithm searches for a distinct set of weighting matrices ($\mathbf{Q}$, $\mathbf{R}$) to minimize one of the aforementioned objective functions (IAE, ITAE, or ISE). Consequently, the output of this design phase consists of three different LQR controller variants, each tuned according to a specific performance criterion, ready for comparative performance evaluation in the subsequent stage.

Table 2. Configuration parameters for the PSO algorithm.

Parameter	Symbol	Value/limit
Population size	N	50
Maximum iterations	$iter_{max}$	50
Cognitive constant	c_1	2.0
Social constant	c_2	2.0
Inertia weight	w	0.9 $\searrow$ 0.4
Q search space	q_1, q_2, q_3, q_4	$[0.01, 1000] \times \mathbf{Q}_{quanser}$
R search space	r	$[0.01, 1000] \times \mathbf{R}_{quanser}$

Specifically, a population of $N = 50$ particles undergoes $k = 50$ iterations, integrated with a linearly decreasing inertia weight w from 0.9 to 0.4. This configuration facilitates a smooth transition from the exploration phase to the exploitation phase within the search space. Notably, instead of imposing rigid search boundaries, this study proposes a relative search range for the weighting matrices $\mathbf{Q}$ and $\mathbf{R}$, spanning from 0.01 to 1000 times the nominal values provided by Quanser. This scaling strategy plays a pivotal role, enabling the swarm to utilize the default controller—which inherently guarantees baseline stability—as the search focal point. Consequently, the PSO algorithm possesses sufficient degrees of freedom to either relax or tighten the penalty coefficients for tracking errors and control effort. This approach significantly accelerates convergence while preventing the particles from gravitating toward unstable regions that could jeopardize the hardware platform.

To ensure strict mathematical compliance during this heuristic search phase, the state weighting matrix $\mathbf{Q}$ is explicitly restricted to a diagonal structure, $\mathbf{Q} = \text{diag}(q_1, q_2, q_3, q_4)$, which inherently guarantees the symmetry requirement. Furthermore, the search space boundaries for all swarm particles are restricted strictly within the positive domain ($q_i > 0$ and $r > 0$), with the specific lower bounds (LB) and upper bounds (UB) defined explicitly in Table 2. Because these lower search boundaries are selected as strictly positive values greater than zero, as shown in Table 2; every candidate matrix solution generated by the swarm population automatically satisfies the positive-definiteness criteria. This mathematically guarantees the existence of a unique, stabilizing solution for the ARE at each optimization iteration, preventing any unstable control gains during the

heuristic evolution.

As previously discussed, the PSO algorithm plays a pivotal role in the control structure's offline optimization loop. To further elucidate its mechanism for seeking optimal $\mathbf{Q}$ and $\mathbf{R}$ matrices, this section outlines the fundamental principles of the PSO algorithm. Originally proposed by Eberhart and Kennedy in 1995, PSO is an intelligent optimization heuristic inspired by the social behavior of biological swarms, such as bird flocks or fish schools [13,36]. The PSO principle revolves around a population of agents, called particles, each representing a potential solution within the search space. In this specific problem, each particle serves as a candidate $(\mathbf{Q}, \mathbf{R})$ matrix pair. These particles "fly" and adjust their trajectories based on a dual learning mechanism: remembering the best position it has personally attained and learning from the best position discovered by any member of the entire swarm (*gbest*) [19]. The position vector and the velocity vector of the i -th individual are represented as follows:

$$\mathbf{X}_i = (x_{i1}, x_{i2}, \dots, x_n); \mathbf{V}_i = (v_{i1}, v_{i2}, \dots, v_n) \quad (27)$$

The behavior of each particle is mathematically described through two primary equations used to update its velocity and position at each iteration k [11,36]:

$$\mathbf{V}_{id}^{k+1} = v_{id}^k + c_1 \cdot \text{rand}_1(pbest_{id}^k - x_{id}^k) + c_2 \cdot \text{rand}_2(gbest^k - x_{id}^k) \quad (28)$$

$$\mathbf{X}_{id}^{k+1} = \mathbf{X}_{id}^k + \mathbf{V}_{id}^{k+1} \quad (29)$$

Based on Eq (28), the new velocity of a particle is a synthesis of its current velocity, a component directed toward its personal best experience (*pbest*), and a component directed toward the swarm's global best experience (*gbest*). This iterative process—comprising fitness evaluation, updates, and position adjustment according to Eq (29)—is repeated until the termination criteria are satisfied [11], as summarized in Figure 6.

