

Research article

Incorporating systematic carbon risk into global business portfolios for carbon-intensive industries

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Abstract: In the contemporary economy, carbon-intensive multinational corporations must function as portfolio managers, balancing growth against non-diversifiable systematic carbon risk driven by globally synchronized policies like the Paris Agreement. Analyzing 181 jurisdictions from 1985–2024, I employed a double-index model to isolate carbon beta (a distinct climate transition sensitivity) from traditional market risk through hierarchical grouping and rolling window optimization. The findings demonstrated that risk-adjusted returns could be significantly enhanced by concentrating capital (half of the total portfolio weight) within the three sorted groups exhibiting the lowest systematic risk. Notably, the 2015 Paris Agreement acted as a structural pivot, shifting optimal strategies toward defensive concentration in low-carbon regions to mitigate regulatory volatility. Sensitivity analysis further revealed that the opportunity cost of ignoring carbon risk was highest during periods of low carbon-index volatility, providing a robust, analytically grounded framework for constructing resilient business portfolios in a carbon-constrained world.

Keywords: international business portfolios; growth potential; strategic carbon exposure; double index model; simple and extended criteria; risk-adjusted return

JEL Codes: C15, G11, G12, G13, G22, G33, H61, H80

1. Introduction

In the contemporary global economy, multinational corporations (MNCs) are increasingly conceptualized as portfolio managers who must strategically allocate business units across geographies to maximize growth while navigating non-diversifiable constraints. While traditional expansion strategies focused primarily on financial indicators like market share and return on investment, the rise of globally synchronized climate policies such as the Paris Agreement and carbon border adjustment mechanism (CBAM) has established carbon exposure as a fundamental systematic risk factor. Because these regulatory shocks exhibit high cross-country correlation, they represent a market parameter that affects firm value globally and cannot be neutralized through simple geographic diversification.

The central importance of this research lies in its framework for identifying and isolating global systematic carbon risk through a double-index model. By disentangling the specific carbon beta from traditional market beta, the model enables MNCs to treat climate transition sensitivity as a distinct, manageable dimension of their global operations. This isolation is essential for strategic capital allocation, enabling firms to navigate an optimal frontier where any increase in carbon-intensive exposure must be justified by a proportional increase in risk-adjusted returns. Through a two-layer grouping procedure and rolling window optimization, the framework captures the non-stationary nature of transition risks, enabling the portfolio to adapt to evolving regulatory regimes and structural discontinuities.

Empirical results derived from a 40-year analysis of 181 countries demonstrate that proactively rebalancing international exposure significantly improves a firm's risk-adjusted performance. The optimal portfolio achieves this by shifting weights toward low-risk jurisdictions, concentrating roughly half of the total portfolio weight within the three groups exhibiting the lowest systematic risk. Furthermore, the analysis identifies the 2015 Paris Agreement as a pivotal structural shift that stabilized carbon risk exposures. In the subsequent period, carbon-aware optimization achieved a statistically significant reduction in risk and a move toward defensive concentration in low-carbon jurisdictions. Ultimately, isolating systematic carbon factors provides a rigorous, analytically grounded solution for MNCs to build resilient business portfolios in an increasingly carbon-constrained world.

2. Literature review

2.1. Foundations of modern portfolio theory in global strategy

Modern portfolio theory (MPT), established by Markowitz (1952), provides the mathematical foundation for balancing expected returns against risk, defined as the variance of returns. Elton and Gruber (1997) emphasized that the core of this theory is the insight that assets cannot be selected based on unique characteristics alone; rather, an investor must consider how each security co-moves with others to construct an efficient frontier. In the context of international business, MNCs are conceptualized as portfolio managers who must strategically allocate business units across geographies to optimize global growth potential against systematic constraints. Moreover, scholars like Buckley and Casson (2020) argued that the headquarters function must pragmatically reconfigure internal and

external resources to respond to a volatile global environment, effectively treating international expansion as a complex collection of assets requiring sophisticated risk-return trade-offs.

2.2. The emergence of carbon risk as a systematic factor

Climate change introduces new dimensions of risk that the Task Force on Climate-related Financial Disclosures (2017) categorizes into physical risk (direct damage from acute or chronic weather events) and transition risk (risks arising from the process of adjustment toward a lower-carbon economy). Transition risk is driven by globally synchronized shifts in policy, technology, and social preferences. Bos and Gupta (2019) defined stranded assets as investments that lose economic value ahead of their anticipated useful life due to such legislative or market shocks. Because these regulatory interventions, such as the Paris Agreement and CBAM, apply across borders and exhibit high cross-country correlation, carbon exposure has transitioned from an idiosyncratic firm concern into a non-diversifiable systematic risk factor.

2.3. Quantifying carbon sensitivity: The BMG factor and carbon beta

To measure this systematic dimension, Görgen et al. (2020) and Roncalli et al. (2020) proposed the construction of a Brown-Minus-Green (BMG) factor-mimicking portfolio. This zero-cost portfolio is long in carbon-intensive (brown) firms and short in low-carbon (green) firms, capturing the market's actual pricing of transition risks. The sensitivity of an asset to this factor is defined as its carbon beta. Empirical studies by Bolton and Kacperczyk (2021) and Hsu et al. (2023) revealed a pollution premium, where investors demand higher returns for high-emitting firms to compensate for their exposure to policy regime shift risks. Ilhan et al. (2021) further demonstrated that this risk is priced into option markets, particularly as protection against downside tail risks.

2.4. Integrating MNC decision-making and capital allocation

The literature suggests that multinational firms have not fully incorporated climate risks into their location decisions, which Gu and Hale (2023) argued creates a potential for large and abrupt capital reallocations as transition shocks intensify. Li and Gallagher (2022) further concluded that little is known about how MNCs incorporate specific physical and transition risks into overseas investment strategies. By failing to isolate the specific carbon-driven risk premium, firms cannot target transition sensitivity for strategic rebalancing. In this study, I address these gaps by utilizing a double-index model to isolate jurisdictional carbon risk factor exposure across 181 countries, providing an analytically grounded framework for constructing resilient business portfolios in an increasingly carbon-constrained world.

2.5. Identifying the research gap: Isolation of global systematic carbon risk

Despite these advancements, a critical research gap remains regarding the identification and isolation of global systematic carbon risk for constructing resilient multinational business portfolios.

Most empirical studies, such as those by Bolton and Kacperczyk (2021), remain primarily firm-centric, focusing on listed companies within developed market indices. There is a profound lack of comprehensive research establishing a jurisdiction-level systematic risk factor applicable to a broad spectrum of global economies. Furthermore, Roncalli et al. (2020) noted that the financial market often conflates fundamental metrics (like carbon intensity) with market-based risk. There is a vital need to statistically isolate carbon beta from traditional market beta to enable MNCs to treat climate transition sensitivity as a distinct, manageable dimension of their global operations.

