



Research article

Improved vehicle state estimation under low friction using a modified dragonfly optimization algorithm

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Abstract: Vehicle state estimation accuracy in the Adaptive Unscented Kalman Filter (AUKF) is highly dependent on the proper tuning of the process and measurement noise covariance matrices. Conventional optimization techniques often produce suboptimal covariance values, increasing the risk of estimation divergence and compromising vehicle stability. To overcome this limitation, this paper proposes a Modified Dragonfly Optimization Algorithm-based Adaptive Unscented Kalman Filter (MDOA-AUKF), in which adaptive covariance tuning is formulated as an optimization problem. By incorporating elite solution preservation and multi-behavioral swarm dynamics, the proposed MDOA effectively balances exploration and exploitation, thereby enabling adaptive updates of the process (Q) and measurement (R) noise covariance matrices at each iteration. This improves the convergence, estimation accuracy, and robustness under nonlinear driving conditions and varying vehicle mass. The proposed approach is implemented using a two-degree-of-freedom bicycle model considering lateral and yaw dynamics under a low-friction road condition (tire–road friction coefficient of 0.4) and evaluated using Fishhook (FH) and Double Lane Change (DLC) maneuvers. During the FH maneuver, the proposed method achieves root mean squared error (RMSE)/ normalized root mean squared error (NRMSE) values of 0.0008/0.21% and 0.0010/0.03% for the sideslip angle and yaw rate, respectively, while the corresponding values during the DLC maneuver are 0.0011/0.24% and 0.0009/0.02%, respectively. Compared with Particle Swarm Optimization (PSO), Ant Colony Optimization (ACO), and Grey Wolf Optimization (GWO), the proposed approach provides significantly lower estimation errors and faster convergence. Robustness is further validated under vehicle mass variations from 1800 kg to 2300 kg, demonstrating consistently low estimation errors. These results confirm that the proposed MDOA-AUKF is an accurate, robust, and computationally efficient framework for intelligent vehicle state estimation.

Keywords: bicycle model; sideslip angle; yaw rate, adaptive unscented Kalman filter (AUKF); modified dragon fly optimization algorithm (MDOA)

1. Introduction

Within Industry 4.0, hybrid machine learning and metaheuristic optimization models have improved decision-making and project scheduling [1], while similar approaches have been applied to healthcare resource allocation [2]. In the energy sector, optimization techniques have been utilized for renewable energy project portfolio selection and sustainable supply chain design under uncertain conditions [3–6]. Likewise, optimization-assisted estimation methods have enhanced battery state estimation, where particle swarm-assisted particle filtering and square-root unscented Kalman filtering with online parameter identification have improved the estimation accuracy and reliability under varying operating conditions [7,8].

Kalman filter based methods have been widely utilized for the estimation of vehicle states due to their recursive nature and ability to mitigate measurement noise. In particular, the extended Kalman filter (EKF) and unscented Kalman filter (UKF) have been extensively exploited to estimate the sideslip angle and yaw rate under different driving patterns. Recent research on vehicle sideslip angle and overall state estimation has focused on enhancing the estimation accuracy and robustness. Recent studies have proposed steering torque-based sideslip angle estimation for intelligent commercial vehicles [9] and nonlinear interacting multiple model-based Kalman filtering to improve the estimation under varying vehicle dynamics and severe road conditions [10]. The EKF method depends on local linearization and may exhibit reduced accuracy in highly nonlinear systems, whereas UKF-based estimators provide enhanced performances by propagating sigma points through nonlinear systems. Thus, the UKF has attracted significant attention for estimating various vehicle states during complex maneuvers including double lane changes (DLC), fishhook (FH), and slalom maneuvers [11–13]. However, the estimation performance of the UKF is strongly dependent on the appropriate selection of the process (Q) and measurement (R) noise covariance matrices, which are typically chosen empirically or assumed to be constant [14]. Such assumptions often neglect the time-varying uncertainties that arise from tire dynamics, road friction coefficients, and simplifications in the mathematical model, which can result in a reduced estimation accuracy or filter instability in highly dynamic situations.

To improve the adaptability of Kalman filter-based estimators, several adaptive and optimization-assisted approaches have been proposed. Bayesian optimization has been employed for automatic covariance tuning to improve the estimation accuracy with reduced computational effort [15], while adaptive approaches, such as the Sage–Husa and innovation-based UKF, dynamically update the noise statistics under uncertain conditions [16]. Furthermore, improved UKF variants have demonstrated enhanced estimation performances in highly nonlinear and dynamic applications, including maneuvering target tracking [17]. Observer-based and hybrid estimation frameworks have also been developed for simultaneous vehicle state estimation, trajectory tracking, and lateral dynamics monitoring, thus contributing to improved vehicle handling and stability [18–20]. Furthermore, parallel UKF architectures have enabled the concurrent estimation of vehicle states and tire–road friction, while adaptive UKF methods incorporating time-varying attenuation mechanisms have demonstrated improved performances in dynamic and uncertain environments [21,22]. To further

enhance the estimation robustness, adaptive filtering techniques such as interacting multiple model-based AUKF (IMM-AUKF), adaptive fading UKF, adaptive covariance tuning, forgetting factor-based methods, and maximum likelihood-based adaptive Kalman filtering have been proposed to accommodate varying driving conditions and unknown noise characteristics [23–28].

Recently, metaheuristic optimization algorithms, including Particle Swarm Optimization (PSO), Ant Colony Optimization (ACO), Grey Wolf Optimization (GWO), and Ant Lion Optimization (ALO), have been incorporated into Kalman filter frameworks for adaptive tuning of process and measurement noise covariance matrices, resulting in improved estimation accuracies [29–32]. Although these methods improve the estimation accuracy, they primarily employ offline or quasi-static covariance tuning and are validated under limited operating conditions. Furthermore, their reliance on high-degree-of-freedom vehicle models results in high computational costs, thus restricting real-time implementation. Among recent optimization techniques, the Dragonfly Algorithm (DA) has attracted considerable attention because of its strong exploration and exploitation capabilities, and improved variants have demonstrated enhanced convergence and optimization performances across diverse engineering applications [33–37]. Motivated by these advantages, this work employs a Modified Dragonfly Optimization Algorithm (MDOA) to adaptively tune the process and measurement noise covariance matrices of the UKF for accurate and robust vehicle state estimations. Since vehicle dynamic states, such as the sideslip angle, yaw rate, and tire–road friction coefficient, cannot be directly measured using production sensors, reliable state estimation remains essential for advanced vehicle safety and control systems, including ESC, ABS, AFS, and ADAS.

Based on the above literature review, the following research gaps are identified:

- Existing UKF-based vehicle state estimation methods rely on fixed or empirically tuned noise covariance matrices (Q and R), which limit their adaptability under time-varying and uncertain driving conditions;
- Most adaptive and optimization-based approaches either rely on strong statistical assumptions or perform offline/quasi-static tuning, making them less effective for real-time implementation in highly nonlinear environments; and
- The application of the dragonfly algorithm for the dynamic tuning of noise covariance matrices in vehicle state estimation remains largely unexplored, while existing methods often employ computationally intensive high-order models.