The optimization procedure for the LQR weighting matrices $(\mathbf{Q}, \mathbf{R})$ utilizing the PSO algorithm is summarized in the flowchart shown in Figure 6. Upon executing three independent search processes, the optimal parameter sets for the LQR controller—corresponding to the IAE, ITAE, and ISE performance criteria—were successfully identified. Detailed results are presented in Table 1. Notably, each optimization criterion converges to a distinct and unique $(\mathbf{Q}, \mathbf{R})$ matrix pair. This observation underscores the inherent complexity and the heavy reliance on experience associated with manual selection methods. By automating the tuning process, the PSO algorithm not only systematically identifies optimal values but also provides an objective basis for comparing performance across design variants. This task is exceptionally difficult to execute accurately using traditional trial-and-error approaches.

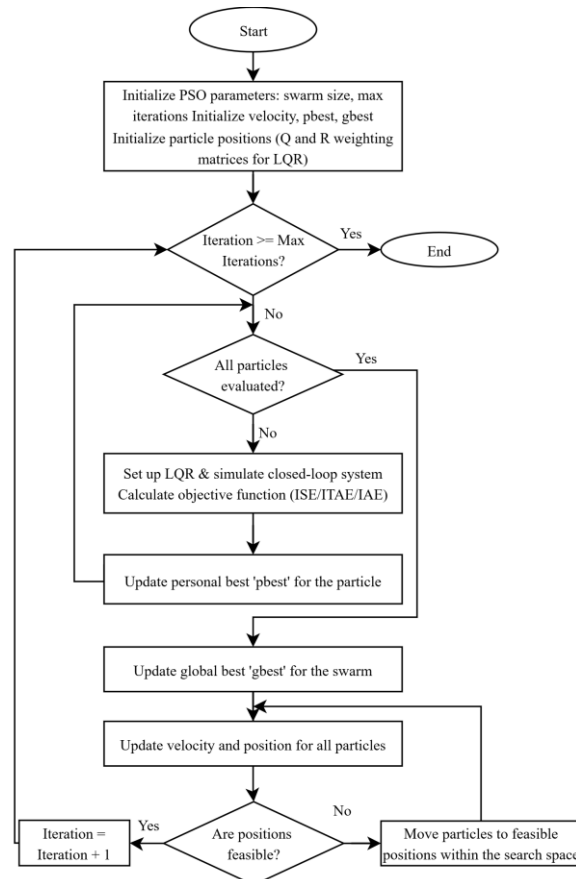


Figure 6. Flowchart of the PSO algorithm.

2.4. Experimental setup

To evaluate the practical performance of the proposed algorithms, a real-time control architecture was implemented, as illustrated in Figure 7.

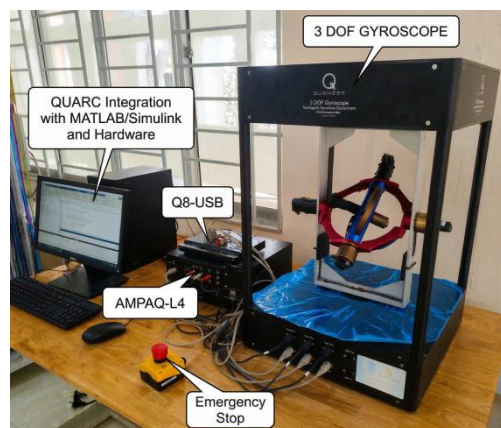


Figure 7. Connection diagram of the experimental system.

Within this platform, the Q8-USB data acquisition (DAQ) card serves as the core communication bridge between the computational environment (PC) and the physical plant (Quanser gyroscope). Specifically, state-feedback signals from the optical encoders are digitized by the Q8-USB and interfaced with the MATLAB/Simulink environment. Here, the controller—compiled

via the QUARC library—calculates and issues control commands, which are subsequently current-amplified through the AMPAQ-L4 power module before being transmitted to the actuators. This entire closed-loop cycle is executed at a stringent sampling frequency of 1 kHz (a 1-ms sampling period) to minimize phase lag and ensure dynamical stability [31,32]. Furthermore, to ensure the highest reliability of the collected data, an initial setup procedure (Figure 8) is required to perform mechanical calibration and establish precise equilibrium points before system operation.