3. Analysis

3.1. Research structure

3.1.1. Theoretical background

My primary purpose of this analysis is to adapt MPT to the strategic management of carbon-intensive MNCs. Under this framework, MNCs are conceptualized as global portfolio managers who must balance the growth potential of business units, analogous to expected returns, against their strategic carbon exposure, which serves as a systematic risk factor. Portfolio efficiency is achieved by navigating an optimal frontier where marginal increases in carbon exposure are justified only by sufficiently higher expected growth. This enables a rigorous evaluation of the trade-offs between high-growth, carbon-intensive emerging markets, and lower-growth, resilient jurisdictions.

The significance of identifying global systematic carbon risk lies in its non-diversifiable nature. Although idiosyncratic risks can be neutralized through geographic expansion, systematic carbon risk can arise from global policy synchrony, such as the Paris Agreement and CBAM. Because these regulatory shocks exhibit high cross-country correlation, they represent a fundamental market parameter that affects firm value across all jurisdictions. For instance, climate policy shocks (like a sudden carbon tax) create liquidity drains and correlated asset devaluations. Even green assets are subject to the resulting systematic volatility and shifts in the equity risk premium. Additionally, carbon transition involves structural shifts in energy prices, supply chains, and inflation (e.g., greenflation). These factors impact the discount rates and input costs of all firms, regardless of their individual carbon footprint. Formally demonstrating this non-diversifiable nature requires isolating the commonality of these risks to show they persist even within a globally diversified portfolio.

To capture transition risk dynamics, the model acknowledges that systematic carbon risk shocks are inherently path-dependent and non-linear. The impact of carbon policies in specific jurisdictions propagates over time, reflecting the evolving nature of stranded asset risk. When these non-linear shocks cause the systematic risk factor to exhibit non-linear dynamics, the resulting country-specific growth rates follow suit. While many models treat these as stable factors, I employ rolling window regression analysis to capture these regime shifts and evolving sensitivities, providing a more realistic representation of the fluid climate policy landscape.

The technical implementation utilizes factor analysis via the Double-Index Model to isolate this carbon-driven risk premium. By defining country-specific growth rates as a linear combination of systematic factors, the model disentangles the global carbon risk from traditional market risk. This

isolation is essential for strategic rebalancing, as it enables MNCs to proactively reweight their international exposures, divesting from high-gamma assets or investing in low-carbon innovations, much like dynamic portfolio rebalancing in financial markets. This ensures that the methodological workflow of the paper can quantify a MNC's vulnerability to global transition shocks.

3.1.2. Empirical design

My focus of this analysis is placed on the specific analytical workflow of the double-index model. This approach prioritizes the identification, isolation, and integration of systematic carbon risk into a rigorous portfolio optimization framework for multinational corporations.

For the research design, I employ a double-index model that treats systematic carbon risk not as absolute emissions, but as a global risk factor derived from a factor-mimicking portfolio. To address cross-country heterogeneity, a two-step hierarchical clustering method is used to classify 181 jurisdictions into distinct groups based on their systematic risk sensitivities. This enables the model to capture the non-linear adaptation characteristics of various jurisdictions as they move from voluntary disclosure to mandatory compliance regimes. By structuring the carbon index as a zero-cost portfolio (long low-carbon/short high-carbon), the design ensures the factor reflects the carbon policy-driven risk premium independent of broad market fluctuations.

A critical component of the research design is the statistical isolation of the carbon risk factor from the market risk factor. While economic growth and carbon emissions are historically coupled, this model utilizes a Gram-Schmidt orthogonalization process via OLS regression. By regressing the market index against the carbon risk index and extracting the residuals, the design creates two mutually orthogonal factors. This ensures that the carbon beta (γ) estimated for each group represents a distinct parameter of transition risk exposure, preventing multicollinearity and enabling precise strategic hedging in the optimization phase.

The double-index model is flexible enough to incorporate other systemic risk factors beyond carbon risk. For example, it is possible to construct a zero-cost portfolio based on national credit ratings, adjust it to be orthogonal to other factors, and then incorporate that variable into the model as another systematic risk factor. However, the corresponding decision rules for such adjustments become more complex, making it difficult to derive and present closed-form solutions, potentially necessitating reliance on mathematical optimization.

To account for the non-stationary nature of climate transition risks, such as sudden carbon tax increases or regime shifts following the Paris Agreement, the design utilizes a rolling window optimization technique. Instead of a static 40-year average, the model employs a 20-year look-back period that updates at each optimization point. Between 2004 and 2024, the model repeats 21 iterations of portfolio rebalancing. This ensures that the input parameters (means and covariances) reflect evolving regulatory environments, capturing structural discontinuities that a standard stationary model would overlook.

The research design evaluates the efficacy of carbon risk integration using two major metrics: Effective Number of Constituents (ENC) and Shannon Entropy. These metrics enable a quantitative comparison between the optimal portfolio (carbon-integrated) and the suboptimal portfolio (market-only). To ensure the reliability of these empirical results, the design generates heteroskedasticity-

consistent standard errors and t-statistics through the rolling regression framework. This enables a robust assessment of whether reallocating capital toward low-gamma jurisdictions significantly alters the portfolio's breadth and resilience against systematic transition shocks.

3.2. Empirical methodology

3.2.1. Double-index model

The single-index model (Elton et al., 1976) can be modified to be the double-index model (Jin, 2022b, 2022c) by incorporating the systematic ESG risk factor. Here, I utilize the double-index model while replacing the systematic ESG risk with the systematic carbon risk and applying it to countries instead of funds. It is assumed that the following double-index model is an adequate description of reality:

$$R_i = \alpha_i + \beta_i I_m + \gamma_i I_s + e_i \quad \text{for } i = 1, \dots, N \quad (1)$$

$$I_m = \alpha_m + e_m \quad (2)$$

$$I_s = \alpha_s + e_s \quad (3)$$

$$E(e_m e_s) = 0 \quad (4)$$

$$E(e_m e_i) = 0 \quad \text{for } i = 1, \dots, N \quad (5)$$

$$E(e_s e_i) = 0 \quad \text{for } i = 1, \dots, N \quad (6)$$

$$E(e_i e_j) = 0 \quad \text{for } i = 1, \dots, N \quad ; \quad j = 1, \dots, N \quad ; \quad \text{and } j \neq i \quad (7)$$

where R_i = the return on country i (a random variable)¹

I_m = the return on a market risk factor (a random variable)

I_s = the return on a carbon risk factor (a random variable)

β_i = the responsiveness of country i to changes in the market risk factor

γ_i = the responsiveness of country i to changes in the carbon risk factor

α_i = the return on country i that is independent of both changes in the market risk factor and changes in the carbon risk factor

α_m = the mean of returns on the market risk factor

α_s = the mean of returns on the carbon risk factor

e_i = a variable with a mean of zero and variance σ_{ei}^2

e_m = a variable with a mean of zero and variance σ_m^2

e_s = a variable with a mean of zero and variance σ_s^2

σ_{ei}^2 = the variance of returns on country i 's idiosyncratic component

σ_m^2 = the variance of returns on the market risk factor

σ_s^2 = the variance of returns on the carbon risk factor

In our setting, the last four equations characterize the approximation of the double-index model to the covariance structure. The assumption implied by these equations is that the only joint movement between countries comes about because of common responses to two systematic risk factors.

