



Research article

Detecting eccentricity levels in induction motors via persistent homology

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Abstract: We propose a persistent homology procedure to estimate the severity of mixed eccentricity in three-phase induction motors from line-current measurements. Currents are generated in ANSYS RMxprt/Maxwell under no-load operation for both star and delta connections across six eccentricity levels. Each steady-state frame is embedded as a point cloud in $\mathbb{R}^3$ and approximated with alpha complexes to compute persistence diagrams and Betti curves. We define descriptors based on the topological properties and behaviors of the Betti curves. Under the delta connection, we add a new descriptor associated with noise in 2-homology. All descriptors exhibit monotonic trends with increasing eccentricity and allow quadratic fits that enable interpolation of intermediate eccentricity levels. We also show that alpha complexes implemented in GUDHI provide substantial computational advantages over Vietoris-Rips complexes implemented in Ripser.

Keywords: fault diagnosis; three-phase induction motors; eccentricity; topological data analysis; persistent homology; Betti curves

1. Introduction

According to the 2023 report of the International Energy Agency, electric motor systems accounted for approximately 53% of the global electricity consumption, making them the largest electricity-consuming technologies worldwide. Their widespread use in industrial, commercial, agricultural, and transportation applications highlights the importance of improving their reliability and implementing efficient fault diagnosis techniques. Three-phase induction motors are among the most widely used electrical machines in industrial and commercial applications due to their robustness, reliability, and relatively low maintenance requirements [1]. These machines play a fundamental role in modern production systems, where continuous operation is essential to maintain productivity and reduce

operational costs. Consequently, unexpected motor failures may lead to production interruptions, increased maintenance expenses, reduced efficiency, and significant economic losses [2, 3]. Although induction motors exhibit high reliability, they are continuously subjected to mechanical, electrical, and thermal stresses that may gradually degrade their performance. Common faults include bearing defects, broken rotor bars, stator winding failures, and air-gap eccentricity. Previous studies indicate that eccentricity is associated with a large percentage of mechanical failures in induction motors and can generate unbalanced electromagnetic forces, excessive vibration, acoustic noise, overheating, accelerated bearing wear, and, in severe situations, rotor-stator contact [4, 5]. Therefore, the timely detection and quantification of eccentricity-severity constitute an important objective in predictive maintenance programs.

Several techniques have been developed for induction motor condition monitoring and fault diagnosis. Classical approaches include vibration analysis [6, 7], thermal monitoring [8, 9], acoustic measurements [10, 11], and motor current signature analysis (MCSA) [12, 13], which has become particularly attractive because it is non-invasive and can be implemented using signals that are readily available during normal operation [5]. More recently, advances in machine learning and artificial intelligence have enabled the development of automated diagnostic systems capable of identifying different fault conditions from large volumes of measured data [14]. Despite their success, many of these methods require extensive feature engineering, large training datasets, multiple sensing devices, or complex computational architectures.

In recent years, topological data analysis (TDA) has emerged as a powerful mathematical framework for extracting meaningful information from complex datasets. Unlike conventional signal-processing techniques, TDA characterizes the global geometric structure of data and provides properties that are inherently robust to noise and small perturbations [15, 16]. Recently, Wang et al. demonstrated that TDA can be used to detect and quantify eccentricity faults in induction motors [13]. Their methodology shows that persistent homology, one of the main tools of TDA, is capable of distinguishing different eccentricity severities without relying on complex machine-learning models. However, their analysis was restricted to Vietoris-Rips complexes, and only star-connected motors were considered, leaving open questions regarding the behavior with different connection configurations. Motivated by these limitations, this work proposes a TDA-based framework for estimating eccentricity-severity in three-phase induction motors using line-current measurements obtained from ANSYS RMxprt and ANSYS Maxwell simulations. The proposed methodology analyzes both star and delta connections and employs alpha complexes to compute persistent homology. We show that alpha complexes implemented in the GUDHI library of Python substantially reduce computational time and memory consumption compared with Vietoris-Rips complexes implemented in the Ripser package. The general idea is that, using persistent homology, we identify topological descriptors that exhibit monotonic behavior with respect to eccentricity-severity, thereby enabling the interpolation of intermediate fault levels. The obtained results demonstrate that the proposed methodology provides an accurate, computationally efficient, and non-invasive tool for eccentricity-level estimation in induction motors. Furthermore, its low computational cost makes it suitable for online monitoring applications, while the scalability of the alpha complexes suggests that this methodology can be effectively extended to the analysis of substantially larger datasets.

The remainder of this paper is organized as follows: Section 2 introduces persistent homology. Section 3 provides the necessary background and describes the proposed methodology. Section 4

details the data acquisition and preparation procedures. Section 5 presents the results obtained for both star and delta connections and discusses the proposed interpolation strategy. Finally, Section 6 summarizes the main conclusions of this work.

2. Topological data analysis

In this section, we provide a brief and intuitive introduction to TDA, which enables the extraction of geometric/topological features from data clouds in any dimension. Some advantages that make TDA highly attractive for challenging data-analysis tasks are as follows: topological features are invariant under small deformations, the extracted information does not depend on the data's dimensionality or coordinates, and the technique is remarkably robust to noise compared with other geometric methods. Evidence of this is the recent and growing use of TDA across a wide range of scientific and technological problems, including fault diagnosis [13, 17], medicine [18, 19], chemistry [20, 21], biology [22, 23], dynamical systems [24, 25], and business [26, 27].

In this work, we use persistent homology, a TDA tool that identifies holes of different dimensions “surrounded” by a data cloud. Useful references on persistent homology include [15, 16, 28]. Next, we provide a brief and intuitive introduction to persistent homology in three steps. Assume that $N = \{p_1, p_2, \dots, p_\ell\} \subset \mathbb{R}^3$ is a point cloud with ℓ samples, and denote by $|p_i - p_j|$ the Euclidean distance between points $p_i, p_j \in N$.

Step 1. Approximate N with simplicial complexes.

To extract information from N using persistent homology, we first approximate the data with mathematical objects for which homology is straightforward to compute. For this purpose, simplicial complexes are used; they are constructed by gluing geometric building blocks along shared faces. These building blocks are called j -dimensional simplices, with $j \geq 0$, and are determined by subsets of N . They include: points (0-dimensional), line segments determined by two points (1-dimensional), triangles determined by three points (2-dimensional), tetrahedra determined by four points (3-dimensional), and so on. In this work, we use alpha complexes and Vietoris-Rips complexes, two algorithms for approximating N with simplicial complexes based on point proximity. If

$$r_{\min} = \min_{p_i, p_j \in N} |p_i - p_j| \quad \text{and} \quad r_{\max} = \max_{p_i, p_j \in N} |p_i - p_j|,$$

denote the minimum and maximum distances between points in N , then for each $r \in [r_{\min}, r_{\max}]$ we construct the alpha complex $\mathcal{A}_r(N)$ and the Vietoris-Rips complex $\mathcal{VR}_r(N)$ associated with N at construction radius r ; see Figure 1. Technical details can be found in [16].

Although we work with both alpha and Vietoris-Rips complexes, for simplicity and clarity, for the remainder of this section, we explain the ideas using alpha complexes only.

Step 2. Compute the persistent homology of N .

Homology is a tool from algebraic topology that detects n -dimensional holes ($n \geq 0$) enclosed by a mathematical object (topological space). The number of n -holes enclosed by an object is called the n -th Betti number and is denoted by β_n . An object contained in $\mathbb{R}^3$ satisfies $\beta_0, \beta_1, \beta_2 \geq 0$ and $\beta_n = 0$ for $n \geq 3$. This is because such an object can enclose only 0-holes (connected components), 1-holes (loops), and 2-holes (voids or cavities).

The point cloud $N = \{p_1, p_2, \dots, p_\ell\} \subset \mathbb{R}^3$ has ℓ connected components, one per point, but it cannot enclose loops or cavities by itself. Therefore, if for some $r \in [r_{\min}, r_{\max}]$, the alpha complex

$\mathcal{A}_r(N)$ encloses a 1- or 2-hole γ , then we say that γ is a hole of N (Figure 1). In general, there is no privileged radius $r_0 \in [0, r_{\max}]$ such that $\mathcal{A}_{r_0}(N)$ reflects all holes of N . For this reason, we consider the homologies of $\mathcal{A}_r(N)$ for all radii in $[0, r_{\max}]$. For each radius r , we compute the Betti numbers β_0, β_1 , and β_2 of $\mathcal{A}_r(N)$, corresponding to the number of connected components, loops, and cavities, respectively. Suppose γ is a hole of N . Its birth radius $r_{\text{birth}}(\gamma)$ and death radius $r_{\text{death}}(\gamma)$ are the smallest and largest construction radii for which γ is enclosed by $\mathcal{A}_r(N)$, respectively. The persistence of γ is the difference $r_{\text{death}}(\gamma) - r_{\text{birth}}(\gamma)$. Persistent homology consists of computing the pairs $(r_{\text{birth}}, r_{\text{death}})$ of all holes of N .