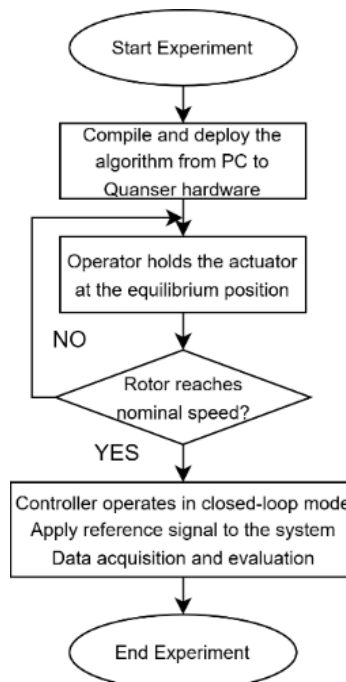


Figure 8. Experimental operational procedure.

3. Results

3.1. Experimental setup and optimal parameters

To validate the efficacy of the proposed methodology, evaluation scenarios were conducted directly on the real-time hardware platform using the MATLAB/Simulink R2023a environment integrated with the QUARC library [31,32]. The gyroscope system was investigated using a 20-degree-step reference signal to assess its trajectory-tracking capability and dynamical stability. To ensure objectivity and consistency of the boundary conditions, the controllers—comprising the default LQR and the three PSO-LQR configurations—were sequentially compiled and executed independently on the same mechanical setup with an identical input signal profile.

The operational parameters of the PSO algorithm were established to balance global convergence and computational cost, as previously summarized in Table 2 (Section 2.3). To demonstrate the efficacy of this search mechanism, the evolution of fitness values over 50 iterations for the IAE, ITAE, and ISE criteria is shown in Figure 9.

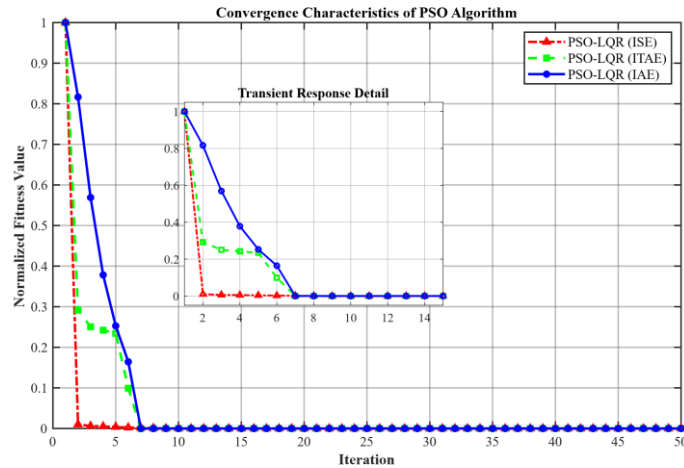


Figure 9. Convergence characteristics of the PSO algorithm under different objective functions.

Observations from the convergence plots indicate that the PSO swarms effectively identify the correct search direction and progress toward the global optimal region within the first 15–20 iterations. Notably, the ISE convergence curve exhibits a significantly steeper decline during the initial stage. This reflects the mathematical nature of the ISE criterion, which heavily penalizes large errors at the system's startup, thereby forcing the algorithm to suppress the tracking error rapidly. In contrast, the IAE and ITAE functions exhibit smoother, more asymptotic convergence, allowing the swarm greater degrees of freedom to explore optimal control effort configurations (R) rather than focusing solely on instantaneous error elimination.

After independent execution of the three objective functions, the resulting optimal parameter sets and their corresponding state-feedback gain matrices K are detailed in Table 3.