¹ Note that returns are used interchangeably with growth rates in this analysis.

3.2.2. Proxies for systematic risk factors

The double-factor model identifies two systematic risks: market risk and carbon risk. To identify proxies for these factors, the growth rate of the global economy is used as a proxy for overall market risk. Next, the proxy return of the systematic carbon risk factor can be constructed as a zero-cost portfolio (Bolton & Kacperczyk, 2021; Jin, 2018, 2022a). By taking long positions in low-carbon countries and short positions in high-carbon countries, the common growth component is neutralized, focusing the index on the carbon policy-driving risk premium. The 181 sample countries are sorted in descending order by carbon intensity (carbon emissions as a percentage of GDP). The 90 most (least) carbon-intensive countries are categorized as the high-emitting (low-emitting) group. The growth rate of each group is calculated by weighing the size of each country's economy, and the growth rate of the low-emitting group is subtracted from that of the high-emitting group to derive the return on systemic carbon risk. The systematic carbon risk represents the market movement triggered by the aggregate of carbon issues only. The assumption $\sigma_{ms} = 0$ indicates that two systematic risks should be orthogonal to each other. For the assumption, returns on the market risk index are regressed for returns on the carbon risk index, and then the residuals plus the alpha estimate are derived as the adjusted market risk factor.

Regarding the orthogonality of risk factors, I acknowledge that while raw carbon emissions are historically coupled with economic growth, the double-index model utilizes a factor-mimicking portfolio designed to isolate the carbon policy-driven risk premium from broader market movements. Rather than assuming that these factors are naturally independent, the methodology treats them as latent components that must be disentangled to enhance portfolio optimization. By taking long positions in low-carbon jurisdictions and short positions in high-carbon counterparts, the common growth component is neutralized, enabling the resulting index to focus on the systematic carbon risk premium. To ensure statistical rigor and eliminate multicollinearity, I identify two mutually orthogonal factors by performing a Gram-Schmidt process through OLS regression analysis, where the market index is regressed against the carbon risk index to extract the residual, purely idiosyncratic market growth component. This orthogonalization is essential for the extended decision rule, as it enables the precise estimation of carbon beta (gamma) as a distinct parameter for strategic risk hedging.

3.2.3. Decision rules for an optimal portfolio

Like the simple criteria (Elton et al., 1976, 1978) and the extended criteria (Jin, 2022b), the portfolio with the highest excess return to standard deviation, the Sharpe ratio, is explored. The criteria first find an unconstrained vector of the relative weights for each group to maximize the portfolio's Sharpe ratio. Then, these weights are scaled to ensure that they are fully invested. Additionally, it is assumed that a riskless asset exists, and short sales are not allowed. When short sales are not allowed, Kuhn-Tucker conditions are required to determine an optimum. If unlimited lending and borrowing can take place at rate (R_f), the optimum amount to invest in group i is determined as:

$$X_i^* = \frac{Z_i}{\sum_{j \in k^*} Z_j} \quad \text{for all } Z_i > 0 \quad (8)$$

where

$$\pi^* = (\beta_i; \gamma_i; \sigma_{ei}^2; \alpha_m; \alpha_s; \sigma_m^2; \sigma_s^2) \text{ are estimated from the double-index model.} \quad (9)$$

$$k^* \text{ is the set of groups in the optimal portfolio under the double-index model.} \quad (10)$$

$$Z_i = \frac{\omega_i}{\sigma_{ei}^2} (RAR_i - \Phi_k) \quad (11)$$

$$\omega_i = \sqrt{\beta_i^2 \sigma_m^2 + \gamma_i^2 \sigma_s^2} \quad (12)$$

$$RAR_i = \frac{(\bar{R}_i - R_f)}{\omega_i} \quad (13)$$

$$\Phi_k = \frac{\beta_i \sigma_m^2}{\omega_i} \left(\frac{\phi_{1,k} - \phi_{2,k} \phi_{3,k}}{1 - \phi_{2,k} \phi_{4,k}} \right) + \frac{\gamma_i \sigma_s^2}{\omega_i} \left(\frac{\phi_{3,k} - \phi_{1,k} \phi_{4,k}}{1 - \phi_{2,k} \phi_{4,k}} \right); \quad \phi_{1,k} = \frac{\sum_{j \in k} [(\bar{R}_j - R_f) \beta_j / \sigma_{ej}^2]}{1 + \sum_{j \in k} (\beta_j^2 \sigma_m^2 / \sigma_{ej}^2)}; \quad (14)$$

$$\phi_{2,k} = \frac{\sum_{j \in k} (\beta_j \gamma_j \sigma_s^2 / \sigma_{ej}^2)}{1 + \sum_{j \in k} (\beta_j^2 \sigma_m^2 / \sigma_{ej}^2)}; \quad \phi_{3,k} = \frac{\sum_{j \in k} [(\bar{R}_j - R_f) \gamma_j / \sigma_{ej}^2]}{1 + \sum_{j \in k} (\gamma_j^2 \sigma_s^2 / \sigma_{ej}^2)}; \quad \text{and} \quad \phi_{4,k} = \frac{\sum_{j \in k} (\beta_j \gamma_j \sigma_m^2 / \sigma_{ej}^2)}{1 + \sum_{j \in k} (\gamma_j^2 \sigma_s^2 / \sigma_{ej}^2)}$$

The detailed derivation of the ranking device is presented in Jin (2022a) and the Appendix. A summary of the decision rules is as follows: First, all groups are ranked from 1 to N by the excess return to systematic risk ratio RAR_i : highest to lowest. Then, I compute a value for the cut-off point Φ_k as if the set k contains only the first group. Next, Z_i is calculated while setting $i = 2$. If $Z_2 < 0$, the algorithm stops. The optimal portfolio contains only the first group. Otherwise, the cut-off point is computed while letting the set k contain the first two groups. The algorithm proceeds for $i = 3, 4, \dots, N$ until Z_i turns negative. If it turns negative for the $j+1$ st group, then set k^* contains the first j groups. Set k^* arriving this way satisfies the Kuhn-Tucker conditions, as described in Elton, Gruber, and Padberg (1976; 1978). Thus, all groups in set k^* can be identified in this way. Once these groups are found, the X_i^* value for each group in the set can be determined by calculating the Z_i for each group and dividing each Z_i by the sum of the Z_i over set k^* . Note that the desirability of any group and the amount invested in it will be uniquely related to its RAR_i . Although the decision rules are the same as those described in Elton et al., (1976; 1978), the original simple criteria can be considered a particular case of the extended ones, as explained in Jin (2022a).