To address these challenges, this study proposes a novel framework that combines a MDOA with an AUKF for vehicle state estimation. In the proposed approach, the process and measurement noise covariance matrices (Q and R) are adaptively tuned by formulating their adjustment as a performance-driven optimization problem. The MDOA incorporates mechanisms such as elite solution preservation and adaptive inertia weighting to effectively balance exploration and exploitation, thereby ensuring stable convergence under nonlinear and uncertain driving conditions. The performance of the proposed framework is evaluated using the root mean square error (RMSE) between estimated and reference states, with a focus on accurately estimating the vehicle sideslip angle and yaw rate during aggressive driving maneuvers. This work improves existing UKF-based estimation methods by enabling real-time adaptive tuning of noise covariance matrices using the MDOA, thereby overcoming the limitations of offline tuning and enhancing robustness under nonlinear and uncertain driving conditions.

The key contributions of this research are outlined below:

- Unlike existing metaheuristic approaches (e.g., PSO, GWO, and ALO) that rely on offline tuning, a performance-driven adaptive UKF is developed where noise covariance matrices (Q and R) are dynamically optimized using MDOA;
- In contrast to the high-order models employed in prior studies, a computationally efficient 2-degree of freedom (DOF) nonlinear bicycle model is adopted for real-time applicability; and
- Compared with methods validated under limited operating conditions, the proposed approach is tested under aggressive maneuvers (DLC and FH) on low-friction roads, demonstrating improved robustness.

The subsequent sections of this paper are structured in the following manner: Section 2 outlines the system modeling; Section 3 provides details of the filter design; Section 4 examines the simulation results and provides a comparative analysis; and Section 5 describes the conclusion by highlighting key findings and suggesting avenues for future research.

2. System modelling

This section describes the simplified mathematical model used for sideslip angle and yaw rate estimation. Subsection 2.1 deals with assumptions made to formulate the vehicle mathematical model and the nonlinear state space model. Furthermore, the required tire model is explained in the Subsection 2.2.

2.1. Vehicle mathematical modelling

For the purpose of simulation in MATLAB, instead of considering all dynamics involved in the motion of a vehicle, a simplified 2-DOF nonlinear mathematical model, known as the bicycle model, is utilized [38]. The aim of considering this simplified model is to avoid complexity of the model resulting in high computational load. This model is developed based on the assumptions mentioned below and considering the important dynamics, including the lateral and yaw motion of the vehicle:

1. The left and right tires are assumed to have the same slip angles characteristics;
2. The vehicle is assumed to be a point mass;
3. The vehicle un-sprung mass is neglected;
4. In this model, the inclination angle is neglected; and
5. The aerodynamic drag is not considered

Assume that the motion of the vehicle is defined on a global 2-dimensional (X, Y) axis system. This axis is utilized to define the positioning and velocity of the vehicle. Figure 1 represents the bicycle model in the 2- dimensional axis to observe the lateral and yaw motions.

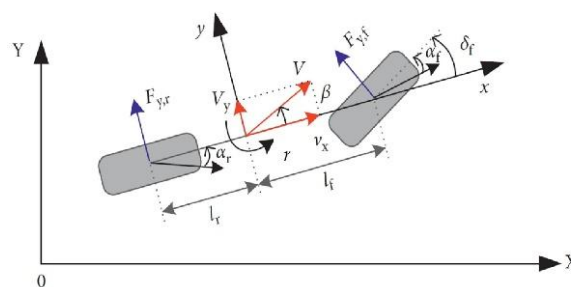


Figure 1. 2-DOF bicycle model.

The mathematical equations of the bicycle model are defined utilizing Newton force and Euler inertia equations of motions for linear and angular motion, respectively. For the linear motion, the sideslip angle (β) of the vehicle is considered and the yaw rate (r) is considered for angular motion.

The lateral dynamics is described in terms of the sideslip angle as mentioned in Equation (1):

$$mV_x(\dot{\beta} + r) = \sum F_y, \quad (1)$$

$$F_y = F_{yf} \cos \delta_f + F_{yr}, \quad (2)$$

where m is the mass of the vehicle, V_x is the longitudinal velocity of the vehicle assumed constant, r is the yaw rate, and F_y is the total lateral force experienced by the tires. F_{yf} , F_{yr} are the front and rear tire lateral forces, respectively. Equation (2) describes the total lateral forces in terms of the front and rear.

The yaw dynamics of the bicycle model is defined by the Eq. (3):

$$I_z \dot{r} = l_f F_{yf} \cos \delta_f - l_r F_{yr}, \quad (3)$$

where I_z is the moment of inertia of the vehicle, l_f is the distance of front tire to the center of gravity, and l_r is the distance of the rear tire to the center of gravity.

Equations (1) and (3) are further simplified and expressed in a state space model in Eqs. (4) and (5):

$$\dot{\beta} = -r + \frac{F_{yf} \cos \delta_f}{mV_x} + \frac{F_{yr}}{mV_x}, \quad (4)$$

$$\dot{r} = \frac{l_f F_{yf} \cos \delta_f}{I_z} - \frac{l_r F_{yr}}{I_z}, \quad (5)$$

2.2. Tire forces

This paper utilizes a simplified lateral force of the magic formula for the smaller values of longitudinal velocity. The vehicle is considered to be a front wheel drive; therefore, the steering input is only provided to the front tires. The governing equations for the front and rear tire lateral forces are given in Eqs. (6) and (7):

$$F_{yf} = C_{\alpha f} * \alpha_f, \quad (6)$$

$$F_{yr} = C_{\alpha r} * \alpha_r, \quad (7)$$

where F_{yf} , F_{yr} , $C_{\alpha f}$, $C_{\alpha r}$, α_f , and α_r are the front and rear tire lateral forces, front and rear tire stiffness coefficient, and tire front and rear slip angle, respectively. The expressions of slip angles are mentioned in Eqs. (8) and (9):

$$\alpha_f = \delta_f - \beta - \frac{l_f r}{V_x}, \quad (8)$$

$$\alpha_r = -\beta + \frac{l_r r}{V_x}, \quad (9)$$

where δ_f is the input steering angle. Therefore, after utilizing the Eqs. (6) and (7) in the Eqs. (4) and (5), the obtained nonlinear relation between the states and inputs are stated in the Eqs. (10) and (11):

$$\dot{\beta} = -r + \frac{C_{af}(\delta V_x - \beta V_x - l_f r) \cos \delta_f - \beta V_x + l_r r}{m V_x^2}, \quad (10)$$

$$\dot{r} = \frac{l_f C_{af}(\delta V_x - \beta V_x - l_f r) \cos \delta_f - l_r(l_r r - \beta V_x)}{V_x I_z}. \quad (11)$$

Equations (10) and (11) are the required nonlinear state equations. The state variables are defined as $x_1 = \beta$, $x_2 = r$. The state equation can be represented as in Eqs. (12) and (13):

$$\dot{x}(t) = g(x(t), u(t)), \quad (12)$$

$$\begin{bmatrix} \dot{\beta} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} -r + \frac{C_{af}(\delta V_x - \beta V_x - l_f r) \cos \delta_f - \beta V_x + l_r r}{m V_x^2} \\ \frac{l_f C_{af}(\delta V_x - \beta V_x - l_f r) \cos \delta_f - l_r(l_r r - \beta V_x)}{V_x I_z} \end{bmatrix}, \quad (13)$$

where $\dot{x}(t) = [\dot{x}_1 \ \dot{x}_2]^T = [\dot{\beta} \ \dot{r}]^T$. Equation (14) is the matrix form representation of the state equation. The input is denoted by $u(t) = \delta_f(t)$. The output variables are chosen as $y_1 = r$, $y_2 = a_y$. Similarly, the measurement equation is described in Eqs. (14) and (15):

$$y(t) = h(r(t), a_y(t)), \quad (14)$$

$$\begin{bmatrix} r \\ a_y \end{bmatrix} = \begin{bmatrix} r \\ V_x(r + \beta) \end{bmatrix}, \quad (15)$$

where a_y is the lateral acceleration. For the nonlinear state estimation, Equations (14) and (15) are utilized in the further section.