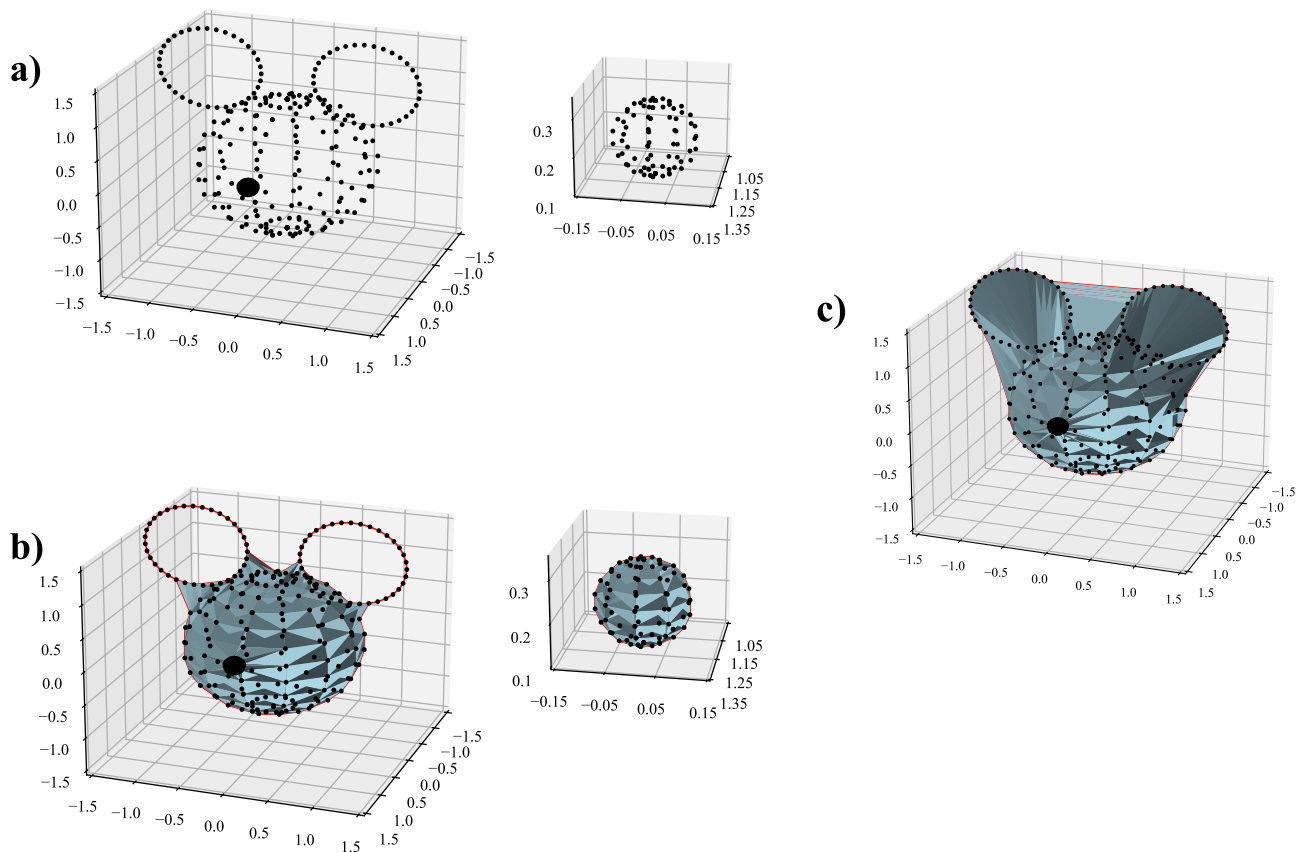


Figure 1. a) Point cloud $N \subset \mathbb{R}^3$ resembling a mouse head, together with a magnified view of the small spherical structure representing the nose. In b) and c), the 1-simplices and 2-simplices generated with the corresponding radius are shown in red and gray, respectively. b) Alpha complex $\mathcal{A}_r(N)$ for $r = 0.2664$. At this radius, the two 1-holes associated with the ears and the 2-holes corresponding to the head have already appeared. The cavity associated with the nose has already been created and subsequently filled. c) Alpha complex $\mathcal{A}_r(N)$ for $r = 1$. At this stage of the filtration, the 1-holes corresponding to the ears have disappeared, and the 2-holes associated with the head have been completely filled.

Step 3. Interpreting persistent homology.

All the holes and their persistence provide information about the shape of the point cloud N . Holes with large persistence correspond to significant topological features of N . In contrast, holes with small

persistence represent less relevant features and are often interpreted as noise in the data. Consequently, persistent homology provides a summary of the number, size, and significance of the holes present in the cloud N . To present persistent-homology results, we use persistence diagrams and Betti curves.

- *Persistence diagrams*: Each hole γ defines a pair $(r_{\text{birth}}, r_{\text{death}}) \in \mathbb{R}^2$ with $0 \leq r_{\text{birth}} < r_{\text{death}}$. These points lie in the first quadrant and above the diagonal $r_{\text{birth}} = r_{\text{death}}$; see Figure 2. The persistence diagram of N is the set of all the pairs that represent holes of any dimension. For simplicity, pairs corresponding to holes of different dimensions are presented in different colors (Figure 2).

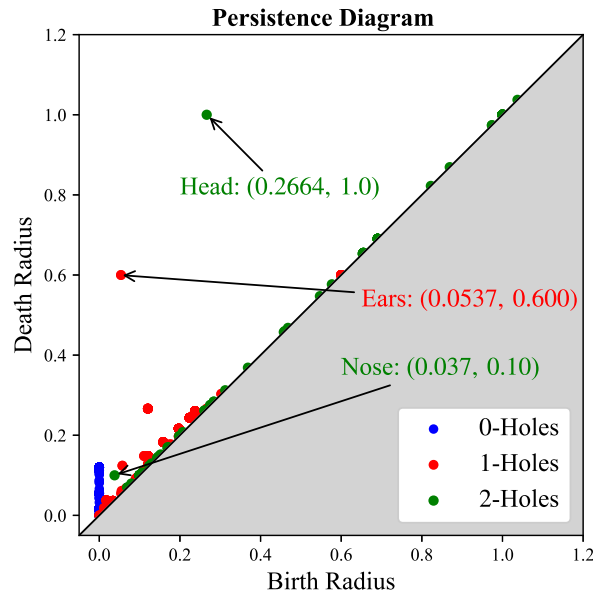


Figure 2. Persistence diagram of the cloud in Figure 1. The points in blue, red, and green correspond to 0-, 1-, and 2-holes of the data cloud, respectively. Notice that the points corresponding to the ears, nose, and head are labeled. Red and green pairs near the diagonal represent 1- and 2-holes that can be regarded as noise in the 1- and 2-homology.

- *Betti curves*: Fix $n \in \{0, 1, 2\}$. For each radius r , the Betti number $\beta_n(r)$ counts the number of n -dimensional holes in $\mathcal{A}_r(N)$. The n -th Betti curve is the function $r \mapsto \beta_n(r)$. Betti curves are represented with the same color used for the corresponding pairs in the persistent diagram; see Figure 3.

3. Preliminaries and methodology

In an ideal induction motor, the rotor and stator share the same geometric axis, producing a uniform air gap referred to as the nominal air gap. Eccentricity arises when these axes become displaced relative to one another, resulting in a non-uniform air gap distribution. Depending on the nature of the displacement, eccentricity can be classified as static (EE), dynamic (ED), or mixed. Its severity is commonly expressed as the ratio of the rotor displacement to the nominal air gap length. Although eccentricity levels below 10% are generally considered tolerable in industrial applications [29], this fault is of particular concern because it can develop either independently or as a secondary effect of approximately 80% of mechanical faults occurring in induction motors [4]. Therefore, the accurate and timely identification of eccentricity-severity is of great importance in industrial applications.

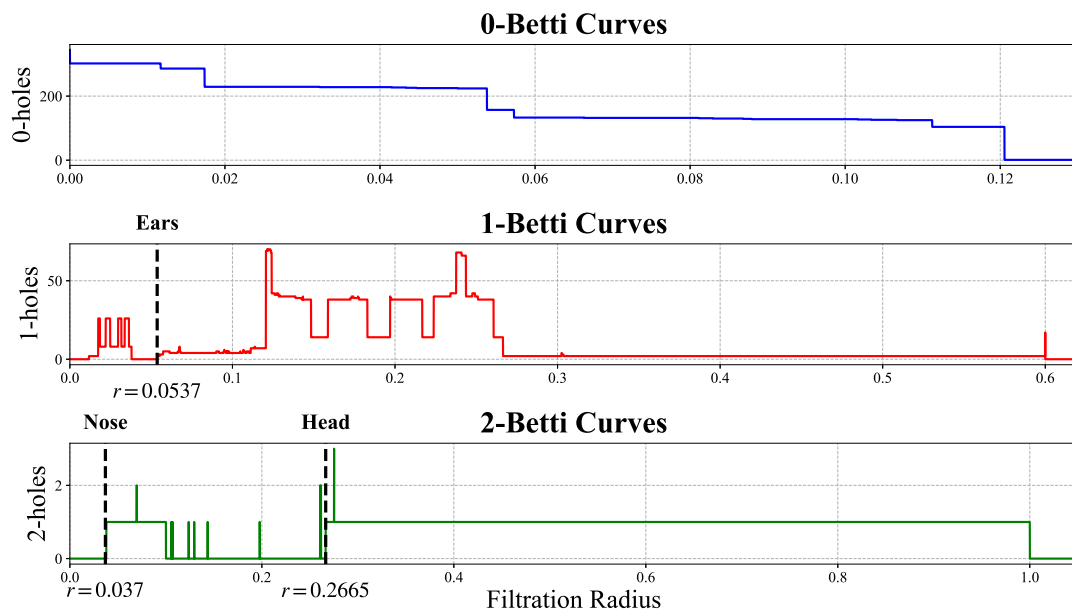


Figure 3. 0-, 1-, and 2-Betti curves of the point cloud shown in Figure 1. The 0-Betti curve decreases because connected components merge as the construction radius r increases. The 1- and 2-curves describe the appearance and disappearance of 1- and 2-holes. The highest peaks of the curves are reached when many holes form, although many of them correspond to noise. Note that the scales are different, and the black dashed lines emphasize the radii at which the holes corresponding to the ears, nose, and head are formed.