While I utilize static mean-variance optimization as a foundational framework, this choice is a purposeful starting point designed to address the literature gap regarding global carbon beta estimation and the isolation of systematic carbon risk. I recognize that a static approach may not fully encapsulate the fluid, regime-shifting nature of climate policy; however, the methodology mitigates this by employing a rolling window technique that enables risk parameters to update over time. To further align the technical implementation with the paper's dynamic conceptual claims, the framework could be transitioned to a state-space model utilizing a Kalman filter, which enables the global carbon beta to evolve continuously in response to national regulatory shifts. Furthermore, the model can be extended into a multi-period stochastic optimization framework, ensuring that strategic capital allocation remains resilient across the varying transition paths and long-term horizons characteristic of carbon-intensive global business strategies.

The double-index model fundamentally transcends uniform risk assumptions. As revealed through its extended criteria, this model quantifies cross-country differences (such as national risk preferences regarding global systemic carbon risk) via gamma estimates. This parameter weights the likelihood of abrupt regulatory shifts; for instance, high-commitment nations (e.g., the EU) exhibit lower sensitivity (carbon beta), reflecting their proactive transition efforts. As the gamma parameter is periodically re-estimated, countries may be reclassified into different groups during the grouping phase if changes occur in their national carbon intensity (reflecting emissions-GDP decoupling rates). This analytical process ensures that the model reflects the geopolitical reality that carbon risk is perceived and priced differently across sovereign borders. As a result, adjustments to the carbon risk premium are allowed, based on economic transition resilience (i.e., a country's structural capacity to absorb transition shocks without systemic default).

3.2.4. Workflow

Incorporating the systematic carbon risk factor can cause a material change in the optimal portfolio composition and its performance. The impact can be examined by following three steps: estimating parameters, combining countries into sorted groups, and constructing optimized portfolios. As the first step, the systematic risk exposure is estimated as the coefficient of a time-series regression for a 20-year estimation period. In the double-index model, equation (1) is estimated for each i of 181 sample countries, and the parameter estimates of $\pi^* = (\hat{\alpha}_i, \hat{\beta}_i, \hat{\gamma}_i, \hat{\sigma}_{ei}^2, \hat{\alpha}_m, \hat{\alpha}_s, \hat{\sigma}_m^2, \hat{\sigma}_s^2)$ are collected. In the single-index model, equation (19) without the carbon risk factor is estimated, and the parameter estimates of $\pi^o = (\hat{\alpha}_{o,i}, \hat{\beta}_{o,i}, \hat{\sigma}_{o,ei}^2, \hat{\alpha}_{o,m}, \hat{\sigma}_{o,m}^2)$ are collected:²

$$R_i = \alpha_{o,i} + \beta_{o,i}I_{o,m} + e_{o,i} \quad \text{for } i = 1, \dots, N \quad (15)$$

As the next step, sorted groups are constructed to be used in place of individual countries to alleviate the errors-in-variables problem. For the single-index model, the standard procedure in the empirical literature (Fama & MacBeth, 1973; Jensen et al., 2006) is followed. According to the market risk exposure $\hat{\beta}_{o,i}$, 40 beta-sorted groups of countries are formed. For the double-index model, a two-layer grouping criterion is employed to accommodate the systematic carbon risk (Jin, 2022a). Specifically, the combined systematic risk metric $\hat{\kappa}_i$ is defined as:

$$\hat{\kappa}_i = \sqrt{(\hat{\beta}_i^2 \hat{\sigma}_m^2 + \hat{\gamma}_i^2 \hat{\sigma}_s^2) / (\hat{\sigma}_m^2 + \hat{\sigma}_s^2)} \quad (16)$$

The sample countries are divided into 20 kappa-sorted groups at the first layer according to the kappa estimates $\hat{\kappa}_i$. Countries within each kappa-sorted group are then sorted into two gamma-sorted sub-groups at the second layer, according to the gamma estimates $\hat{\gamma}_i$. The two-layer procedure results in 40 sorted groups. Note that $\hat{\kappa}_i = \hat{\beta}_{o,i}$ in the single-index model since $\hat{\sigma}_s^2$ is restricted to zero. Moreover, note that 20 kappa-sorted groups at the first layer are identical between single-index and double-index cases, enabling the comparison between the optimized results in the double-index model with those in the single-index model. When necessary, the systematic risk metrics are averaged with economy size weights across countries within each group j to obtain $\hat{\beta}_j$ and $\hat{\gamma}_j$.

² Note that we add sub-scripts “o” in the single-index model for comparative purposes.

As the last step, two optimized portfolios are constructed. In the double-index model, the extended criteria are applied to 40 groups sorted by two systematic risk exposures, the set of groups k^* is selected, and the weight of each group X_i^* is calculated. Then, the optimized portfolio P^* is constructed using the weights X_i^* , and portfolio returns R_P^* are computed by combining the weights with parameter estimates π^* . Similarly, in the single-index model, the original simple criteria are applied to 40 groups sorted by the only market risk exposure $\hat{\beta}_{o,i}$ to derive the corresponding metrics: k^o , X_i^o , P^o , and R_P^o using π^o .

In this paper, I compare the double-index model against a single-index model (the CAPM replication model) as a benchmark. The impact of carbon risk integration can be identified by investigating the difference between two optimized portfolios: P^o and P^* . Since it is assumed that the double-index model adequately describes reality, the optimized portfolio in the double-index model is named the optimal portfolio. For comparison, the optimized portfolio in the single-index model is labeled as the quasi-optimal portfolio because it does not consider systematic carbon risk exposure when appropriate. Regarding the decision rule for the optimal portfolio, the systematic carbon risk factor can cause a change in the excess return to systematic risk ratio and the optimal cut-off point. The impact on both aspects can be identified by checking the difference in weights and the risk-adjusted returns.

To complement the static mean-variance framework, I employ a rolling window optimization technique designed to capture the non-stationary evolution of systematic carbon risk while maintaining statistical rigor. Rather than if risk parameters remain constant over the 40-year horizon, the model utilizes a 20-year look-back period to estimate risk exposures relative to each optimization point. This dynamic approach enables the carbon beta and market sensitivities to adapt to sudden regime shifts, such as abrupt carbon tax implementations or the global policy realignment following the Paris Agreement. Furthermore, the impact of technological innovation investments in high-carbon industries is primarily reflected in changes to the country's sensitivity to systemic carbon risk. Over time, once technology has sufficiently spread to many other countries, this impact transforms into changes in the factors of systemic carbon risk. Beginning with the first optimization point in 2004, the process is repeated for 21 iterations through to the final point in 2024, ensuring that the resulting portfolio weights reflect the most recent transition paths and structural discontinuities inherent in the green transition.

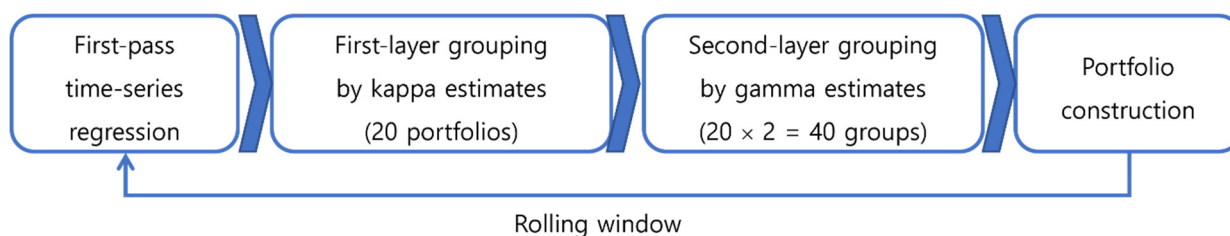


Figure 1. Workflow.