3. Filter design

The performance accuracy of the UKF depends on the several factors, including the selection of sigma points, the process noise covariance matrix (Q), and the measurement noise covariance matrix (R). The accuracy of the UKF can be improved by incorporating adaptive tuning of the Q and R . This section elaborates about the adaptive nature of the UKF-based modified dragonfly optimization to optimize the covariance matrices.

3.1. Unscented Kalman filter (UKF)

In the EKF approach, the nonlinear model is linearized by computing the Jacobian matrix at each linearization point. This Jacobian-based approximation reduces the filter's accuracy by introducing errors in the true posterior mean and covariance. To overcome this problem, the probability distribution in the UKF is represented by a set of sigma points generated using the Unscented Transform algorithm [15]. These sigma points have the same mean and covariance as the original state distribution.

The steps of the UKF include the following initialization of states.

1. The initial values of mean and covariance defined by the estimated state are described in Eqs. (16) and (17):

$$\bar{x}_0 = E(x_0), \quad (16)$$

$$P_0 = E[(x - x_0)(x - x_0)^T], \quad (17)$$

where $x = [\beta \ r]^T$, and the initial values are set as $x = [\beta(0) \ r(0)]^T = [0 \ 0]^T$.

2. Calculation of sigma points and weights: the formula to calculate the number of sigma points is $2n+1$. Using $2n+1$ χ_i column vectors, a matrix χ is created that contains the locations of the sigma points. The following set of equations is used to determine the symmetrical sigma points given in Eq. (18):

$$\chi_i = \begin{cases} \bar{x} + (\sqrt{(n + \lambda)} P_x)_i, \\ \bar{x} - (\sqrt{(n + \lambda)} P_x)_i \end{cases} \quad (18)$$

where λ is the adjustment parameter defined as $\lambda = \alpha^2(n + \kappa) - n$, and α, κ are the scaling factors that affect the spread of the sigma points away from the mean. The value is taken as $\alpha \in (0,1]$ and for $\kappa \geq 0$. Furthermore, the calculation of weights and sigma points can be referred to [11,13].

3. Propagation of sigma points: the following equation performs each sigma point's non-linear transformation given in Eq. (19):

$$\chi_{k+1|k} = g(\chi_{k|k}, u_k) \quad (19)$$

4. Time Update: one step ahead forecasts of the states and error covariance matrix are given in Eqs. (20) and (21):

$$x_{k+1|k} = \sum_{i=0}^{2n} \omega_i^m \chi_{i,k+1|k}, \quad (20)$$

$$P_{k+1|k} = \sum_{i=0}^{2n} \omega_i^c (\chi_{i,k+1|k} - x_{k+1|k})(\chi_{i,k+1|k} - x_{k+1|k})^T + Q. \quad (21)$$

5. Measurement Update: in this step, the sigma points are propagated through the measurement equation defined in Eq. (22):

$$[\Psi_{i,k+1|k} = h(\chi_{i,k|k}, u_k). \quad (22)$$

The mean of the measurement is calculated in Eq. (23):

$$y_{k+1|k} = \sum_{i=0}^{2n} \omega_i^m \Psi_{i,k+1|k}. \quad (23)$$

The error covariance matrix is obtained in Eq. (24):

$$P_{yy} = \sum_{i=0}^{2n} \omega_i^c (\Psi_{i,k+1|k} - y_{k+1|k})(\Psi_{i,k+1|k} - y_{k+1|k})^T + R. \quad (24)$$

The cross-correlation covariance matrix is given in Eq. (25):

$$P_{xy} = \sum_{i=0}^{2n} \omega_i^c (\Psi_{i,k+1|k} - y_{k+1|k})(\chi_{i,k+1|k} - x_{k+1|k})^T. \quad (25)$$

The Kalman gain feedback matrix is calculated in Eq. (26):

$$K_{k+1} = P_{xy} P_{yy}^{-1}. \quad (26)$$

The updated state and error covariance matrix are given in Eqs. (27) and (28):

$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + K_{k+1}(y_{k+1|k} - \hat{y}_{k+1|k}), \quad (27)$$

$$P_{k+1|k+1} = P_{k+1|k} + K_{k+1} P_{k+1|k+1} K_{k+1}^T. \quad (28)$$

3.2. Adaptive unscented Kalman filter (AUKF)

The noise covariance matrices in the Kalman filter Algorithm for the prediction and correction of the mean and covariance of states are often dynamic or unknown for the real time implementation. The filter algorithm incorporates this time-varying noise characteristic to enhance its adaptability to dynamic covariance matrices. Equation (40) makes it clear that if the process noise matrix Q is inaccurately estimated, then the anticipated state covariance P_k and $P_{k+1|k}$ will be affected, and the estimated P_k and $P_{k+1|k}$ will also be biased. The state estimate derived from Equations (24) and (46) will deteriorate as a result of the Kalman gain K_k and K_{k+1} being erroneous. Likewise, for the measurement noise R , it may be inferred that the filters' solution may degrade or diverge in the absence of precise system noise variance.

To incorporate the adaptive nature of noise covariance matrices in the UKF, a MDOA is proposed to improve the state estimation accuracy. Based on the static and dynamic swarming behavior of dragonflies (DFs), MDOA is a swarm intelligence based evolutionary optimization algorithm method within the provided boundary. Researchers from a variety of disciplines have shown considerable interest in DOA because of its effectiveness and simplicity [34,36].

3.3. Modified dragonfly optimization algorithm (MDOA)

A static swarm is a small group of DFs that forage for food in a constrained area. This movement is characterized by abrupt and rapid changes in the individual flight path. In contrast, a dynamic swarm consists of a large group of flies that move in a consistent direction over a considerable distance to migrate from one location to another. The exploitation phase is characterized by the static behavior of swarm DFs, whereas the exploration phase is characterized by the dynamic behavior of swarm DFs, since the goal of these DFs in a swarm is comparable to the optimization problems. These characteristics form a foundation of the DOA.

In a MDOA [34,35], the optimal solution (Q, R) values are calculated based on the selection of elite values of (Q, R) based on the RMSE. Swarm behaviors such as maintaining separation to prevent collisions, aligning their velocities with neighboring individuals, maintaining cohesive by moving toward the group's center, being drawn toward food sources, and steering away from potential predators are responsible for optimal solution. The elite values of (Q, R) , along with the adaptive inertia factor (w_j) and behavioral coefficients of swarms of the MDOA, dynamically balance exploration and exploitation during the optimization process. Higher inertia factor values at the beginning of the algorithm encourage DFs to explore a larger search space. As the inertia factor gradually decreases with increasing iterations, DFs can concentrate on refining solutions near the current best solution. This mechanism improves convergence toward the optimal solution.