3.1. Preliminary work

The idea of detecting the degree of eccentricity in a three-phase induction motor by analyzing supply current data with TDA was previously explored in [13]. In this reference, currents were sampled at 10 kHz, and steady-state frames of 1,024 samples were extracted, corresponding to approximately 0.1 s. Five load conditions and six mixed-eccentricity cases were considered, with a constant ED of 6% and a variable level of EE; see Table 1. The motor characteristics are summarized in Table 2.

In [13], it was shown that the harmonics in the motor supply currents generated by the presence of eccentricity are reflected as noise for small construction radii in the 0- and 1-Betti curves. To estimate eccentricity levels in a motor with a star connection, samples were captured and segmented into frames of 1,024 data points, which were then studied with persistent homology using Vietoris-Rips complexes in the Ripser package of Python. In order to validate the consistency of the results, multiple steady-state data frames were analyzed, although the reported results correspond to only one of those frames. The authors observe that the area under the 0-Betti curves and the radius at which the 1-Betti curves reach their highest peak increase with eccentricity magnitude. These two properties were used as parameters in basic models such as k-NN and quadratic regression. To analyze the robustness of the results, the training and evaluation of those models were carried out by considering the five load conditions. To test the methods, two analysis scenarios were considered: “motor manufacturing” and “diagnosis of motors in operation”. The results show that the properties of the Betti curves help the models achieve good accuracy in eccentricity detection. This evidences that persistent homology can quantify the severity of eccentricity in a motor while avoiding the use of complex machine-learning models.

Table 1. Eccentricity levels considered for the analysis.

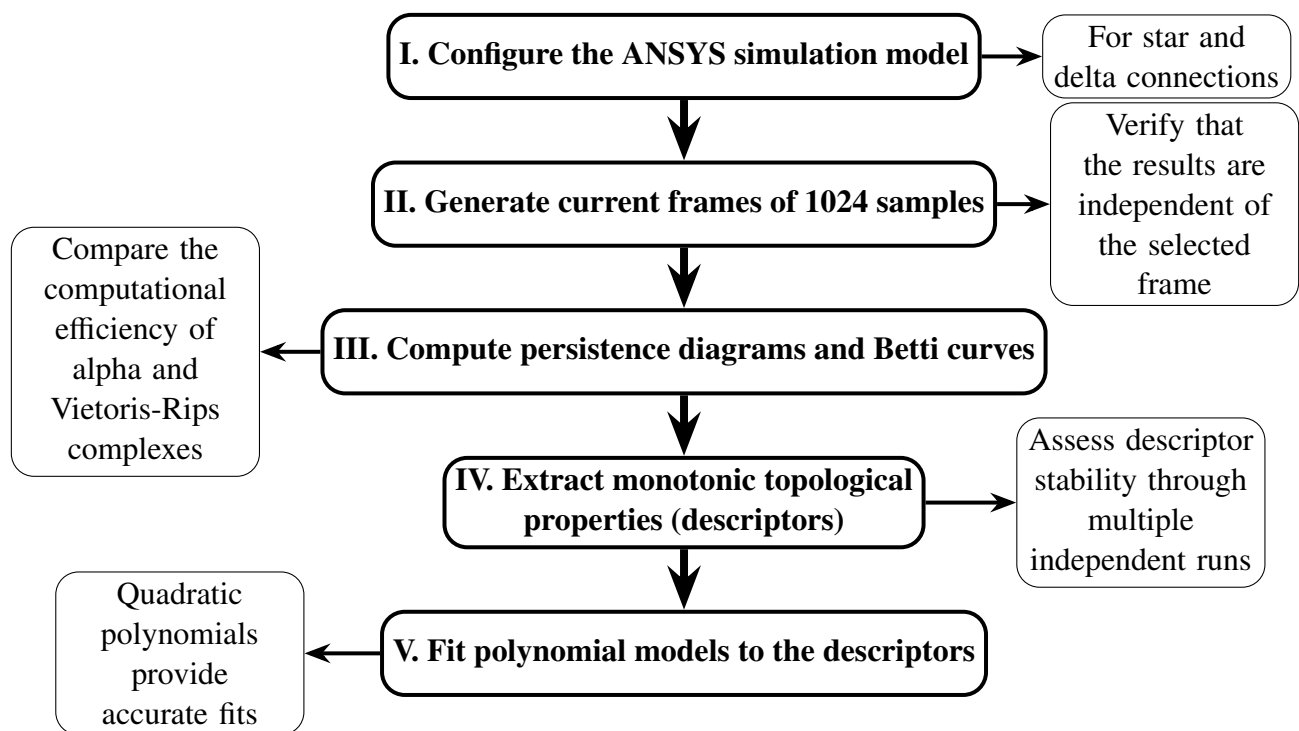
Cases	% EE	% ED
<i>I</i>	7.1	6
<i>II</i>	16.5	6
<i>III</i>	31.1	6
<i>IV</i>	42.5	6
<i>V</i>	47.5	6
<i>VI</i>	57.3	6

Table 2. Characteristics of the induction motor analyzed in [13].

Parameter	Value
Number of poles	4
Rated power	0.75 kW
Number of stator slots	36
Number of rotor bars	28
Air gap	0.28 mm
Line voltage	200 V
Frequency	60 Hz

3.2. Methodology

In this article, we consider no-load three-phase induction motors under both star and delta connections. The eccentricity levels and motor specifications used throughout the study are listed in Table 1 and Table 2, respectively. All current signals analyzed in this paper were generated from a theoretical motor model implemented in ANSYS. The proposed methodology consists of the five steps summarized in the flowchart of Figure 4.

**Figure 4.** Flowchart summarizing the proposed five-step methodology. The side boxes highlight the methodological considerations and associated validation procedures.

Compared with the work of Wang et al. [13], which constitutes the main reference for the application of TDA to induction-motor eccentricity diagnosis, the present study introduces the following three significant extensions:

1. We investigate both star and delta motor connections and analyze the differences in the resulting topological behavior.
2. Persistent homology is computed using two different simplicial-complex constructions: alpha complexes implemented in the GUDHI package and Vietoris-Rips complexes implemented in Ripser. A comparative analysis is then performed, showing that alpha complexes provide a more suitable framework in terms of computational efficiency and fault characterization.
3. We introduce monotonic topological features that differ from those considered in [13], allowing us to define new descriptors related to eccentricity severity.

4. Data acquisition and preparation

4.1. Data acquisition

The supply current data analyzed here were obtained using ANSYS RMxpert and ANSYS Maxwell. For modeling, ANSYS RMxpert allows one to vary 70 motor parameters, including electrical characteristics (RMS voltage, power, frequency, etc.), geometric characteristics of the stator and rotor (such as number of slots and bars, position, size, and geometric shape of their components), speed, torque, connection type, and construction materials [30]. Once the motor model is available, the desired percentage of eccentricity is implemented in ANSYS Maxwell by determining the position of the rotor and stator. Afterwards, current data are extracted under steady-state conditions, since the motor exhibits constant speed and torque. We use an integration step of 0.0001 s to obtain the frames $i_a(t)$, $i_b(t)$, $i_c(t)$ of 1,024 data points corresponding to the supply currents measured in amperes (A).

The use of ANSYS offers several advantages for induction motor modeling. First, it allows one to represent the geometry and electromagnetic behavior of the motor with high accuracy, obtaining a virtual model that faithfully reproduces the operation of a real system. In addition, the ACT Extension tool in ANSYS Maxwell allows one to configure different eccentricity levels accurately by means of controlled modification of rotor position with respect to the stator.

A technical disadvantage of ANSYS is the high computational burden, since the simulation requires several hours to obtain the data. Another limitation is that, when comparing with experimental data from a real motor, the simulator requires all 70 parameters in order to obtain a model that reproduces that motor. In practice, manufacturers usually provide datasheets with only basic information (voltages, RMS currents, active power, connection, ...), but omit data such as the geometry of stator slots and rotor bars, detailed dimensions of the magnetic core, type and properties of the magnetic and conductive materials, and internal construction parameters. Therefore, the user must adjust these values in ANSYS RMxpert until the simulation displays behavior consistent with the motor.