Figure 1 symbolizes the workflow. First, the adjusted market risk factor exposure ($\hat{\beta}_i$) and systematic carbon risk factor exposure ($\hat{\gamma}_i$) from the first-pass time-series regression are used. Next, a two-layer grouping is conducted based on these risk exposures. Then, portfolio optimization for the

40 groups is performed via the extended decision rule in Equations (12)–(18). Finally, the rolling window optimization technique is applied.

3.2.5. Methodological positioning relative to modern literature

Climate-related transition risk is characterized by path-dependency, nonlinearity, and fat-tailed distributions that traditional Gaussian models often struggle to capture. In the context of global business portfolios, the impact of carbon policies in specific jurisdictions propagates across the network over time, creating stranded asset risks that are inherently skewed and prone to sudden regime shifts. While the double-index model assumes a linear relationship between country-specific growth and systematic factors, these complex statistical characteristics are reflected through the dynamics of the systematic carbon risk factor itself. These sophisticated dynamics can be partially captured through rolling window regression analysis, though they may necessitate the use of path-dependent Markov-switching models to fully account for the non-linear shocks inherent in global climate regulation.

To better situate this research within contemporary literature, several sophisticated techniques offer promising avenues for integration or comparison. Bayesian frameworks, such as the Black-Litterman approach (Black & Litterman, 1992), could combine historical carbon data with forward-looking transition scenarios to mitigate estimation errors. Furthermore, replacing standard variance with Conditional Value-at-Risk (CVaR) would enable optimization specifically against catastrophic left-tail losses and stranded asset risks. Integrating entropy-based constraints or Second-Order Stochastic Dominance (SSD) frameworks would further enhance robustness against concentration risk and model ambiguity, ensuring the portfolio is diversified across uncorrelated transition paths rather than being overly exposed to a few green jurisdictions.

Despite the availability of these modern frameworks, the double-index model remains highly appropriate as it is designed to identify and isolate the systematic carbon risk factor (γ) from traditional market risk (β). A genuine performance comparison with entropy or SSD-based models is feasible only once systematic carbon factors are isolated within those broader frameworks. In this study, I demonstrate that the double-index approach accurately captures the impact associated with carbon shocks (e.g., 2015 Paris agreement) through the rolling-window technique. By providing a clear methodological workflow to quantify carbon beta, the model offers a practical and analytically grounded solution for multinational corporations to manage nonlinear transition risks in an increasingly carbon-constrained world.

3.3. Data

3.3.1. Country data

My data comprises a sample of 181 countries retrieved from the World Bank Development Indicators in December 2025. Focusing on the countries that have observations on GDP and carbon emissions for 40 years provides more precision to the research results by isolating business strategies exposed to country risk factors. The data consist of the total annual growth rates, GDP weights, and carbon intensities over 40 years (1985–2024).

3.3.2. Systematic risk factors

The set of systematic risk factors comprises a pair of indices: the market risk index (growth rates of the world economy) and the carbon risk index (returns on a zero-cost portfolio). Descriptive statistics of their returns are presented in Table 1. The first column shows that the average return of the high-carbon economy is materially larger than that of the low-carbon economy. In addition, the average return of the carbon risk index is smaller than that of the market risk index. In the next column, the return volatility of the carbon risk index is smaller than that of the market risk index, indicating that the orthogonalization of the two systematic risk factors removes some of the common drivers of volatility. The table also shows that the skewness and kurtosis³ of the carbon risk index are close to zero, while the statistics for market risk are significantly different from zero.

Table 1. Systematic risk indices.

Index	Mean	SD	Skew	Kurt	S-W	K-S	K-S2
Market risk index	3.11	1.49	−2.10	7.79	0.79 ***	0.20 *	---
High-emission index	4.00	1.48	−1.17	3.59	0.91 ***	0.14	0.43 ***
Low-emission index	2.03	1.70	−1.82	6.14	0.81 ***	0.16	0.45 ***
Carbon risk index	2.55	1.02	0.05	0.97	0.97	0.14	0.45 ***

***, **, and * denote statistical significance at the 1%, 5%, and 10% level.

As an extension of this difference, I conduct normality tests to evaluate the unique distributional properties of both risk factors. While the Shapiro–Wilk test rejects normality for the market risk index, highlighting potential skewness or kurtosis, it does not reject it for the carbon risk index, suggesting a more symmetric return profile for that specific factor. To provide a more robust comparison, I also implement Kolmogorov–Smirnov (K-S) tests, which are less sensitive to extreme tail deviations; these results indicate that the standardized returns of the market risk index do not significantly deviate from a standard normal distribution in their central tendency. Furthermore, a secondary suite of K-S2 tests is conducted to demonstrate that the carbon risk and market risk factors possess statistically distinct distributions, confirming that the systematic carbon risk factor captures a unique dimension of volatility that justifies its isolation in the double-index model.

Table 1 provides the descriptive statistics and distributional analysis for the four indices analyzed over 40 years (1985–2024): the market risk index, high-emission index, low-emission index, and the carbon risk index. It reports the annual mean return, standard deviation (SD), skewness (Skew), and kurtosis (Kurt), alongside the results of Shapiro–Wilk (S-W) and Kolmogorov–Smirnov (K-S) normality tests.

3.3.3. Exposures to systematic risk factors

Figure 2 presents the annual distribution of market risk exposure (beta) for the 40 sorted groups from 2004 to 2024, representing their responsiveness to the global market risk factor. Derived from first-pass time-series regressions, the violin plots reveal that while median sensitivities typically cluster

³ We use Fisher's definition of kurtosis, and thus the kurtosis of a normal distribution is zero. Kurtosis figures are normalized by $N-1$.

near 1.0, the sample contains significant variability, with some groups exhibiting extreme sensitivities above 4.0, particularly around 2007, and others showing negative responsiveness. The vertical dotted line in 2015 serves as a reference point for observing shifts in systematic risk, highlighting the diverse risk profiles that form the basis for the subsequent portfolio optimization workflow.