The position of each DF is updated utilizing the adaptive inertia factor of weighted sum of separation, alignment, cohesion, attraction, and distraction with the previous positions. The adaptive inertia factor is defined in Eq. (29):

$$w_j = w_{max} - \left(\frac{j (w_{max} - w_{min})}{\text{total number of iterations}} \right), \quad (29)$$

where j is the current iteration. Equation (30) defines the position of the DF:

$$\Delta x_{i,j+1} = \Delta x_{i,j} + w_j (aS_{i,j} + bA_{i,j} + cC_{i,j} + dF_{i,j} + ePr_{i,j}), \quad (30)$$

where a , b , and c are the separation, alignment, and cohesion behavioral coefficients, respectively, j is the number of iterations, and d and e represent the attraction and distraction coefficients, respectively. S_i , A_i , C_i , F_i , and Pr_i are utilized to depict the separation, alignment, cohesion, attraction, and distraction characteristics of i^{th} single DF in a group, respectively.

The negative sum of the i^{th} DFs distances from each neighbor in the j^{th} iteration is denoted by $A_{i,j}$. The average velocity in the j^{th} iteration yields $S_{i,j}$. The distance of the i^{th} DF from the neighborhood's average location in the j^{th} iteration yields $C_{i,j}$. $F_{i,j}$ is found by adding the locations of the i^{th} DF and the location with the worst objective value so far in the j^{th} iteration to move away from it, and $Pr_{i,j}$ is calculated as the distance of i^{th} DF from a location with the best objective value so far in j^{th} iteration to move towards the best location. The exploratory and exploitative behaviors of Dfs can be adjusted by varying the parameter values.

A flow chart depicting the workflow of the MDOA is shown in Figure 2.

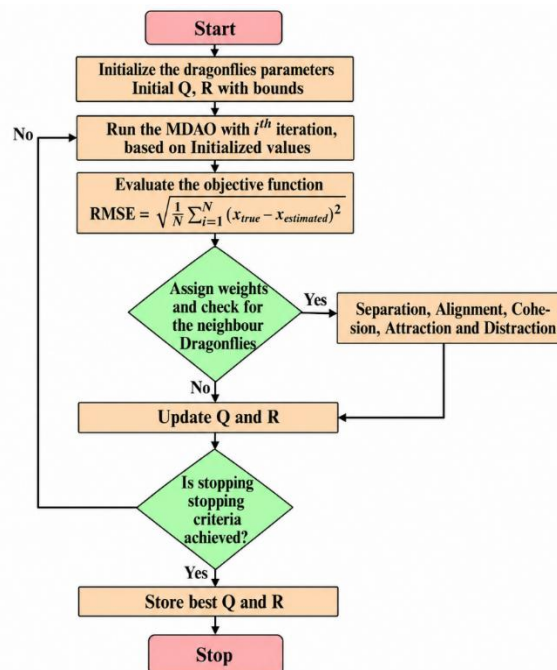


Figure 2. Flow Chart for calculation of Optimal Q, R .

The steps are mentioned as follows:

1. Determine the DFs fitness by giving them random values between the boundary limits of Q and R . The values of attraction and distractions are assigned.
2. Start the iterations after deciding on a stopping condition. Depending on the number of iterations and unpredictability, update the factor and coefficients.
3. The parameters of each fly are calculated and the position is updated.
4. Lastly, this algorithm will terminate if the termination condition (the best optimal position for DFs to minimize the RMSE) is fulfilled or satisfied.

3.4. MDOA based adaptive UKF

In this section, the estimation of the sideslip angle and yaw rate utilizing the MDOA based Adaptive is explained. Using a nonlinear vehicle model and noisy sensor readings, the UKF calculates the vehicle's side slip angle and yaw rate. It models uncertainties in the dynamics of the system using Q (process noise covariance), and it models the uncertainty in the sensor data using R (measurement noise covariance). How well Q and R are tuned determines the UKF's accuracy. A metaheuristic optimization technique called the DOA looks for the ideal values of Q and R . Q and R are iteratively updated by MDOA as follows:

1. Creating a population of potential values for Q and R .
2. Executing the UKF for every solution.
3. The RMSE between the true states (x_{ture} or x_k) and the estimated states (x_{est} or $\hat{x}_k$) is calculated as in Eq. (31):

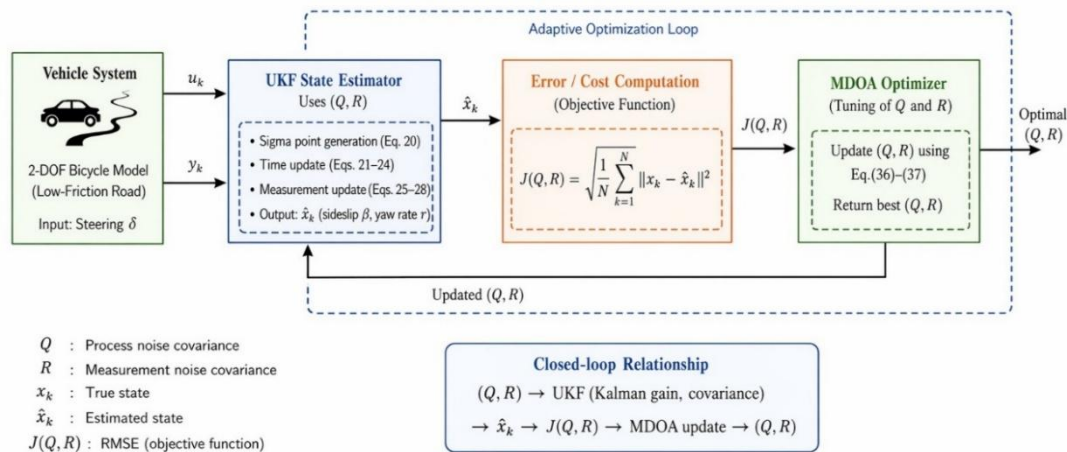
$$RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^N (x_{i,tur} - x_{i,est})^2}. \quad (31)$$

4. Use the DOA's movement guidelines (distraction, attraction, cohesion, separation, and alignment). Create fresh (Q, R) value candidates.
5. Maintain values within reasonable bounds by enforcing boundary requirements.
6. Until convergence, continue running the UKF and modifying Q and R .
7. The ideal noise covariance is determined by choosing the best (Q, R) pair that minimizes the RMSE.

The proposed MDOA based AUKF fundamentally differs from PSO, GWO, and ACO-based adaptive filtering methods in its mathematical framework and behavioral representation, as summarized in Table 1 [36,37]. In contrast to PSO and GWO, which utilize a limited number of attractors and implicit balancing of exploration and exploitation, the MDOA distinctly breaks down the swarm behavior into components of separation, alignment, cohesion, attraction, and distraction. This approach allows for independent management of diversity maintenance, smoothness of convergence, and robustness, which is especially crucial for tuning covariance in nonlinear Kalman filtering. Additionally, the strategy of repelling poor solutions and preserving elite solutions helps to avoid premature convergence and prevents overconfidence in the estimator. These theoretical attributes render the MDOA more effective for optimizing adaptive noise covariance in nonlinear vehicle dynamics compared to current swarm-based approaches. This architecture allows for consistent convergence of noise covariance matrices under severe nonlinearities and low friction conditions, thus leading to higher estimate accuracies and robustness.

Table 1. Comparison between different optimization techniques.