The results presented in the following sections are based on a single frame containing 1,024 data points. To evaluate the robustness of the proposed methodology, 15,000 current samples were acquired for each of the six eccentricity levels considered (Table 1), and the last six frames were analyzed. As a representative example, Figure 5 displays the corresponding 0- and 1-Betti curves for the case EE = 7.1%, where each color represents a different frame. The mean 0- and 1-Betti curves are also shown as dashed black lines. A strong agreement can be observed among the six Betti curves, all of which closely follow their respective mean curves. Since the area under the Betti curves will play a central role in the proposed methodology, the corresponding values for the six frames are reported in Table 3,

together with the area of the mean Betti curve and the standard deviations. For the H_0 Betti curves, the standard deviation is 0.0194 compared with a mean area of 12.2744, while for the H_1 Betti curves, the standard deviation is 0.0092 compared with a mean area of 3.1649. These values correspond to relative variations of approximately 0.16% and 0.29%, respectively. Such small deviations indicate that the topological features are highly stable across different frames extracted from the same signal. Therefore, the observed variability introduced by the frame selection is negligible compared with the magnitude of the descriptors themselves. This result justifies the use of a single frame of 1024 samples in the subsequent analysis, as it provides representative topological information while significantly reducing the computational cost.

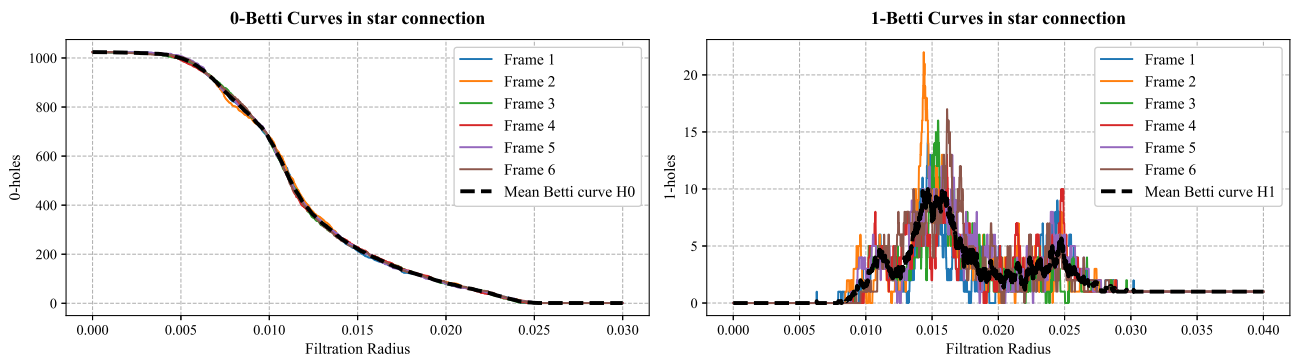


Figure 5. 0- and 1-Betti curves computed from six data frames with the eccentricity-severity of $EE = 7.1\%$ under star connection. The mean curve is presented in a black dashed line.

Table 3. Statistical analysis of the areas under the H_0 and H_1 Betti curves for the six analyzed frames with the eccentricity-severity of $EE = 7.1\%$.

Frame number	H_0		H_1	
	Area	Mean Betti curve area	Area	Mean Betti curve area
1	12.2601	12.2744	3.1582	3.1649
2	12.3064		3.1758	
3	12.2524	Standard deviation 0.0194	3.1542	Standard deviation 0.0092
4	12.2656		3.1622	
5	12.2834		3.1790	
6	12.2783		3.1598	

4.2. Data preparation

From the acquired supply current data, the array $\mathcal{I}(t) = (i_a(t), i_b(t), i_c(t)) \in \mathbb{R}^3$ was formed, thus determining a frame of 1,024 points. This dataset is then studied by persistent homology, using alpha and Vietoris-Rips complexes. In this way, we generate persistence diagrams and Betti curves associated with the six eccentricity levels considered in Table 1. In the ideal case, the three-phase system is

balanced, and therefore, the currents should ideally have the form

$$I_a(t) = I_m \sin(2\pi ft), \quad I_b(t) = I_m \sin(2\pi ft + 120^\circ), \quad I_c(t) = I_m \sin(2\pi ft + 240^\circ),$$

where I_m is the amplitude and f is the fundamental frequency. In our case, $I_m \approx 2.58425$ A. Then, $I(t) = (I_a(t), I_b(t), I_c(t)) \in \mathbb{R}^3$ is an ideal trajectory satisfying

$$I_a(t) + I_b(t) + I_c(t) = 0 \quad \text{and} \quad |I(t)| = \sqrt{\frac{3}{2}} I_m \approx 3.16504 \text{ A}.$$

We conclude that the curve $I(t)$ describes a circle $\mathcal{C}$ of radius $\sqrt{3/2} I_m \approx 3.16504$ A, centered at the origin and contained in the plane

$$\mathcal{P} = \{(x, y, z) \in \mathbb{R}^3 : x + y + z = 0\}.$$

Due to eccentricity, simulated frames $I(t)$ are not perfectly balanced and exhibit harmonics. Consequently, for the six eccentricity levels considered, the point clouds will be distributed around $\mathcal{C}$. We study the topological properties of this distribution via persistent homology.

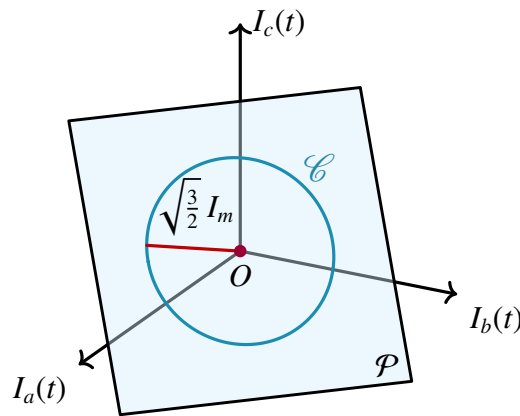


Figure 6. Circle $\mathcal{C}$ traced by $I(t) = (I_a(t), I_b(t), I_c(t)) \in \mathbb{R}^3$ in the ideal balanced case.

5. Results

Since the datasets lie in $\mathbb{R}^3$, for both star and delta connections, we consider 0-, 1-, and 2-holes. All computations were performed in Python on an MSI GF63 Thin 10SCXR-222 laptop equipped with an Intel Core i5-10300H processor, 32 GB RAM, and an NVIDIA GeForce GTX 1650 Max-Q GPU.

5.1. Computational cost of data approximation

Persistent homology computations for the six eccentricity cases were performed by approximating the point cloud in two ways: by alpha complexes using the GUDHI package and by Vietoris-Rips complexes using the Ripser package. To compare the computational cost of both methods, we measured computation time and RAM consumption in both libraries. The obtained results are shown in Table 4.

Table 4. Comparison of processing times (seconds) and RAM usage (MB) for computing 0-, 1-, and 2-hole persistences under star and delta connections.

% EE	Time (s)				RAM (MB)			
	Star		Delta		Star		Delta	
	Ripser	GUDHI	Ripser	GUDHI	Ripser	GUDHI	Ripser	GUDHI
7.1	218.9543	0.5127	196.6140	0.0600	15.2617	0.0820	15.2812	0.0195
16.5	191.2018	0.5631	188.8373	0.0600	17.2539	0.0507	11.3008	0.0234
31.1	182.3229	0.5220	186.7771	0.0610	18.8711	0.0429	18.3906	0.0234
42.5	184.4018	0.5712	181.4258	0.0630	20.7188	0.0468	17.7461	0.0195
47.5	181.5168	0.5998	159.2522	0.0670	18.0234	0.0312	17.8945	0.0195
57.3	157.0123	0.5561	159.6336	0.0650	20.2344	0.0234	18.2656	0.0195

Although the results with both methods are adequate, observe that the computation times associated with alpha complexes are three to four orders of magnitude smaller than those corresponding to Vietoris-Rips complexes. On the other hand, RAM consumption with alpha complexes is three orders of magnitude smaller than with Vietoris-Rips complexes. This comparison is consistent with the results reported in [31]. It is noteworthy that, with alpha complexes, a one-order decrease in computation time is observed in delta connection with respect to star connection, whereas with Vietoris-Rips complexes, the same time order is maintained.

The acquisition time for a single data frame is approximately 0.1 s. However, as shown in Table 4, processing a frame using Vietoris-Rips complexes with the Ripser package requires between 150 and 220 s, which is several orders of magnitude longer than the acquisition time itself. Moreover, as the size of the dataset increases, the computational burden becomes even more pronounced: Ripser requires several hours to process planar data clouds containing approximately 2000 points and fails to complete computations for datasets exceeding 3000 points. These limitations make Vietoris-Rips complexes impractical for online monitoring and large-scale applications. In contrast, as Table 4 shows, alpha complexes implemented in the GUDHI package process the same data frames in only 0.06–0.5998 s and maintain stable performance as the dataset size increases. In our experiments, GUDHI successfully processed 102,400 data points in approximately 14 s with a memory consumption of about 311 MB. These results demonstrate the superior computational efficiency and scalability of alpha complexes, making them a more suitable choice for real-time fault-monitoring systems and large-scale persistent homology computations.

In view of these computational advantages, all results presented in the following sections were computed using alpha complexes.