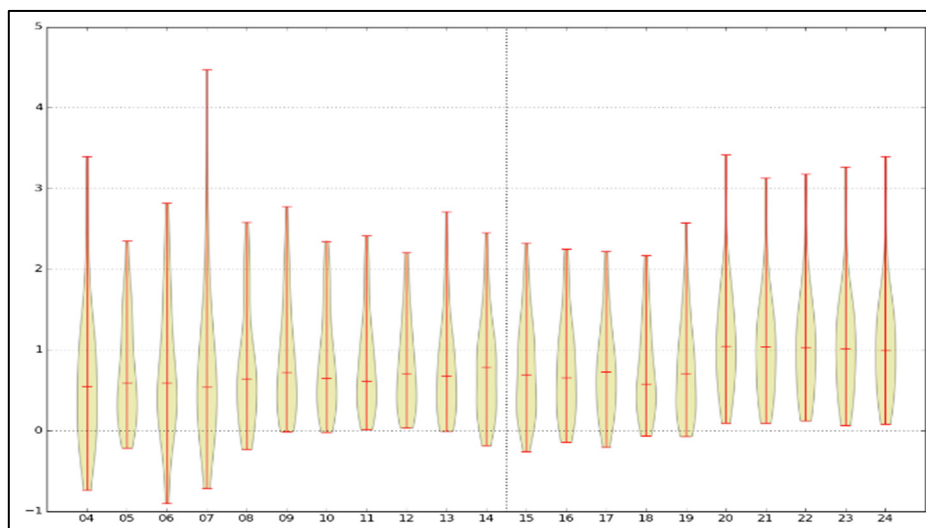


Figure 2. Distribution of market risk exposure (beta) by year.

Figure 3 illustrates the cross-sectional distribution of systematic carbon risk exposure (gamma), which measures group-level responsiveness to a zero-cost carbon risk factor constructed from the spread between high-and low-emission groups. The data shows a wide range of carbon sensitivities in the early years (2004–2008), with extreme values reaching above 6.0 and below -2.0 , followed by a notable trend toward convergence and stabilization after 2015. These gamma estimates are essential for the two-layer grouping procedure, as they enable countries to be further categorized into gamma-sorted sub-groups to more effectively manage non-diversifiable environmental risks.

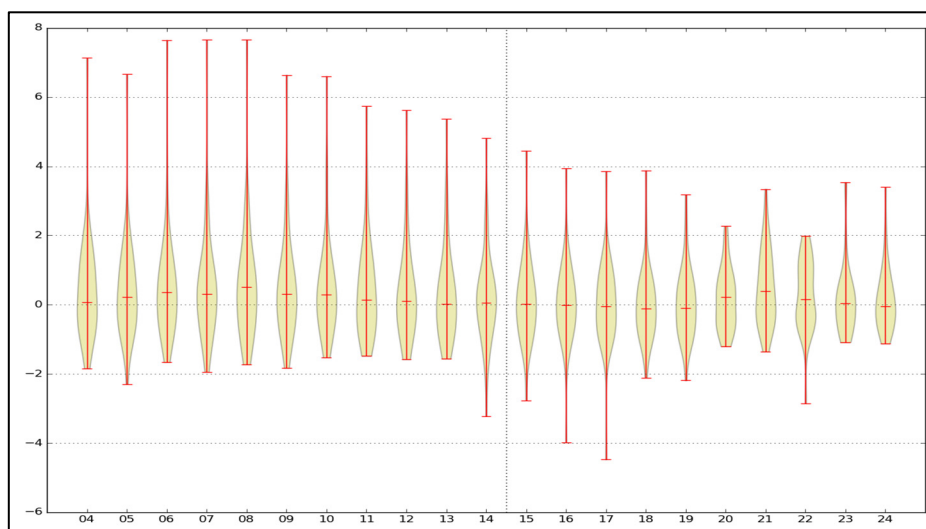


Figure 3. Distribution of systematic carbon risk exposure (gamma) by year.

3.4. Empirical results

3.4.1. Time-varying exposures to systematic risk factors

As shown in Figures 2 and 3, exposure to systematic risk factors can fluctuate over time, so a rolling window technique is applied.⁴ Specifically, exposure in 2004 is estimated using data from the preceding 20 years (1985–2004), and grouping is performed based on this exposure. Then, moving forward one year at a time, exposure in 2005 is estimated using data from 1986 to 2005 and used for grouping at that point. This method estimates exposure for a total of 21 years (2004–2024), thereby constructing 21-year panel data for 181 countries. Finally, a random effects panel regression analysis is conducted to determine whether each of the three exposures (beta, gamma, and the systematic risk indicator kappa) changed around 2015, when the Paris Agreement was adopted. In this analysis, the dependent variable is the estimated exposure value, and the independent variables consist of a constant term and a dummy variable set to 1 for the period after 2015. Table 2 below presents the estimation results of the panel regression analysis. When accounting for country-level heterogeneity through random effects, the results suggest that while beta increased around 2015, gamma and kappa decreased.

Table 2. Shift of exposures to systematic risk factors.

Variable	Beta		Gamma		Kappa	
Constant	0.858	***	0.255		1.183	***
Dummy (since 2015)	0.092	***	−0.129	***	−0.084	***

***, **, and * denote statistical significance at the 1%, 5%, and 10% level.

Table 2 presents the estimation results of a random effects model, applied to exposure panel data for 181 countries, which are estimated annually over 21 years (2004–2024) using observations from the preceding 20 years. Country-specific heterogeneity is assumed via random effects.

3.4.2. Weight comparison between optimized portfolios

Integrating the carbon risk factor into portfolio optimization can alter the optimized weights across sorted groups. By examining the difference in optimized weights, it can be shown that systematic carbon risk integration has a material influence. To make the comparison feasible, once weights for 40 sorted groups are determined through the optimization process, the weights for 20 first-layer kappa-sorted groups are computed by summing the weights of the two second-layer gamma-sorted sub-groups belonging to each first-layer kappa-sorted group. Now, 20 constituents are identical between the double-index model and the single-index model, but those constituents have different weights.

The boxplot in Figure 4 depicts the distribution of differences between the optimal portfolio weights (X^*) and quasi-optimal weights (X^0) for each of the 20 kappa-sorted groups, where a positive value indicates that the optimal portfolio increases the weight of that group. Consistent with the theoretical shift toward lower systematic risk, the groups with the lowest kappa estimates, specifically

⁴ The cross-country variance observed in each exposure may vary by year, and the dynamic changes in these fluctuations can be examined through the violin plots presented in Figures 2 and 3.

K00, K01, and K02, show substantially positive weight differences, confirming that the optimal model prioritizes these jurisdictions to capture roughly half of the total portfolio weight. In contrast, several subsequent groups (K03 through K06) display negative medians, revealing that the quasi-optimal model over-allocates to these segments by failing to account for their carbon risk exposures. As systematic risk increases toward the higher-kappa groups (K11 through K19), the weight differences stabilize near zero. The figure indicates that, since the input required for the optimization is estimated from the historical returns, the results reflect the market conditions during the sample period.