Table 3. Optimal PSO-LQR controller parameters compared to the default Quanser LQR

Controller	Weighting matrix Q	Factor R	Feedback gain K
LQR (Quanser)	diag(2.00,16.00,0.01, 0.0001)	5.00	[0.65, 1.79, 0.12, 0.0045]
PSO-LQR (ISE)	diag(0.10,50.00,0.001, 0.004)	0.10	[1.09, 22.37, 1.20, 0.19]
PSO-LQR (ITAE)	diag(1.15, 894.01, 0.31, 0.02)	1.26	[1.06, 26.65, 1.38, 0.13]
PSO-LQR (IAE)	diag(0.07, 676.79, 0.11, 0.02)	0.30	[0.69, 47.51, 1.53, 0.26]

The obtained data indicate that the PSO algorithm identified a gain matrix K with significantly higher amplitudes than those of the default design, particularly in the velocity feedback components. Specifically, in the PSO-LQR (IAE) configuration, the coefficient reached 47.51 (approximately 26 times the default design). Notably, although the gain matrix K for the IAE configuration possesses a substantially larger amplitude than that of the ISE configuration, the control effort penalty R for IAE (0.30) was maintained by the PSO at a level three times higher than that of ISE (0.10). This superior weighting distribution provides the mathematical rationale for why the IAE control signal responds decisively without driving the actuators into saturation, as observed with the ISE approach. This reflects the global learning strategy of PSO: maximizing the system's dynamical limits to accelerate tracking while maintaining awareness of hardware safety thresholds—a stark contrast to the manufacturer's conservative design approach.

3.2. Evaluation of trajectory tracking performance and dynamical stability

To comprehensively validate the performance of the proposed algorithms, the gyroscope system was evaluated using a square-wave tracking scenario with a maximum amplitude of $\pm 20^\circ$. A notable aspect of this methodology is the bifurcation of the experiment into two distinct operating frequency ranges, enabling an independent analysis of transient characteristics and steady-state stability under physical constraints.

During the transient response investigation phase, the system was tested with a high-frequency square-wave reference (short switching periods). This scenario aims to assess the convergence speed and the risk of control saturation when the actuators are forced to operate near their dynamical limits. The response plots are visualized in Figure 10, and the corresponding quantitative parameters are summarized in Table 4.

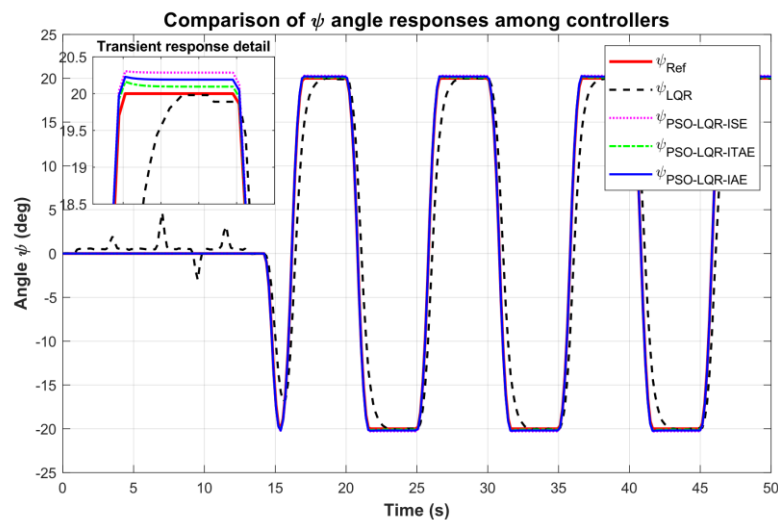


Figure 10. High-frequency square-wave trajectory tracking responses.

Table 4. Quantitative comparison of performance indices among the controllers.

Controller	Criterion	Overshoot	Settling time (s)	Steady-state error (°)	Control RMS (Nm)
LQR (Quanser)	Default	0.0%	2.50 ± 0.10	0.92 ± 0.05	0.0872
PSO-LQR	ISE	~0.5%	1.62 ± 0.05	0.25 ± 0.08	0.3184
PSO-LQR	ITAE	~0.2%	1.85 ± 0.06	0.15 ± 0.04	0.1446
PSO-LQR	IAE (proposed)	0.0%	1.68 ± 0.02	0.10 ± 0.02	0.0962