5.2. Star connection

The results of this section were obtained by including in the ANSYS simulator the motor properties listed in Table 2. To validate our method, and because ANSYS simulations require the 70 motor parameters, we undertook the task of searching for the parameters needed to reconstruct the results reported in [13].

In Figure 7, we present the persistence diagrams for the cases $EE = 7.1\%$ and $EE = 57.3\%$. Figure 8 shows the 0- and 1-Betti curves for the six eccentricity levels under study (Table 1). By analyzing the results produced by TDA, the following three descriptors are obtained, which make it possible to identify the eccentricity level.

- I. The death radius of the most persistent 1-hole is $r = 3.1271$ for $EE = 7.1\%$ and $r = 2.9965$ for $EE = 57.3\%$ (Figure 7). We denote this death radius by H_1R_{mue} .
- II. From Figure 8, it is observed that the area under the 0-Betti curves increases as the eccentricity percentage increases. We denote this area by H_0Ar .
- III. Figure 8 shows that the area under the 1-Betti curves, estimated from the appearance of noise until the death of the most persistent 1-hole, increases as the eccentricity percentage increases. We denote this area by H_1Ar .

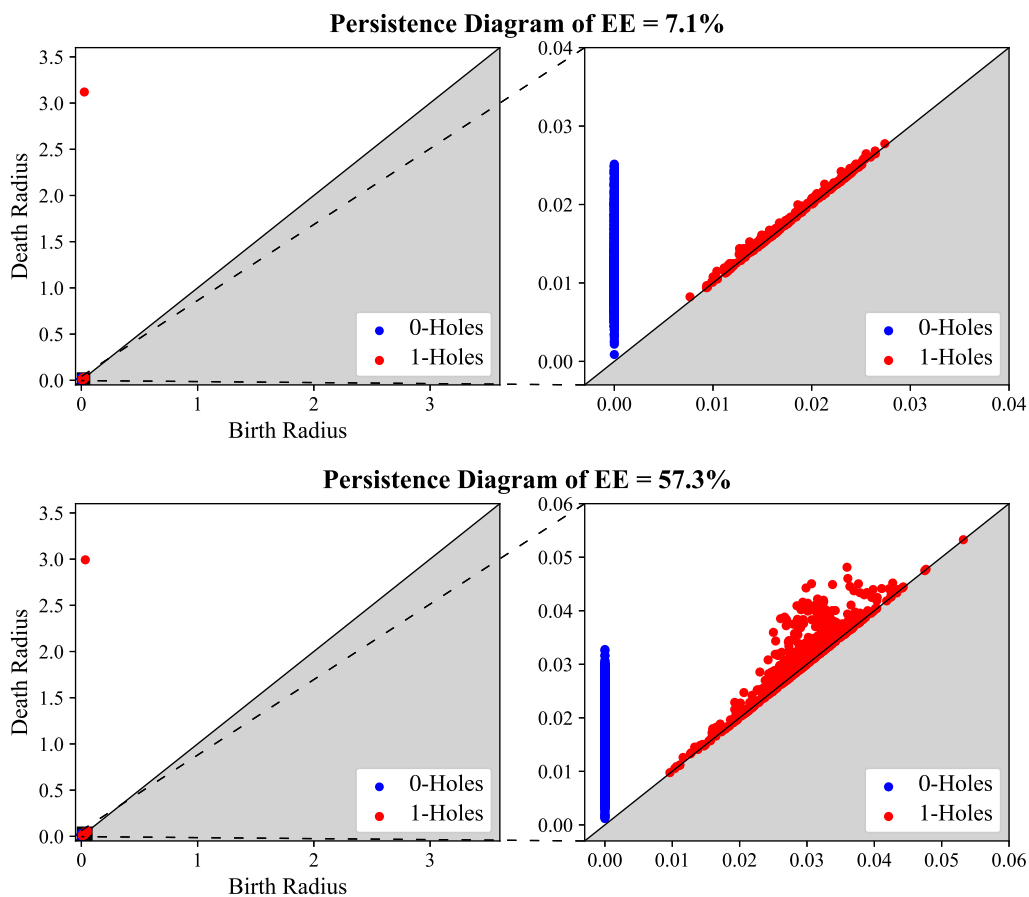


Figure 7. Persistence diagrams and zoomed views of the noise region for cases $EE = 7.1\%$ and $EE = 57.3\%$ under star connection.

Table 5 presents the values of the descriptors H_1R_{mue} , H_0Ar , and H_1Ar for the six eccentricity levels studied. Observe that these descriptors follow a monotone pattern, which will allow us to detect the eccentricity level in the motor.

It should be mentioned that noise in the 1-holes also increases as the eccentricity level increases (Figure 8). Consequently, the construction radius at which the noise in 1-homology ends follows an increasing pattern; however, this parameter has high sensitivity and therefore high variability, which makes its use as a robust descriptor of eccentricity difficult. A similar behavior was observed for the radius at which the 1-Betti curve reaches its highest peak (a parameter used in [13]), the maximum value of the noise in 1-homology; see Figure 5.

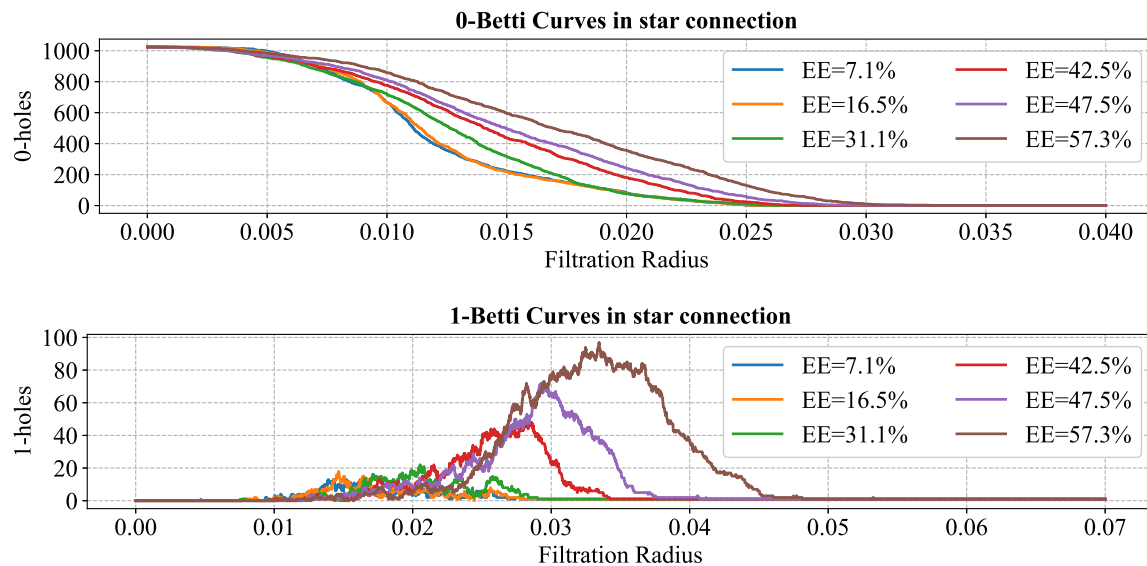


Figure 8. 0- and 1-Betti curves for the six eccentricity levels studied under star connection. For the 1-Betti curves, a zoomed view of the region where noise appears is shown.

Table 5. Descriptors obtained for the studied eccentricity levels under star connection.

% EE	$H_1 R_{mue}$	$H_0 Ar$	$H_1 Ar$
7.1	3.11988	12.29770	3.17634
16.5	3.11499	12.36709	3.17958
31.1	3.09012	12.97179	3.20039
42.5	3.05757	14.53984	3.40653
47.5	3.03461	15.44958	3.58394
57.3	2.99254	17.27690	4.11215

5.3. Delta connection

This section presents the results obtained after changing the motor connection from star to delta in the ANSYS RMxprt simulator while keeping all other parameters unchanged. To obtain line currents with amplitudes comparable to those of the star connection, the delta-connected motor was supplied with a line voltage of $\frac{200}{\sqrt{3}}$ V, where V is the voltage used in the star connection.

As in the previous case, Figure 9 presents the persistence diagrams corresponding to $EE = 7.1\%$ and $EE = 57.3\%$, while Figure 10 shows the Betti curves for all eccentricity levels under study.

As can be observed from the persistence diagrams, a small amount of noise appears in 2-homology for the delta connection. This feature is noteworthy because it indicates that the data frames are no longer entirely contained in the plane $\mathcal{P}$ introduced in subsection 4.2. It is important to emphasize that this noise is detected only when alpha complexes are used; the same data frames analyzed with Vietoris-Rips complexes do not exhibit 2-dimensional homological features. This difference can be explained by the fundamental theories used by the two methods to construct simplicial complexes [16]. In addition to the three descriptors defined for the star connection, the presence of noise in 2-homology enables the definition of an additional descriptor for the delta connection:

- IV. The construction radius at which the noise in 2-homology ends is $r = 0.0324$ for $EE = 7.1\%$ and $r = 0.0526$ for $EE = 57.3\%$ (Figure 9). We denote this radius by H_2R_{rui} .