Aspect	PSO	GWO	ACO	MDOA
Behavioural Component	2	3	Pheromone	5
Repulsion from poor solutions	Absent	Absent	Absent	Present
Diversity Preservation	Weak	Weak	Moderate	Strong
Adaptive filtering suitability	Moderate	Moderate	Low	High
Noise-robust search	Limited	Limited	Low	High

**Figure 3.** Block Diagram of the proposed methodology.

The block diagram in Figure 3 illustrates the overall operation of the proposed MDOA-based AUKF framework in a closed-loop system. The vehicle system provides the input signal u_k (steering) and the measured outputs y_k to the UKF, which estimates the system states $\hat{x}_k$, including the sideslip angle and yaw rate, based on the nonlinear vehicle model (Eqs. 10–13). The UKF performs the estimation through the sigma points generation (Eq. 20), prediction or time update (Eqs. 20–21), and measurement correction (Eqs. 22–28). The predicted covariance matrix is affected by the process noise covariance Q , while the Kalman gain K_k (Eq. 26) is influenced by both the matrices Q and R . As a result, the quality of the estimated states $\hat{x}_k$ strongly depends on how well Q and R are selected, as they determine how the filter balances the model prediction and measurement correction.

To improve this balance, the estimation error is evaluated using the RMSE defined in Eq. (31), which is treated as the objective function $J(Q, R)$. Then, this error is used by the MDOA to adjust the values of Q and R . The update of these parameters follows the position update rules of the MDOA (Eqs. 29–30), where the candidate solutions are iteratively refined. The optimization goal can be written as $(Q^*, R^*) = \arg \min_{(Q, R)} J(Q, R)$, thereby representing the algorithm searches for the covariance matrices that minimize the difference between the true states x_k and the estimated states $\hat{x}_k$. The optimized values are fed back into the UKF, creating a continuous feedback loop. This interaction allows the filter to dynamically adapt its parameters, thus leading to a better estimation accuracy, faster response, improved robustness under nonlinear dynamics, and low-friction conditions.

3.5. Mathematical formulation of MDOA based AUKF to tune the Q, R

UKF State Estimation Dependency: the estimated state $\hat{x}_k$ directly depends on the input, measurements, and the covariance matrices Q and R as in Eq. (32):

$$\hat{x}_k = f_{UKF}(u_k, y_k, Q, R). \quad (32)$$

Any change in Q or R alters the Kalman gain and covariance update, thereby affecting the estimation accuracy.

Objective Function (RMSE Minimization): this function measures the difference between the true states and estimated states over all time steps as state in Eq. (31). Minimizing this error ensures that the estimated values closely follow the actual system behavior.

Optimization Problem: Equation (33) defines the goal of finding optimal covariance matrices that minimize the estimation error:

$$(Q^*, R^*) = \arg \min_{(Q, R)} J(Q, R). \quad (33).$$

It converts the tuning of Q and R into a formal optimization problem rather than manual selection.

MDOA Update Rule (Link to Optimization): Equation (34) shows how the MDOA updates the candidate solutions using swarm-based behavioral rules. These updates guide the search process toward better values of Q and R that reduce the estimation error:

$$(Q^{t+1}, R^{t+1}) = (Q^t, R^t) + \Delta X^t, \quad (34)$$

$$\Delta X^t = sS + aA + cC + fF + dD + w\Delta X^{t-1}. \quad (35)$$

Final Coupled Relation (UKF + MDOA Integration): Equation (36) represents the complete integration of the UKF and the MDOA in a single framework:

$$(Q^{t+1}, R^{t+1}) = \arg \min \sqrt{\frac{1}{N} \sum_{i=1}^N (x_{i,ture} - x_{i,est})^2}. \quad (36)$$

It shows that the MDOA continuously updates Q and R to minimize the estimation error produced by the UKF.

4. Simulation results and discussion

To evaluate the performance of the proposed technique for state estimation, a nonlinear 2-DOF vehicle mathematical model is considered in this study. The lateral and yaw dynamics are effectively characterized by the sideslip angle and yaw rate, respectively; therefore, these variables are selected as the state variables. The sideslip angle represents the lateral deviation of the vehicle relative to its forward motion. A large sideslip angle signifies a reduced control or instability and largely affects the turning performance, specifically during high-speed maneuvers or under low-friction conditions (such as wet or slippery roads). The yaw rate of the vehicle is the rotational speed of the vehicle about its

vertical axis and is expressed in degrees per second (degree/s) or radians per second (rad/s). It determines how quickly the vehicle responds to steering inputs. An excessive yaw rate, combined with a high sideslip angle, may lead to vehicle instability, including rollover or spin-out.

The two standard vehicle maneuvers, DLC and FH, utilized to evaluate the stability and handling under extreme road conditions are explored as inputs to the vehicle mathematical model in this study. These maneuvers evaluate the ability of the vehicle to change lanes without losing control. In this study, a range of Q and R are chosen as $[Q_{min} \ Q_{max}] = [1.0e^{-6} \ 0.100]$ and $[R_{min} \ R_{max}] = [1.0e^{-6} \ 0.100]$ by dynamically optimizing the measurement noise covariance (Q) and process noise covariance (R) matrices. The Q and R values are initialized within the range of $[0.0124 \ 0.0364]$ and $[0.0803 \ 0.0164]$ for the sideslip angle and yaw rate, respectively. To avoid skewed solutions and ensure diversity, the covariance matrices Q and R are randomly initialized within the specified range. The RMSE and normalized RMSE (NRMSE) of the state variables are used as performance metrics to evaluate the estimation accuracy. A MATLAB environment is exploited to validate the reliability and effectiveness of the proposed method

Figures 4 and 5 represent the steering inputs for the DLC and the FH maneuvers, respectively. State estimation is performed on a low friction road surface with a friction coefficient of 0.4, representing a wet surface road. The vehicle is assumed to have a constant longitudinal velocity V_x of 36 km/h. The parameters of the vehicle utilized in the simulation are mentioned in Table 2.

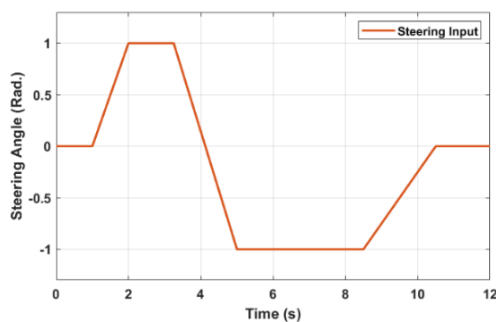


Figure 4. FH maneuver.

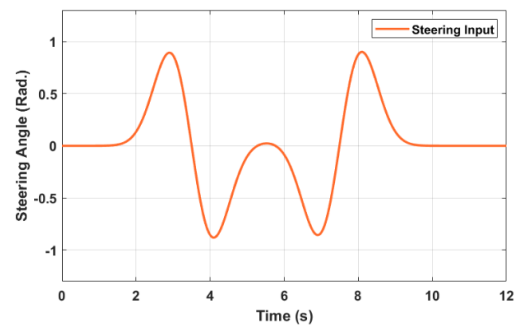


Figure 5. DLC maneuver.

Table 2. Vehicle parameters.

Symbol	Description	Value
m	Mass of the Vehicle	2020 kg
l_f	Distance of C.G. from front tire	1.40 m
l_r	Distance of C.G. from rear tire	1.65 m
I_z	Moment of Inertia	3234 kgm ²
C_f	Front tire Cornering Stiffness	81360 N/Rad.
C_r	Rear tire Cornering Stiffness	81360 N/Rad.