To evaluate the control energy efficiency alongside the transient metrics under physical constraints, the calculated control torque root mean square values (τ_{RMS}) are shown in Table 4. As observed from the empirical data, the default LQR configuration exhibits the minimum control effort ($\tau_{\text{RMS}} = 0.0872$ Nm) due to its conservative state-feedback gains, which consequently compromises tracking accuracy. Conversely, the PSO-LQR (ISE) variant demands the highest control effort ($\tau_{\text{RMS}} = 0.3184$ Nm). This high energy expenditure is inherently linked to its quadratic error penalty mechanism (e^2); during the initial transient stages, where the tracking error is large, the quadratic function aggressively amplifies deviations, driving the physical DC motors into hard voltage saturation boundaries and inducing severe high-frequency chattering. However, as the system approaches the steady-state boundary and the tracking error diminishes ($e < 1$), this quadratic

penalty decays at a parabolic rate toward zero. This mathematical attenuation renders the controller insensitive to small residual deviations, making it incapable of commanding sufficient continuous torque to overcome the nonlinear static Coulomb friction inherent in the gyroscope's gimbal bearings.

In contrast, the proposed PSO-LQR (IAE) framework achieves an optimal trade-off between dynamic response agility and actuator constraints. By utilizing a linear absolute error penalty ($|e|$), which maintains a constant penalty derivative magnitude ($\text{sign}(e)$) regardless of the error zone, it successfully circumvents voltage saturation during large transient transitions. This yields a significantly reduced and smooth control effort ($\tau_{\text{RMS}} = 0.0962 \text{ Nm}$) compared to both the ITAE ($\tau_{\text{RMS}} = 0.1446 \text{ Nm}$) and ISE configurations. Crucially, at the steady-state boundary, this constant derivative ensures that even microscopic tracking errors generate a steady, proportional control action. This continuous, optimized torque successfully counteracts the nonlinear joint friction, thereby driving the residual steady-state error down to a minimum of $0.10 \pm 0.02^\circ$ without triggering secondary mechanical or parasitic oscillations. This theoretical mechanism mathematically justifies why the IAE objective function yields superior tracking accuracy and energy efficiency in practical, friction-dominated mechatronic applications.

Shifting to the evaluation of static stability and state maintenance, a low-frequency square-wave trajectory is applied (Figure 11). In this state, where the derivative of the reference signal is zero, unmodeled nonlinear disturbances—particularly static friction at the gimbal axes—exert their most significant impact on the steady-state error.

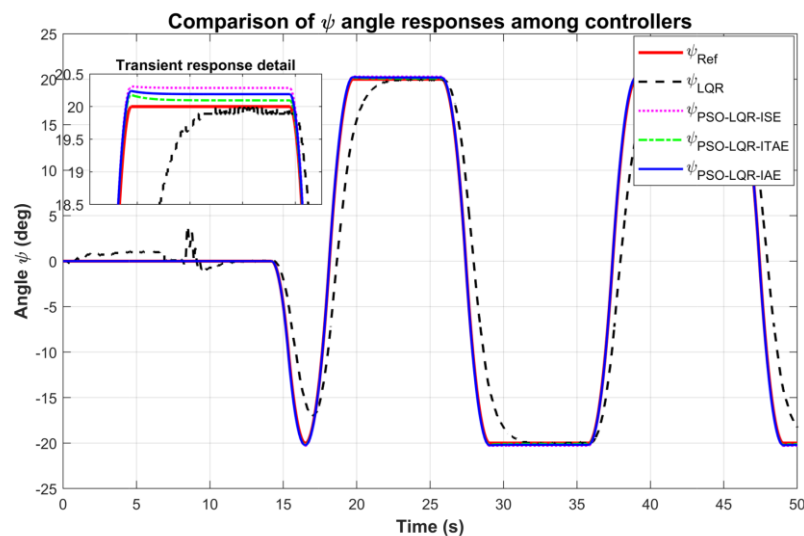


Figure 11. Low-frequency square-wave trajectory tracking responses.

The response plots in Figure 11 and the data in Table 4 further confirm that the conventional LQR solution suffers from significant performance degradation. The system tracks the trajectory with a constant phase lag and struggles to restore the equilibrium point, resulting in a steady-state error of up to 0.92° . In contrast, the PSO-LQR (IAE) controller effectively suppresses nearly all residual errors, smoothly locking the system onto the target position without triggering high-frequency output oscillations.

In summary, the multi-frequency evaluation technique shows that the PSO-LQR (IAE) architecture is superior. It successfully achieves an optimal compromise between high-speed trajectory tracking and robustness against the specific physical disturbances inherent in the gyroscope system.