In Table 6, we report the values of the four parameters H_1R_{mue} , H_2R_{rui} , H_0Ar , and H_1Ar for the six eccentricity severities in delta connection. In this case, the area under the 2-Betti curve increases as eccentricity increases, but again, it is too sensitive to be considered a reliable descriptor of the behavior.

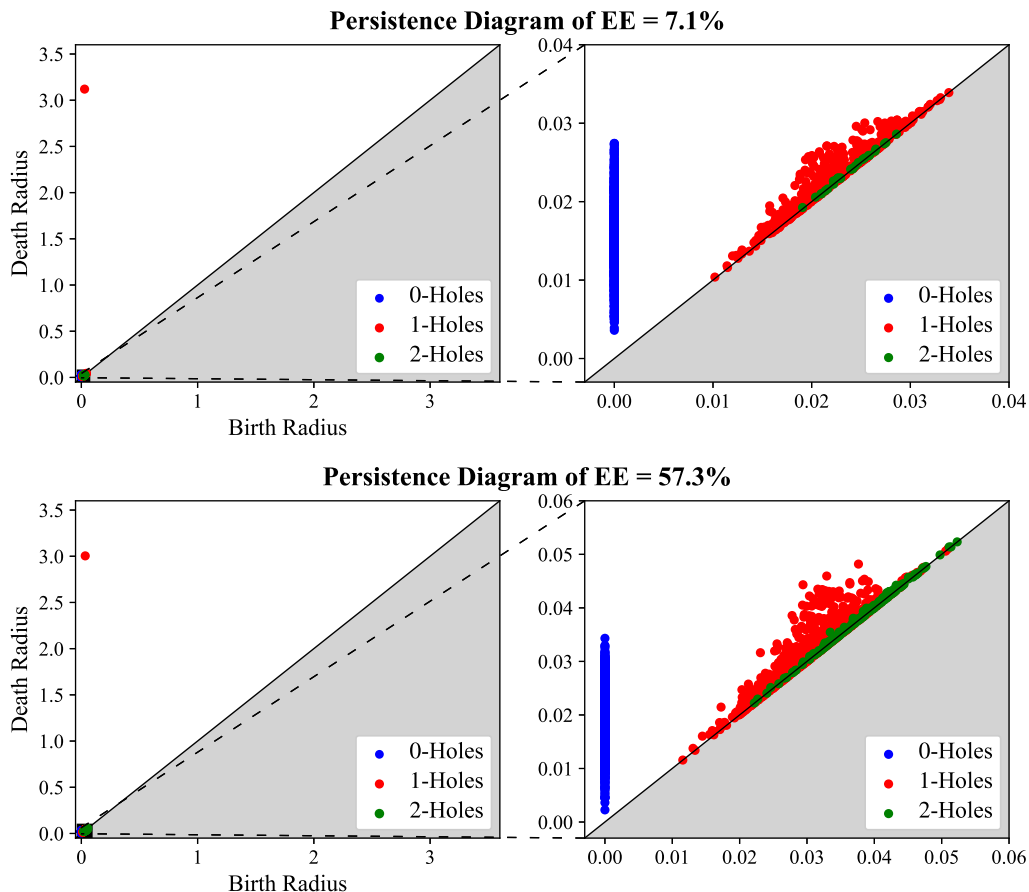


Figure 9. Persistence diagrams and zoomed views of the noise region for cases $EE = 7.1\%$ and $EE = 57.3\%$ under delta connection.

Since the delta-connected motor consumes more power than the star-connected motor, this

difference is reflected in the magnitudes of the descriptors shared by both configurations; see Tables 5 and 6. Finally, it is worth noting that the physical origin of the 2-homology noise observed in the delta connection remains an open question. A plausible explanation may be related to the different current constraints imposed by the connection topology. In a star-connected motor, the presence of a neutral point tends to stabilize the relationships among the phase currents, resulting in a more regular geometric structure of the corresponding point cloud. In contrast, the absence of a neutral point in the delta configuration may allow greater variations in the phase-current relationships and stronger interactions among the line currents. These additional variations may contribute to the formation of small three-dimensional geometric structures within the point cloud, which persistent homology detects as short-lived cavities.

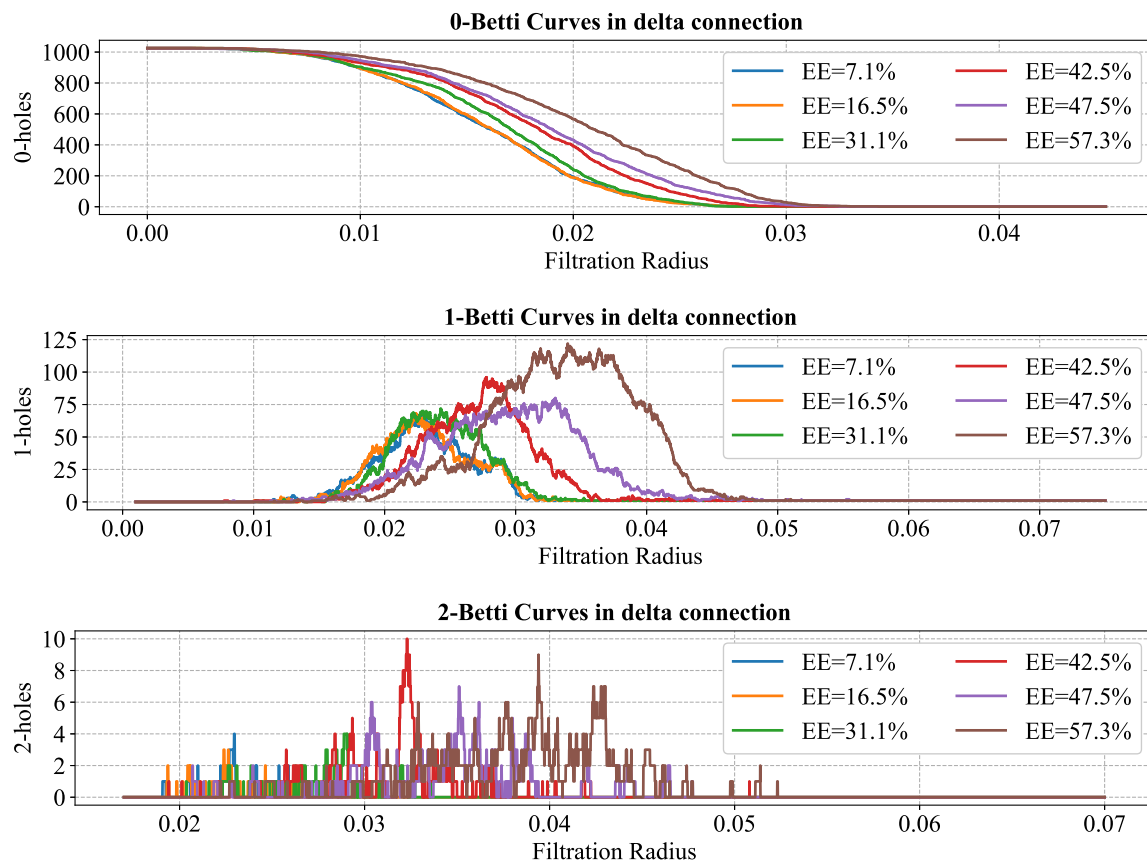


Figure 10. 0-, 1-, and 2-Betti curves for the six eccentricity levels studied under delta connection, including zoomed views of the regions where noise appears.

Table 6. Descriptors obtained for the studied eccentricity levels under delta connection.

% EE	$H_1 R_{mue}$	$H_2 R_{rui}$	$H_0 Ar$	$H_1 Ar$
7.1	3.12010	0.02860	16.20037	3.60648
16.5	3.11518	0.02909	16.23258	3.61404
31.1	3.09026	0.03355	16.93490	3.65796
42.5	3.05774	0.04599	18.37257	3.83415
47.5	3.03138	0.04773	19.16940	3.99586
57.3	3.00415	0.05233	20.90925	4.53796

5.4. Eccentricity-severity prediction for real induction motors

The monotonic behavior exhibited by the proposed descriptors allows the construction of predictive models capable of estimating their values for arbitrary eccentricity levels. To this end, polynomial models were obtained using the Python NumPy library, specifically the functions `polyfit` and `poly1d`, with the eccentricity severity (E) as the independent variable. The eccentricity values 7.1%, 16.5%, 31.1%, 42.5%, 47.5%, and 57.3% were employed as training data. Polynomial models of different degrees were investigated for both descriptor fitting and eccentricity-severity estimation. The results showed that quadratic polynomials provide the most suitable trade-off between predictive accuracy and model complexity. The corresponding fitting curves for both star and delta connections are presented in Figure 11.