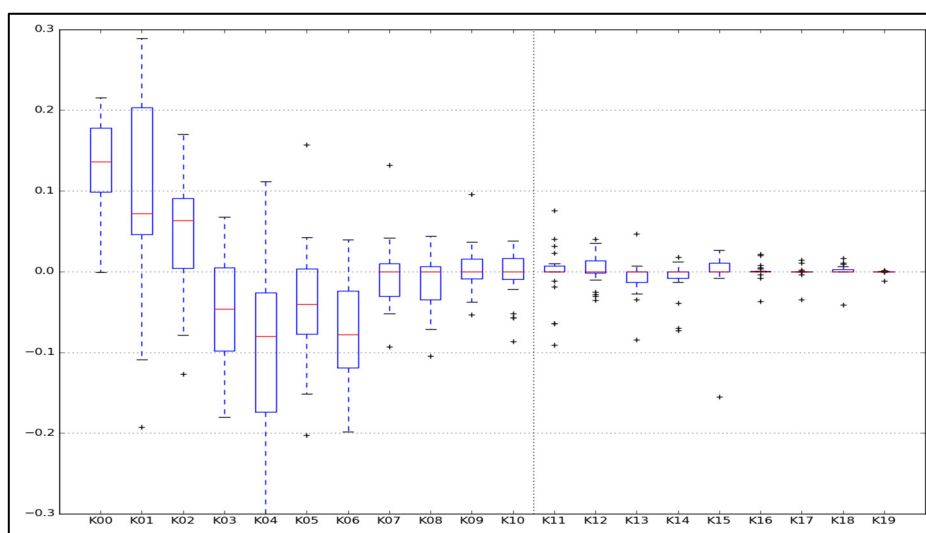


Figure 4. Distribution of weight differences.

Figure 4 illustrates the distribution of differences between the optimal portfolio weights (X^*) and quasi-optimal weights (X^o) for each of the 20 kappa-sorted groups, where a positive value indicates that the carbon-integrated model increases the allocation to that group.

3.4.3. Comparison of diversification

Figure 5 illustrates the annual fluctuations in the entropy measure difference between the optimal and quasi-optimal portfolios, where a positive value indicates that the optimal portfolio has higher entropy, reflecting a more diversified distribution of weights. Data from the earlier sample period (2004–2013) shows that accounting for systematic carbon risk generally increases portfolio diversification, with notable entropy peaks in 2008 and 2012. Conversely, the period following 2015 (marked by the vertical dotted line) shows a shift toward negative values, particularly a sharp decline in 2015, suggesting that the double-index model often opts for a more concentrated allocation in recent years to effectively manage carbon-related volatility. These results highlight that the integration of systematic carbon risk does not always result in broader diversification; instead, it dynamically reconfigures the portfolio, sometimes increasing and sometimes decreasing entropy, to maximize the risk-adjusted return based on the prevailing risk-return profiles of the 40 sorted groups.

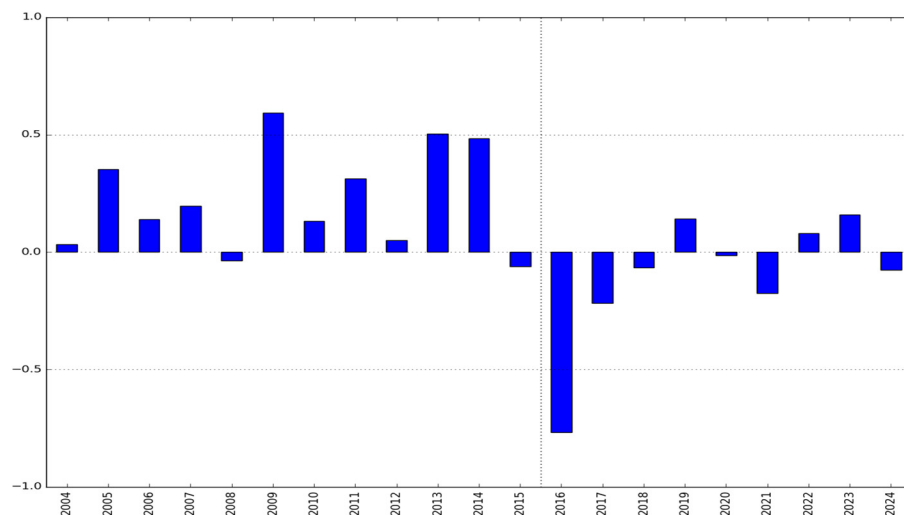


Figure 5. Change in entropy measure.

3.4.4. Comparison of portfolio performance

The size of risk-adjusted return is measured to see what investors would forego if they constructed the quasi-optimal portfolio P^o instead of the optimal portfolio P^* . The evaluation is repeated 21 times for each optimization point. This analysis optimizes the objective function presented in the Appendix and reports the risk-adjusted return metric derived by applying the resulting optimal weights. Specifically, the risk level multiplied by risk aversion is subtracted from the expected return of the optimized portfolio. Risk aversion is measured as the ratio of expected return to the risk level observed in the market risk factor. This risk-adjusted return is chosen as the basis for comparing portfolio performance. The calculated risk level is relatively low, given that the analyzed variable is the country-specific economic growth rate, which generally maintains positive values except for specific periods. Consequently, the calculated Sharpe ratio tends to be higher than levels typically reported in prior performance evaluation studies for a portfolio of financial assets. Consequently, the risk-adjusted return metric of this analysis falls within the more comfortable range. Note that the evaluation results based on this indicator align with the performance assessment results based on the Sharpe ratio.

Table 3. Tests on performance differential.

Period	Return		Risk		RAR		Entropy	
2004~2009	-0.235	**	-0.002		-0.213		0.213	**
2010~2014	-0.242	**	0.001		-0.245		0.297	**
2015~2019	0.208		-0.160	***	0.708	**	-0.194	**
2020~2024	-0.034		0.018		-0.060		-0.005	

***, **, and * denote statistical significance at the 1%, 5%, and 10% level.

Table 3 presents the performance differentials between the optimal and quasi-optimal portfolios across four sub-periods, utilizing OLS regression on the performance gap (optimal minus suboptimal) for return, risk, risk-adjusted return (RAR), and entropy. The empirical results demonstrate that the impact of incorporating systematic carbon risk is highly period dependent. Prior to 2015, the optimal

portfolio exhibited significantly lower returns and higher entropy, indicating that while carbon risk integration enhances diversification levels, it incurs a green premium cost. During these early phases, the global market did not offer financial rewards for carbon-resilient allocations, and the increased entropy suggests a broader, less targeted distribution of capital across jurisdictions.

The 2015–2019 period, immediately following the Paris Agreement, marks a pivotal structural shift in the model's performance. During this window, the optimal portfolio achieves a statistically significant reduction in risk and a corresponding significant improvement in risk-adjusted returns (RAR). Notably, this era saw a reversal in the entropy trend, with a statistically significant decrease in diversification. This suggests that hedging systematic carbon risk in a post-Paris regulatory environment requires a more concentrated strategic reallocation toward a narrowing subset of low-carbon jurisdictions. The model transitions from broad diversification to a more defensive concentration to successfully mitigate heightened carbon volatility.

In the final sub-period (2020–2024), the performance differentials across all metrics become statistically insignificant. This convergence indicates that the influence of systematic carbon risk factors may have been overshadowed by the global contraction of production activities and the overarching market volatility triggered by the COVID-19 pandemic. Consequently, the distinct advantages of carbon-aware optimization are neutralized as global systematic shocks become more generalized, reducing the relative impact of carbon-specific risk premiums on portfolio performance.