To further investigate the control system's robustness, the sinusoidal trajectory tracking performance of the optimized configurations was extensively evaluated under a continuous, time-varying reference profile. This additional experimental scenario explicitly targets the mechatronic system's capability to track smooth, continuous trajectories while simultaneously subjected to nonlinear joint friction and dynamic cross-coupling, with the real-time hardware responses visualized in Figure 12.

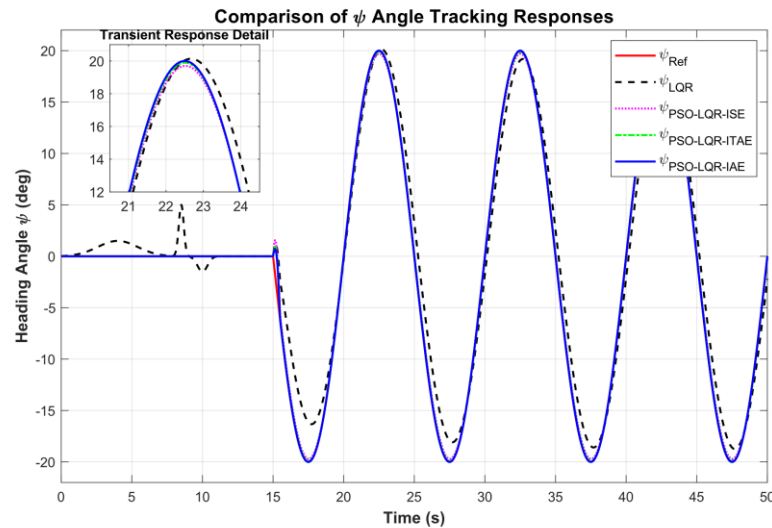


Figure 12. Sinusoidal trajectory tracking responses.

As demonstrated by the empirical curves in Figure 12, the default LQR controller continues to suffer from a noticeable phase lag and systemic latency due to its conservative gain matrix, failing to closely follow the continuous trajectory under the influence of joint friction. In contrast, all metaheuristic-tuned configurations demonstrate superior agility. However, the distinct mathematical formulations of the objective functions yield divergent steady-state behaviors. The PSO-LQR (ISE) configuration shows a rapid initial response but induces micro-oscillations and high-frequency ripples along the sinusoidal path, a direct consequence of its nonlinear penalty mechanism becoming insensitive to small tracking errors. Conversely, the proposed PSO-LQR (IAE) framework achieves the most precise and smooth trajectory-tracking performance. By maintaining a constant penalty derivative, the IAE-driven controller continuously allocates sufficient control torque to counteract nonlinear friction without driving the motor into voltage saturation, confirming its preeminent suitability for practical hardware implementation.

3.3. Evaluation of control effort and actuator constraints

Beyond trajectory-tracking quality, control effort is a pivotal criterion for assessing the practical feasibility of implementation on real hardware. Figure 13 illustrates a distinct divergence in the energy-allocation strategies $u(t)$ among the evaluated controllers in the square-wave tracking scenario.

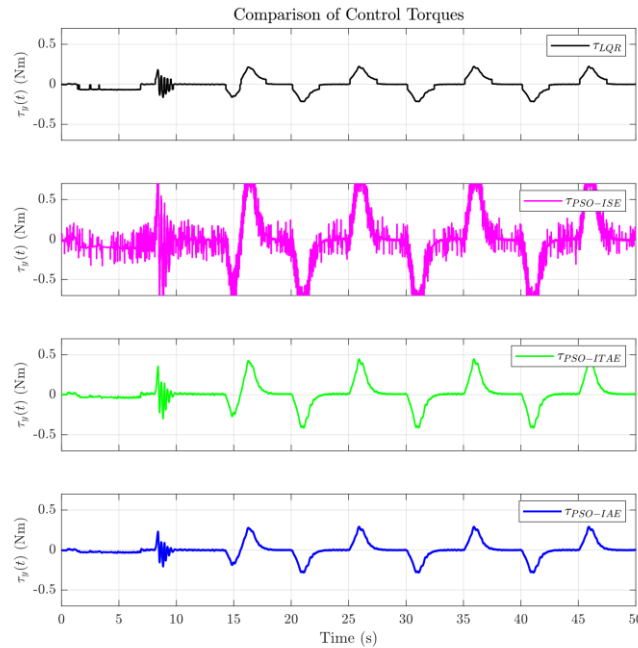


Figure 13. Control torque comparison for square-wave tracking.