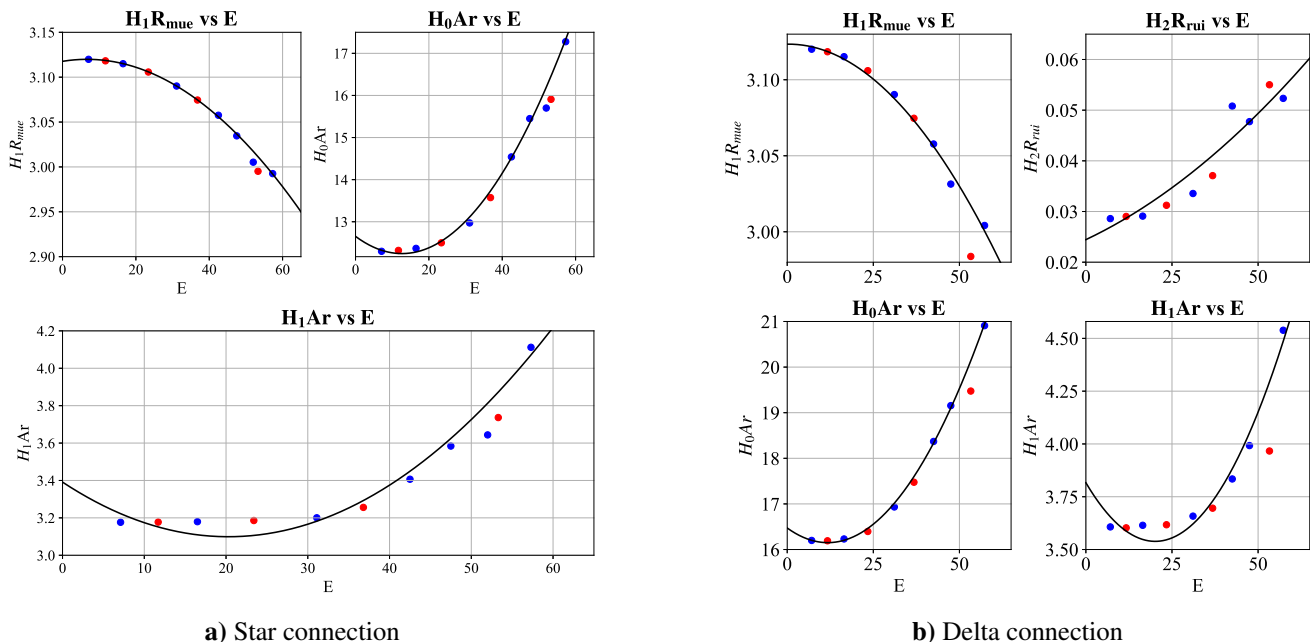


Figure 11. Quadratic polynomial fits of the descriptors. Blue points denote the fitting data, and red points denote the validation data.

To assess the predictive capability of the fitted polynomials, the descriptor values predicted for the

test eccentricities, 11.7%, 23.4%, 36.8%, and 53.3%, were compared with the corresponding values computed using the GUDHI package. The resulting percentage errors are reported in Table 7. It can be observed that, in nearly all cases, the error remains below 10%. The only exception is the descriptor H_2R_{noise} at an eccentricity level of 36.8%, for which the error slightly exceeds this threshold. In Figure 11, the blue points correspond to the eccentricity levels used for fitting, whereas the red points represent the test cases used for validation.

To estimate the eccentricity severity of a three-phase induction motor during operation, the following procedure can be followed:

- I. Configure the ANSYS simulation environment according to the specifications of the motor under study. Then, following the methodology summarized in the diagram of subsection 3.2, obtain the polynomial fitting functions that characterize the behavior of the selected descriptors.
- II. Implement an experimental setup using Hall-effect sensors to acquire the motor supply-current signals. The proposed experimental design is illustrated in Figure 12.
- III. Apply persistent homology to the acquired current data and compute the values of the corresponding descriptors.
- IV. Estimate the eccentricity severity by evaluating the computed descriptor values through the inverse polynomial models. The final prediction is obtained as the average of the eccentricity values estimated from each descriptor.

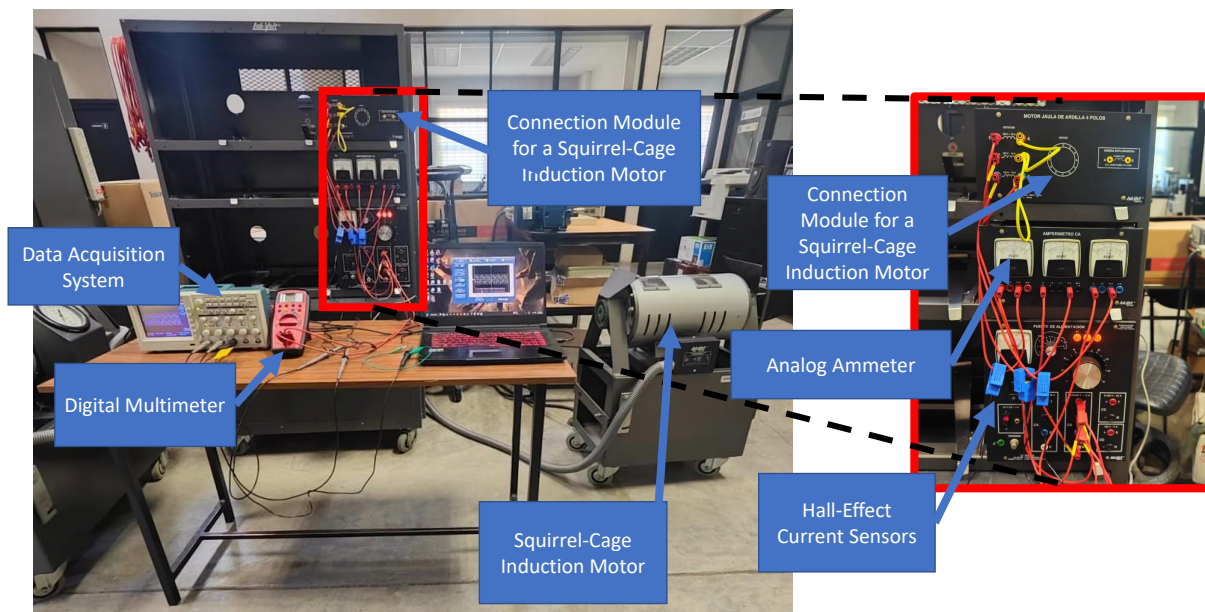


Figure 12. Proposed laboratory experimental setup for acquiring the three-phase supply-current signals used to estimate the eccentricity severity of the induction motor.

To evaluate the predictive capability of the proposed methodology, the eccentricity levels estimated using the inverse polynomial models were compared against the corresponding test eccentricity levels for both star and delta connections. The resulting percentage errors are reported in Table 7. In all cases,

the prediction error remained below 7%, demonstrating that the proposed methodology is capable of providing reliable and accurate estimates of eccentricity severity across different motor configurations.

Table 7. Percentage error of the descriptors and of the predicted eccentricity under star and delta connections.

% EE_{true}	Descriptor error (%)							Eccentricity error (%)	
	Star			Delta				Star	Delta
	H_1R_{mue}	H_0Ar	H_1Ar	H_1R_{mue}	H_2R_{rui}	H_0Ar	H_1Ar	% Error	% Error
11.7	0.0138	0.5928	0.8157	0.0046	0.7906	0.2522	0.4266	6.8193	1.4259
23.4	0.0070	0.3070	2.5044	0.0847	8.2395	0.4107	1.9729	0.5609	5.6775
36.8	0.0030	1.1609	1.1250	0.0502	10.3066	0.7062	0.9083	0.0849	1.5299
53.3	0.5424	3.4626	3.6405	1.1201	6.1840	3.4089	8.2405	5.1565	6.7790

5.5. Advantages and limitations of the proposed methodology

The proposed methodology presents several advantages as well as some limitations that should be considered when assessing its applicability to induction-motor fault diagnosis.

• Advantages

- The proposed approach is closely related to motor current signature analysis (MCSA), since it relies exclusively on stator-current measurements. Consequently, it preserves the main advantages of MCSA, including non-invasive operation, early fault detection, remote monitoring capability, and ease of implementation.
- The use of alpha complexes implemented in the GUDHI package significantly reduces the computational burden compared with Vietoris-Rips complexes computed with Ripser. The obtained execution times indicate that alpha complexes are suitable for near real-time monitoring applications.
- Unlike machine-learning-based approaches, the proposed methodology does not require training datasets, labeled data, or model-training stages. Therefore, it can be applied immediately after the transient regime of the motor has ended.
- The extracted topological descriptors exhibit a stable and monotonic behavior with respect to eccentricity severity, allowing accurate estimation of fault severity. In the experiments considered here, prediction errors remained below 7%.
- In addition to fault detection, the proposed methodology provides a quantitative estimate of eccentricity-severity, which may assist maintenance personnel in evaluating the progression of the fault.

• Limitations

- The methodology has not yet been validated through an online experimental implementation. Such validation requires dedicated hardware and data-acquisition infrastructure that were not available during this study.

- b) The proposed approach relies on a prior motor model implemented in ANSYS. Consequently, detailed knowledge of the motor specifications is required before the severity-estimation stage can be performed.
- c) Persistent homology and, in general, TDA involve mathematical concepts that may not be familiar to practitioners in electrical engineering, making the interpretation of the resulting descriptors less intuitive than traditional signal-processing indicators.
- d) The validation presented in this work is restricted to the motor configurations and no-load operating conditions considered in the study. Additional investigations are required to assess the performance of the methodology under different load conditions and for other motor designs.