Figure 6 depicts the estimation of the sideslip angle during the FH maneuver. In this figure, the green color represents the true states without noise obtained from the vehicle dynamic equations used as reference signal. The cyan color line represents the lateral dynamics affected by noise. The black color line represents the estimated states obtained by simultaneously optimizing the noise matrices

and rejecting the noise. The time interval $4.5s \leq t \leq 5.5s$ highlights the accuracy of the estimated signal during a critical transition phase.

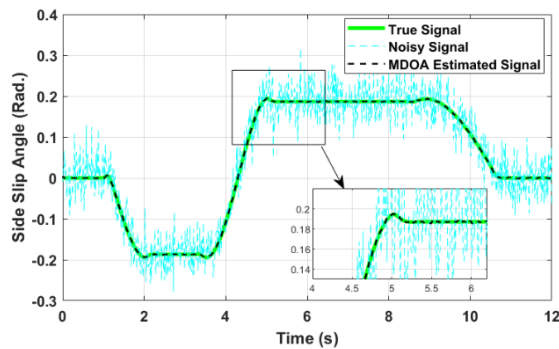


Figure 6. β Estimation for FH maneuver.

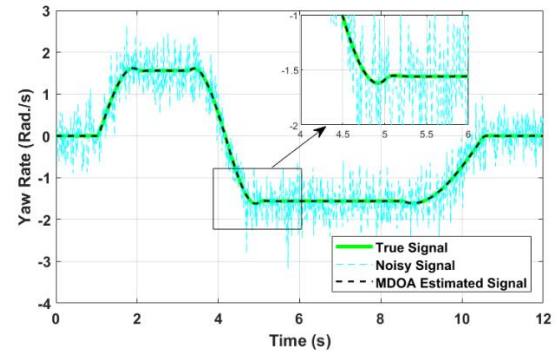


Figure 7. r Estimation for FH maneuver.

The estimated yaw rate is depicted in Figure 7. Despite the presence of noise, the AUKF-estimated signal (black dashed line) smoothly tracks the true yaw rate. The AUKF rapidly responds to abrupt changes ($4.5s \leq t \leq 6s$) without exhibiting any overshoot. The zoomed-in section demonstrates the ability of the AUKF to effectively filter noise while maintaining responsiveness to changes in the actual vehicle dynamics. It can be observed that the MDOA-based estimated signal tracks the true states with a high accuracy and low RMSE as mentioned in Table 3. From the Table, it can be seen that the RMSE values of both the state variables are minimized. Additionally, the values of the NRMSE for the sideslip angle and yaw rate are 0.21% and 0.03%, respectively, representing a better estimation performance. The optimized values of Q and R obtained in MATLAB are [0.083, 0.007] and [0.1, 1e-06], respectively.

Table 3. RMSE values and normalized RMSE for FH maneuver.

State variable	RMSE values	NRMSE values
Sideslip Angle (Rad.)	0.0008	0.21 %
Yaw Rate (Rad./s)	0.0010	0.03%

For the DLC maneuver, the estimation of the sideslip angle is depicted in Figure 8. While suppressing the noise, the AUKF smoothly tracks the actual signal despite the large fluctuations in the noisy signal. The estimated signal closely follows the true signal during abrupt changes ($3s \leq t \leq 6s$, $7s \leq t \leq 9s$), thus suggesting that the AUKF dynamically adjusts to changes. Figure 9 depicts the estimation of the yaw rate, in which it can be seen that the filter precisely estimates the yaw rate while effectively suppressing the noise. Table 4 presents the performance metrics values used to evaluate the estimation accuracy, including the NRMSE values of the sideslip angle (β) and yaw rate (r) for the DLC maneuver. From the Table, it can be observed that the NRMSE of β and r are 0.24% and 0.02%, respectively, indicating an improved estimation performance. Thus, the corresponding optimized value of Q and R are obtained as [0.1, 0.005] and [0.1, 1e-06], respectively.

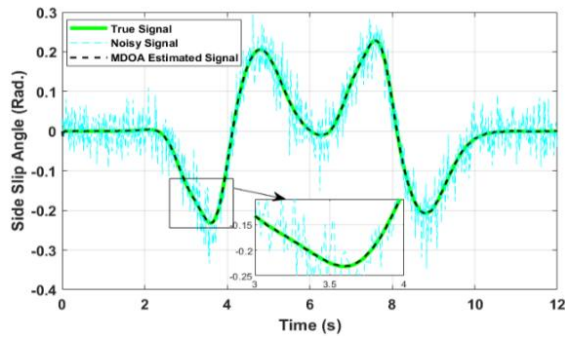


Figure 8. β Estimation for DLC maneuver.

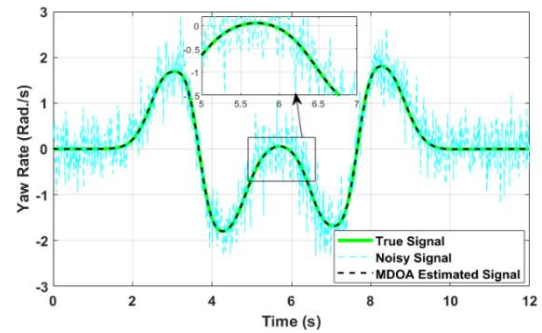


Figure 9. r Estimation for DLC maneuver.

Table 4. RMSE values and NRMSE for DLC maneuver.

State variable	RMSE values	NRMSE values
Sideslip Angle (Rad.)	0.0011	0.24%
Yaw Rate (Rad./s)	0.0009	0.02%

For the purpose of comparison of the accuracy, the MDOA is compared with the other optimization algorithms in the literatures including PSO, GWO, and ACO. Figures 10 and 11 depict the comparison of the sideslip angle and yaw rate estimation, respectively, thereby utilizing the different optimization algorithms. The noisy signal (blue dashed) shows high fluctuations, making direct measurements unreliable. PSO exhibits deviations with respect to the actual signal and a moderate sensitivity to noise. ACO handles noise slightly better than PSO, while small fluctuations are present. From the figure, it is observed that GWO performs better than PSO and ACO.

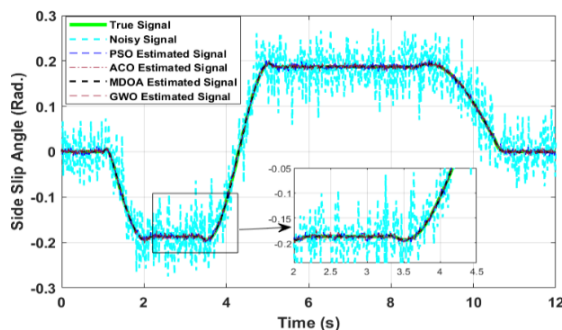


Figure 10. Comparison for FH maneuver.

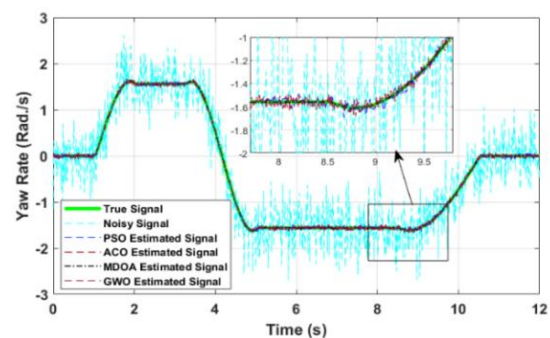


Figure 11. Comparison for FH maneuver.