Analysis of the control plots reveals that the default LQR produces a low-amplitude control signal, with peak values only reaching approximately 0.08 Nm. Although smooth, this lack of instantaneous kinetic energy makes it difficult to overcome static friction, resulting in a sluggish response and an inability to lock the system at the steady-state point.

In stark contrast to the default design, the PSO-LQR (ISE) configuration induces excessive fluctuations in control commands due to the nature of the squared-error penalty mechanism. During trajectory reversal phases, the signal $u(t)$ generates large impulse peaks (Nm) that repeatedly exceed the actuators' physical saturation limits. This characteristic directly triggers high-frequency oscillations, which not only waste energy but also pose a serious threat to the mechanical system's lifespan.

Shifting to the PSO-LQR (ITAE) configuration, the initial control action appears more stable. However, due to the time-weighting factor ($t \cdot |e|$), the actuator is forced to make minor adjustments to continuously eliminate residual errors. This process creates scattered ripples throughout the steady-state phase, significantly reducing the smoothness of the drive system.

Despite the aforementioned drawbacks, the PSO-LQR (IAE) controller demonstrates an optimal energy allocation strategy, stemming from the linear, constant-derivative characteristic of its penalty function ($|e|$). The control signal $u(t)$ responds decisively enough for rapid convergence while remaining safely bounded below 0.3 Nm. Maintaining the kinetic energy below the saturation threshold effectively eliminates chattering, ensuring smooth motion while still providing sufficient torque to firmly lock the system at the target position.

Based on the aforementioned experimental analyses, it can be concluded that the IAE criterion provides the most superior compromise between high-speed tracking optimization and the physical constraints of the control signals. This performance makes it an ideal choice for mechatronic systems with stringent hardware constraints.

3.4. Evaluation of robustness and external disturbance rejection

In practical operations, mechatronic and aerospace systems are consistently subject to uncertain dynamic disturbances, such as mechanical impacts, aerodynamic drag, and structural vibrations. Consequently, evaluating steady-state trajectory tracking alone is insufficient to confirm the reliability of a control algorithm. To validate robustness and disturbance-rejection capabilities, an experimental scenario involving physical impulse impacts was conducted. Specifically, while the system maintained a dynamic equilibrium state, a sudden external force was applied directly to the gyroscope's gimbal assembly. This setup simulates a large-amplitude nonlinear impulse disturbance that deviates the rotation axes from their nominal operating points (see Figure 14a and the segment from 01:10 to 01:20 in the Supplementary Video).