6. Conclusions

In this paper, a technique is proposed for detecting eccentricity-severity in a three-phase induction motor through descriptors generated by persistent homology. We worked with ANSYS simulations of a motor under non-load conditions, and both star and delta connections were considered. Line current data in $\mathbb{R}^3$ were sampled. To apply persistent homology, the data were approximated by alpha and Vietoris-Rips complexes through the Python libraries GUDHI and Ripser, respectively.

From the analysis performed, we conclude that alpha complexes implemented in the GUDHI library significantly outperform Vietoris-Rips complexes in terms of computational efficiency. Their favorable computational properties make them highly suitable for scalability and particularly well suited for large-scale persistent homology computations and online motor monitoring applications. Moreover, alpha complexes proved more effective in capturing features associated with eccentricity-induced noise. In particular, for the delta connection, they revealed 2-dimensional homological features that were not detected by Vietoris-Rips complexes.

For the detection of eccentricity-severity we proposed descriptors based in the topological features of the supply data current that are robust and stable; this was verified with several independent runs. Then, the strategy achieved very good accuracy in the predictions of eccentricity-severity, less than 7%.

The following directions are proposed for future work:

1. Investigate the mechanisms responsible for the emergence of noise in the 2-homology for delta connection.
2. Extend the proposed analysis to induction motors operating under load conditions.
3. Apply the proposed methodology to the real-time monitoring and characterization of induction motor behavior.
4. Develop a control and diagnostic strategy for detecting eccentricity faults and assessing their severity in three-phase induction motors using the proposed descriptors.

Author contributions

Kristian Gochi: Writing, original draft, investigation; Ahtziri González: Methodology, validation, supervision, writing; Juan Anzures: Investigation, review and editing.

Use of Generative-AI tools declaration

The authors declare they have not used any generative Artificial Intelligence (AI) tools in the creation of this manuscript.

Acknowledgments

This research was partially supported by the master's scholarship SECIHTI. The first author also gratefully acknowledges the support and encouragement received during the development of this work, with special thanks to Daniela M. Carrillo Gutiérrez.

Conflict of interest

The authors declare no conflict of interest.

References

1. De Almeida A, Fong J, Brunner CU, Werle R, Van Werkhoven M (2019) New technology trends and policy needs in energy efficient motor systems - a major opportunity for energy and carbon savings. *Renew Sust Energ Rev* 115: 109384. <https://doi.org/10.1016/j.rser.2019.109384>
2. Bonnett AH, and Soukup GC (1992) Cause and analysis of stator and rotor failures in three-phase squirrel-cage induction motors. *IEEE T Ind Appl* 28: 921–937. <https://doi.org/10.1109/28.148460>
3. Simões MG (2023) A concise history of induction motor drives—part 1 [history]. *IEEE Electrif Mag* 11: 5–11. <https://doi.org/10.1109/MELE.2023.3264888>
4. Hamani K, Kuchar M, Kubatko M, Kirschner S (2025) Advancements in induction motor fault diagnosis and condition monitoring: A comprehensive review. *Sensors* 25: 5942. <https://doi.org/10.3390/s25195942>
5. Thomson WT, Fenger M (2001) Current signature analysis to detect induction motor faults. *IEEE Ind Appl Mag* 7: 26–34. <https://doi.org/10.1109/2943.930988>
6. Kim DY, Kim DK, Park MR (2026) Rotor eccentricity in electric machines: An analytical framework with experimental validation for fault diagnosis. *Mech Syst Signal Pro* 244: 113776. <https://doi.org/10.1016/j.ymssp.2025.113776>
7. Mhaske P, Ghosh S, Birajdar R, Gosavi K, Yadav VK (2026) Rotor eccentricity fault in induction motors for special applications: Its effects and detection. *IEEE T Ind Appl* 62: 6277–6287. <https://doi.org/10.1109/TIA.2025.3645016>
8. Glowacz A (2026) Thermal imaging fault diagnosis of three-phase induction motors using neural networks. *Infrared Phys Techn* 155: 106490. <https://doi.org/10.1016/j.infrared.2026.106490>
9. Calderon-Urbe U, Lizarraga-Morales RA, Guryev IV (2024) Fault diagnosis in induction motors through infrared thermal images using convolutional neural network feature extraction. *Machines* 12: 497. <https://doi.org/10.3390/machines12080497>

10. Uzel H, Özüpak Y, Alpsalaz F, Aslan E, Zaitsev I (2025) Acoustic-based fault diagnosis of electric motors using mel spectrograms and convolutional neural networks. *Sci Rep* 16: 3379. <https://doi.org/10.1038/s41598-025-33269-z>
11. Glowacz A, Sulowicz M, Zielonka J, Li Z, Glowacz W, Kumar A (2025) Acoustic fault diagnosis of three-phase induction motors using smartphone and deep learning. *Expert Syst Appl* 262: 125633. <https://doi.org/10.1016/j.eswa.2024.125633>
12. Diversi R, Lenzi A, Speciale N, Barbieri M (2025) An autoregressive-based motor current signature analysis approach for fault diagnosis of electric motor-driven mechanisms. *Sensors* 25: 1130. <https://doi.org/10.3390/s25041130>
13. Wang B, Lin C, Inoue H, Kanemaru M (2024) Induction motor eccentricity fault detection and quantification using topological data analysis. *IEEE Access* 12: 37891–37902. <https://doi.org/10.1109/ACCESS.2024.3376249>
14. Dehnavi VS, Shafiee M (2025) Fault diagnosis of induction motors using novel measurement techniques and data fusion. *Measurement* 256: 118135. <https://doi.org/10.1016/j.measurement.2025.118135>
15. Chazal F, Michel B (2021) An introduction to topological data analysis: Fundamental and practical aspects for data scientists. *Frontiers in Artificial Intelligence* 4: 667963. <https://doi.org/10.3389/frai.2021.667963>
16. Virk Ž (2022) *Introduction to Persistent Homology*. Založba UL FRI. <https://doi.org/10.51939/0002>
17. Gravot E, Torregrosa S, Hascoët N, Kestelyn X, Chinesta F (2025) Topological data analysis for fault classification on transmission lines. *Electr Pow Syst Res* 248: 111915. <https://doi.org/10.1016/j.epsr.2025.111915>
18. Levenson RM, Singh Y, Rieck B, Hathaway QA, Farrelly C, Rozenblit J, et al. (2024) Advancing precision medicine: Algebraic topology and differential geometry in radiology and computational pathology. *Lab Invest* 104: 102060. <https://doi.org/10.1016/j.labinv.2024.102060>
19. Wei Q, Zeng S, Jiang Z, Xu F, Shi S, Wen W, et al. (2025) Topological representation based on wavelet transform as a novel imaging biomarker for tumor diagnosis in ultrasound images: A comprehensive study. *Comput Meth Prog Bio* 269: 108859. <https://doi.org/10.1016/j.cmpb.2025.108859>
20. Bilsky J, Clark AE (2025) Understanding the shape of chemistry data applications with persistent homology. *The Journal of Chemical Physics* 163. <https://doi.org/10.1063/5.0281156>
21. Mandal SK, Kumar R, Rizwee M, Kumar D (2024) Tuning thermal and structural properties of nano-filled pdms elastomer. *Polym Composite* 45: 14832–14844. <https://doi.org/10.1002/pc.28804>
22. Hernández-Lemus E (2025) Topological data analysis in single cell biology. *Front Immunol* 16: 1615278. <https://doi.org/10.3389/fimmu.2025.1615278>
23. Hartsock I, Park E, Toppen J, Bubenik P, Dimitrova ES, Kemp ML, et al. Topological data analysis of pattern formation of human induced pluripotent stem cell colonies. *Scientific Reports* 15: 11544. <https://doi.org/10.1038/s41598-025-90592-1>

24. González A, Hernández B, Acosta-Zamora KP, Ramos E, Núñez J (2025) Topological data analysis of three dimensional orbits in a convective flow. *Physica D*, 134841. <https://doi.org/10.1016/j.physd.2025.134841>
25. Bohlsen N, Robins V, Hole M (2025) Automatic classification of magnetic field line topology by persistent homology. *Physica D* 475: 134595. <https://doi.org/10.1016/j.physd.2025.134595>
26. Sagming M, Heymann R, Visaya MV (2024) Using topological data analysis and machine learning to predict customer churn. *Journal of Big Data* 11: 160. <https://doi.org/10.1186/s40537-024-01020-6>
27. Uray M, Giunti B, Kerber M, Huber S (2024) Topological data analysis in smart manufacturing: State of the art and future directions. *J Manuf Syst* 76: 75–91. <https://doi.org/10.1016/j.jmsy.2024.07.006>
28. Edelsbrunner H, Harer J (2010) *Computational Topology: An Introduction*, American Mathematical Society, 2010. <https://doi.org/10.1090/mbk/069>
29. Hadj Salem M, Mansouri K, Chauveau E, Ben Salem Y, Abdelkrim MN (2024) Multi-power system electrical source fault review. *Energies* 17: 1187. <https://doi.org/10.3390/en17051187>
30. ANSYS, Inc. (2024) Defining a three-phase induction motor. Accessed December 1, 2025.
31. Somasundaram EV, Brown SE, Litzler A, Scott JG, Wadhwa RR (2021) Benchmarking r packages for calculation of persistent homology. *The R Journal* 13: 184. <https://doi.org/10.32614/RJ-2021-033>



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