Table 5 presents a quantitative comparison of the sideslip angle and yaw rate estimation for the FH maneuver, thereby utilizing the three different algorithms with the proposed MDOA. From the Table, it can be seen that the RMSE values of the sideslip angle and yaw rate for PSO are 0.1601 and 0.1118, respectively, showing the deteriorated performance. Additionally, it is found that the NRMSE values are high for both the state variables. For ACO, the NRMSE values are 0.38% and 0.12% for the sideslip angle and yaw rate, respectively. While GWO considerably reduces the estimation error, the corresponding RMSE and NRMSE values are still above those achieved by the proposed MDOA method. The lower error values obtained with the MDOA demonstrate its effectiveness in accurately tracking the vehicle states under the considered driving conditions.

Table 5. Comparison of state variables for FH maneuver.

Optimization Algorithms	RMSE Values		NRMSE Values	
	Sideslip angle (rad.)	Yaw Rate(rad./s)	Sideslip angle (rad.)	Yaw Rate(rad./s)
PSO	0.1601	0.1118	0.43%	0.15%
ACO	0.0487	0.0462	0.38%	0.12%
GWO	0.0028	0.0056	0.32%	0.07%
MDOA	0.0008	0.0010	0.21 %	0.03%

Figures 12 and 13 show the comparison of the sideslip angle and yaw rate estimations, respectively, for the DLC maneuver. The estimation obtained utilizing the PSO is moderately accurate; however, it generates a lag in the estimation during fast changes. ACO provides overshoots during transition time but tracks the reference signal well in comparison to PSO. GWO demonstrates less noise sensitivity and more closely tracks the reference signal. The highlighted time interval $5.0s \leq t \leq 6.6s$ clearly illustrates the comparison of all the algorithms during the dynamic change in steering.

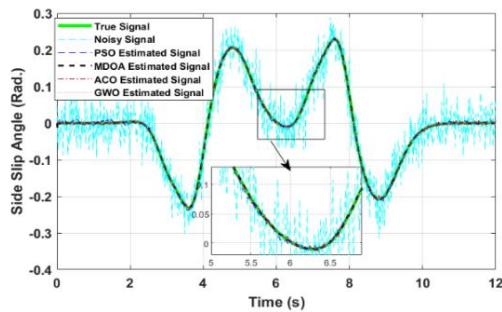
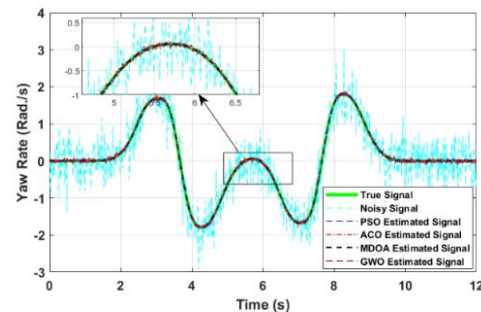
**Figure 12.** Comparison for DLC.**Figure 13.** Comparison for DLC.

Table 6 presents the RMSE and estimation accuracy for the sideslip angle and yaw rate during the DLC maneuver. The sideslip angle RMSE error values of PSO, ACO, GWO, and the MDOA are 0.1512, 0.0361, 0.0032, and 0.0011, respectively. Additionally, the NRMSE values for both the sideslip angle and yaw rate are lowest for the MDOA, thus indicating the better estimation in comparison to other algorithms.

Table 6. Comparison of state variables for DLC maneuver.

Optimization Algorithms	RMSE Values		NRMSE Values	
	Sideslip angle (rad.)	Yaw Rate(rad./s)	Sideslip angle (rad.)	Yaw Rate(rad./s)
PSO	0.1512	0.1212	0.41%	0.18%
ACO	0.0361	0.0391	0.33%	0.14%
GWO	0.0032	0.0049	0.29%	0.09%
MDOA	0.0011	0.0009	0.24%	0.03%

The robustness analysis of the MDOA based AUKF is evaluated by varying the values of the mass of the vehicle. Initially, the mass is considered as $m = 2020$ kg, and the value is subsequently changed to $m = 1800$ kg and $m = 2300$ kg. Figures 14 and 15 illustrate the robustness of the proposed method for state estimation when the mass reduced to 1800 kg. A reduction in value results

in more dynamic and stable responses to steering inputs, as the same force produces greater acceleration. Consequently, faster variations in lateral and yaw dynamics are observed, thus limiting the UKF's ability to track real states. A reduction in the vehicle mass leads to a slight increase in the estimation error. However, by re-tuning the process and measurement noise covariance matrices (Q and R) through the MDOA algorithm, the estimation performance is considerably improved. Under the optimized conditions, the RMSE values of the sideslip angle and yaw rate are reduced to 0.0012 rad and 0.0008 rad/s, respectively. The corresponding optimized covariance matrices are $[0.1, 0.0234]$ and $[0.1, 1e-06]$, respectively.

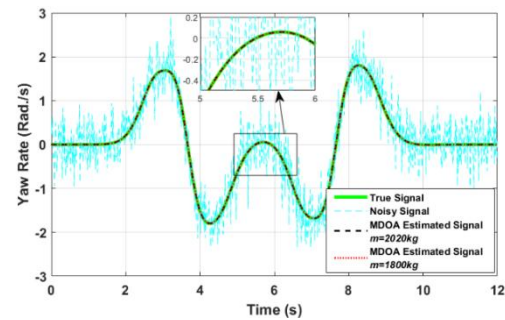
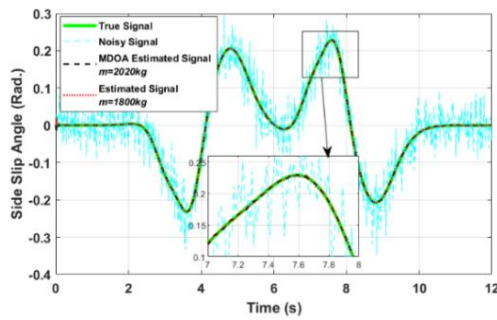


Figure 14. Robustness Analysis ($m = 1800$ kg). **Figure 15.** Robustness Analysis ($m = 1800$ kg).

Increasing the mass $m = 2300$ kg results in an increase in Q , as the system dynamics becomes more sensitive to mass related uncertainties. As Q increases, the AUKF expects higher system noise and relies more on measurements. Following the re-optimization of Q and R , the estimation performance considerably improves. Figures 16 and 17 illustrate the estimation results of the sideslip angle and yaw rate, respectively, for the DLC maneuver. The resulting RMSE values of the sideslip angle and yaw rate are obtained as 0.0013 rad. and 0.0007 rad/s, respectively. The optimized values of Q and R are obtained as $[0.0520, 0.0975]$ and $[0.1, 1e-06]$, respectively.

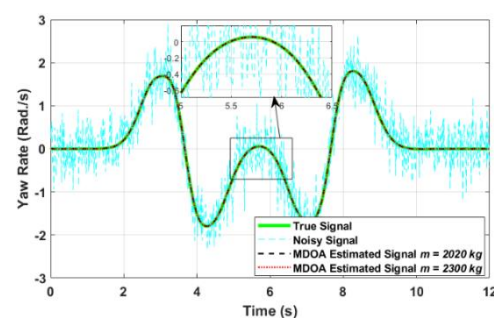
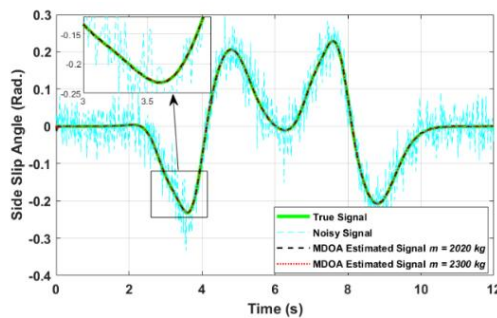


Figure 16. Robustness analysis ($m = 2300$ kg). **Figure 17.** Robustness analysis ($m = 2300$ kg).