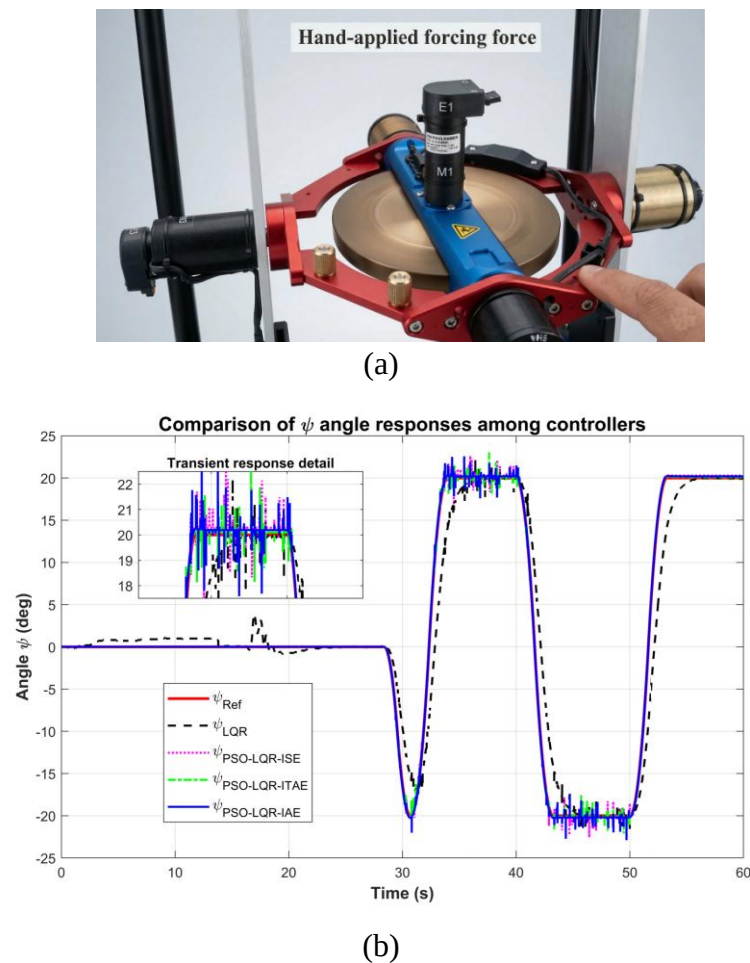


Figure 14. (a) Methodology for applying real-world impulse disturbances. (b) System recovery response under impulse disturbance.

To comply with the physical configuration of the experimental platform, the applied impulse disturbance was introduced via a manual force applied perpendicularly to the inner gimbal frame. Although the instantaneous input torque was not directly measured via a torque sensor, its physical magnitude was strictly quantified by the resulting maximum transient angular deflection, which drove the heading angle ψ to a peak deviation visible in Figure 14b. To ensure the repeatability and statistical reliability of this stress test, the experiment was repeated for five consecutive trials under identical operational boundaries for each controller configuration.

As demonstrated by the empirical curves in Figure 14b, the recovery responses provide clear evidence of the rapid reaction speed and superior robustness of the PSO-LQR architecture. It can be observed that the tracking error reaches its peak value immediately after the external force is released. Instantly, using the optimized state-feedback gain matrix $\mathbf{K}$, the controller generates a maximum counter-torque to dissipate the rotor assembly's excess kinetic energy. Specifically, the proposed PSO-LQR (IAE) configuration exhibits the most outstanding oscillation-damping capabilities, smoothly driving the system back to the reference trajectory with a remarkably short recovery time and eliminating steady-state errors without triggering secondary mechanical oscillations.

The experimental results demonstrate that the PSO-LQR system (particularly the IAE-based configuration) exhibits superior oscillation damping capabilities. The algorithm rapidly drives the system back to the reference trajectory with a remarkably short recovery time. The process of re-establishing equilibrium occurs smoothly, eliminating steady-state errors without triggering residual oscillations around the operating point. This rigorous stress test further confirms that the proposed PSO-LQR controller not only possesses optimal dynamical quality but also ensures high robustness, fully satisfying the stringent requirements of real-world operations.

4. Conclusions

This paper successfully addresses the stabilization and trajectory-tracking problems for a 3-DOF gyroscope system through the experimental implementation of an LQR controller optimized via the PSO algorithm. Moving beyond purely theoretical simulations, the primary contribution of this research lies in the direct experimental validation of how different optimization objective functions diverge under the influence of real-world physical constraints, multi-frequent step profiles, and continuous time-varying sinusoidal trajectories.

The experimental results have elucidated that, from a mathematical perspective, the derivative of the ISE function (2e) induces an excessively aggressive response when large errors occur, leading to chattering phenomena and actuator voltage saturation. In contrast, the IAE function, characterized by its constant derivative ($\text{sign}(e)$), provides a uniform penalty force. This attribute enables the system to achieve rapid response times while maintaining smooth operation and effectively eliminating steady-state errors caused by friction. Quantitative evaluations confirm that the proposed PSO-IAE configuration achieves the most comprehensive compromise, facilitating superior trajectory tracking with an optimal settling time of 1.68 s while minimizing the control energy footprint ($\tau_{\text{RMS}} = 0.0962$ Nm) under physical actuator boundaries and manual impulse disturbances. Ultimately, this study establishes concrete practical engineering benchmarks for constraint-enforced automation in high-level mechatronic mechanisms.

Author contributions

The primary research and manuscript preparation were conducted by the first author, PhD student L. H. Thuong. Assoc. Prof. Dr. P. T. Tung contributed significantly to the critical review and language correction of the paper, while Dr. L. M. Thanh was responsible for the model simulations. The entire research framework was directed by the corresponding author, Prof. Dr. C.-N. Nguyen, who supervised the project and approved the final version.

Use of Generative-AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare no conflict of interest.

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