The estimation accuracy primarily depends on the nonlinear state equations (Eqs. 10–13), in which the sideslip angle and yaw rate are influenced by tire forces and vehicle motion. As these equations are nonlinear, even small errors in the process noise (Q) or measurement noise (R) can affect the predicted states and their corrections. From the UKF equations (20–28), it is clear that (Q) affects the prediction step, while (R) affects the Kalman gain and correction step. If these matrices are not properly chosen, the estimated values may show oscillations or may not accurately follow the true states. This relationship is clearly reflected in Figures 6–9, where accurate tuning of Q and R results in smooth tracking of the true states. The absence of oscillations and overshoot in the estimated curves

indicates that the Kalman gain, governed by Eqs. (20–28), is optimally adjusted. In contrast, the improper tuning would lead to delayed response or fluctuations in the estimated signals.

In this work, the RMSE defined in Eq. (31) is minimized using the MDOA. As a result, the estimated signals closely track the true signals, while the effect of noise is reduced. The smooth estimation curves and fast response, especially during abrupt changes in the vehicle states, indicate that the filter properly balances the prediction and correction process. This demonstrates that the adaptive tuning of Q and R is functioning effectively. The numerical results in Tables 3 and 4 also support this observation. The observed improvement in estimation accuracy can be directly attributed to the adaptive optimization of the noise covariance matrices using the MDOA. Unlike conventional UKF approaches, where fixed or heuristically tuned covariance values are used, the proposed method dynamically adjusts these parameters according to the system behavior. This enables the filter to effectively respond to nonlinearities and time-varying uncertainties, particularly under low-friction road conditions. In comparison to adaptive filtering techniques such as the Sage–Husa and innovation-based UKF methods [10], which rely on statistical assumptions, the proposed method avoids dependence on prior noise characteristics and instead utilizes a performance-driven optimization framework. This results in improved robustness and reduced sensitivity to modeling errors. The lower RMSE and NRMSE values indicate that the difference between estimated and actual values is minimal. Additionally, it is observed that the error is slightly higher in the DLC maneuver compared to the FH maneuver. This is because the DLC involves more rapid steering inputs, which makes the system more nonlinear and difficult to estimate. Even in such conditions, the proposed method maintains good accuracy.

The comparison results shown in Tables 5 and 6 and Figures 10–13 depicts the advantage of the proposed method. The higher errors observed with PSO and ACO indicate that these methods are not able to effectively tune Q and R , which leads to larger deviations from the reference states. GWO performs better than PSO and ACO, but still shows some limitations. The proposed MDOA method performs best because it uses multiple behaviors such as separation, alignment, and attraction (Eqs. 29–30), which help explore the solution space more effectively and avoid suboptimal solutions. This leads to better tuning of the filter parameters and an improved estimation.

These findings are also consistent with observations reported in existing literature. However, compared to existing approaches such as IMM-AUKF [24] and adaptive fading UKF [25], the proposed method offers a distinct advantage by integrating metaheuristic optimization directly into the covariance tuning process. While IMM-based methods improve adaptability by switching between models, they increase the computational complexity and require accurate model selection. Similarly, Bayesian optimization-based tuning [9] provides effective parameter estimation but is generally limited to offline implementation. In contrast, the proposed MDOA-based framework enables continuous real-time optimization, thus resulting in enhanced adaptability under rapidly changing driving conditions. This highlights the novelty and practical relevance of the proposed approach within the broader field of vehicle state estimation.

The robustness analysis (Figures 14–17) further demonstrates how the method behaves under changing conditions. When the vehicle mass is reduced, the system responds faster, which initially increases the estimation error. When the mass is increased, the system becomes slower but introduces more uncertainty. In both cases, the proposed method accordingly adjusts Q and R and maintains good estimation accuracy. This shows that the method can effectively handle changes in the system parameter. The results indicate a strong relationship between the equations, optimization process, and

observed performance presented in the graphs and tables. The adaptive tuning of Q and R based on the RMSE minimization improves the estimation accuracy and stability of the system. Compared to earlier methods that depend on fixed parameters or assumptions, the proposed approach provides greater flexibility and reliability. From a practical perspective, accurate estimation of the sideslip angle and yaw rate is critical for vehicle safety systems. The improved accuracy seen in the results means better performances of systems such as stability control and driver assistance systems, especially in difficult conditions such as low-friction roads.

5. Conclusions

This paper presents a MDOA-AUKF for accurate and robust vehicle state estimation under nonlinear and low-friction driving conditions. The key scientific contribution of this work is the formulation of an adaptive process and measurement noise covariance tuning as an optimization problem and its solution using a MDOA incorporating elite solution preservation and multi-behavioral swarm dynamics. This strategy effectively balances exploration and exploitation, thereby enabling adaptive covariance tuning that improves the convergence, estimation accuracy, and robustness compared with conventional adaptive UKF approaches. The effectiveness of the proposed framework was demonstrated through FH and DLC maneuvers. During the FH maneuver, the proposed method achieved RMSE values of 0.0008 and 0.0010 for the sideslip angle and yaw rate, respectively, corresponding to NRMSE values of 0.21% and 0.03%, respectively, while the sideslip angle RMSE obtained using GWO, ACO, and PSO is 0.0028, 0.0487, and 0.1601, respectively. Similarly, during the DLC maneuver, the proposed approach achieved sideslip angle and yaw rate RMSE values of 0.0011 and 0.0009, respectively, with corresponding NRMSE values of 0.24% and 0.02%, respectively, outperforming GWO (0.0032), ACO (0.0361), and PSO (0.1512) for the sideslip angle estimation. Furthermore, a robustness analysis under vehicle mass variations from 1800 kg to 2300 kg demonstrated consistently low estimation errors, thus confirming the adaptability of the proposed estimator under parameter uncertainties. Additionally, the use of a simplified 2-DOF vehicle model ensures a low computational complexity, making the proposed framework suitable for real-time vehicle applications. Although the proposed method demonstrates superior estimation performance, the present study was limited to MATLAB-based simulations using a simplified vehicle model. Future work will focus on extending the estimator to higher-fidelity vehicle models, validating the approach using hardware-in-the-loop and real vehicle experiments, and integrating the proposed estimation framework with advanced control strategies, such as Model Predictive Control, for next-generation intelligent vehicle systems.

Author contributions

All authors have contributed equally towards the research gap, mathematical modeling and simulations work.

Use of Generative-AI tools declaration

The authors declare they have not used any generative Artificial Intelligence (AI) tools in the creation of this manuscript.

Acknowledgments

Authors would like to acknowledge Sarala Birla University, Ranchi, Jharkhand, India and Ramgovind Institute of Technology Koderma, Jharkhand, India for providing the facilities for the research work. There is no fund associated for this research work.

Conflict of interest

The authors declare that there is no conflict of interest in this manuscript.

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