4. Discussion

4.1. Methodological framework considerations

My analysis is based on static mean-variance optimization. This serves as a meaningful starting point, given the lack of substantive research on global carbon beta estimation and my objective to identify global systematic carbon risk factors and integrate them into the portfolio optimization process.

Despite this significance, the primary issues to be handled involve the inherent statistical complexities of climate-related data, such as non-Gaussian distributions, fat tails, and the risk of over-concentration in specific green assets. Traditional mean-variance optimization often fails to account for these non-linearities, potentially leaving portfolios vulnerable to extreme transition shocks or valuation bubbles. Furthermore, while modern entropy-based (Giunta et al., 2024) and SSD-efficient (Cesarone et al, 2022) frameworks excel at ensuring broad diversification and managing tail dependence, they need to be refined to isolate the specific carbon-risk premium. To bridge this gap, the analysis framework must account for the unique skewness and kurtosis of carbon returns (often modeled via Cornish-Fisher expansions) while ensuring that the resulting portfolio remains operationally feasible for the sticky capital requirements of Foreign Direct Investment.

The double-index model offers a distinct advantage by explicitly architecting a framework that isolates and hedges systematic carbon risk through a dedicated factor analysis. Unlike general entropy measures that prioritize market neutrality, this model identifies the carbon beta (or transition sensitivity) for each jurisdiction, enabling multinational corporations to target transition risk directly. By maintaining the balance between rigorous risk isolation and operational efficiency, the model provides a more practical tool for carbon-intensive industries to navigate global regulatory shifts by providing

framework that can be easily combined with more complex modern techniques (e.g., entropy-based EVaR, SSD, and CVaR).

In future studies, researchers should focus on integrating these complementary techniques to enhance the robustness of the Double-Index framework against structural discontinuities, such as post-Paris Agreement policy shifts. Integrating Conditional Value-at-Risk (CVaR) as a replacement for standard variance could better capture the tail events associated with sudden carbon price hikes. Additionally, researchers could explore the use of Copula-based approaches to model the non-linear tail dependence between national carbon policies, which linear correlations often underestimate. Finally, utilizing Shannon Entropy as a diversification regularizer within the double-index optimization could help mitigate the risk of over-concentration in a few green jurisdictions, ensuring a more resilient global business footprint in an increasingly fragmented regulatory environment.

Table 4 shows the side-by-side comparison based on results from the additional optimization experiments: the EVaR-Entropy and SSD-Efficient strategies. It reveals a consistent and statistically decisive pattern: Both strategies deliver substantially lower risk-adjusted performance than the optimal portfolio based on double-index model over the 2004–2024 panel. The single-index and double-index strategies are statistically indistinguishable from one another; the paired Sharpe difference (optimal minus quasi-optimal) of +0.139 carries a t-statistic of 0.57 ($p = 0.575$) and is not significant. The insignificance confirms that adding the carbon gamma factor to the selection criterion neither helps nor harms in the aggregate. Therefore, optimal and quasi-optimal portfolios serve jointly as a robust benchmark against which the constrained strategies are compared.

Table 4. Side-by-Side portfolio performance.

Metric	Single-index	Double-index	EVaR-Entropy	SSD-Efficient
Sharpe Ratio	+5.6757 (1.1978)	+5.8149 (1.2932)	+3.4647 (0.8796)	+2.4246 (0.4031)
Portfolio Return	+0.0435 (0.0036)	+0.0427 (0.0037)	+0.0378 (0.0017)	+0.0594 (0.0073)
Std Deviation	+0.0080 (0.0016)	+0.0076 (0.0015)	+0.0118 (0.0036)	+0.0250 (0.0044)
Risk-Adj Return	+0.0309 (0.0093)	+0.0313 (0.0086)	+0.0131 (0.0083)	−0.0757 (0.0786)

The EVaR-Entropy portfolio, which minimizes the entropic value-at-risk at $\alpha = 0.95$ with a Shannon entropy regularization term, underperforms the benchmarks materially. Its mean Sharpe ratio of 3.465 is 2.21 Sharpe points below the quasi-optimal ($t = -11.45$, $p < 0.001$) and 2.35 points below the optimal portfolio ($t = -12.49$, $p < 0.001$). The source of this degradation is primarily a volatility expansion: Standard deviation rises from approximately 0.0076–0.0080 for the EG strategies to 0.0118 for EVaR-Entropy, a 48–56% increase, while the portfolio return simultaneously contracts from 0.043 to 0.038. The EVaR at the 95th percentile of −0.0098 and a Shannon entropy of 3.650 confirm that the portfolio achieves meaningful tail-risk coherence and weight diversification by construction, but at a cost that outweighs its benefit in a long-run performance context.

The SSD-Efficient portfolio presents an even sharper trade-off. It achieves a higher raw return (+0.059 versus 0.043 for the optimal portfolio), a testimony to the LP optimizer successfully seeking stochastic dominance over the equal-weight benchmark. However, this comes at the cost of a standard deviation of 0.0250, more than three times the optimal portfolio level, and a risk-adjusted return of −0.076, deeply negative. The resulting Sharpe of 2.425 sits 3.25 points below the quasi-optimal and 3.39 points below the optimal portfolio, both rejections significant at the 0.001 level. Notably, the LP

solver converges successfully in all 21 years and achieves SSD dominance over the equal-weight benchmark in 100% of cases, so the shortfall is not a failure of the optimization but a reflection of the constrained strategy space: Maximizing the return subject to stochastic dominance constraints accepts concentrated, high-variance portfolios that are efficient by the dominance criterion yet inefficient by a conventional Sharpe metric.

In summary, the systematic risk framework, in its single-factor and two-factor forms, remains the dominant approach by the Sharpe criterion over this sample. The EVaR-Entropy and SSD strategies are not inferior by construction: Each enforces a property (tail-risk coherence and stochastic dominance, respectively) that is absent from the double-index approach. What the evidence demonstrates is that these constraints impose a cost in mean–variance space that is measurable, consistent, and highly significant, a finding that motivates the explicit carbon-factor gamma-loading control of the double-index model as the preferred risk management channel.

Table 4. Comparison of the performance of the optimal portfolio derived from the double-index model presented in this paper with the performance of portfolios optimized using other alternative methodologies (entropy-based optimization and SSD-efficient optimization). Values are means over 21 annual observations (2004–2024). Standard deviations are shown in parentheses.

4.2. Sensitivity analysis

It is possible to compare the size of the risk-adjusted return that would be foregone if the quasi-optimal portfolio were chosen instead of the optimal portfolio. The sensitivity analysis evaluates the risk-adjusted return (RAR) gap, calculated as the RAR of the optimal portfolio minus that of the quasi-optimal portfolio, across five scenarios where the carbon risk index's mean and volatility are adjusted by a scaling multiplier (c). As illustrated in the boxplot in Figure 6, the performance differential is highly sensitive to the variability of the carbon risk factor relative to the market. In the “Highly distinguished” scenario ($c=0.2$), where the carbon risk factor exhibits significantly lower variability than historical levels, the optimal portfolio demonstrates its most pronounced advantage, with a positive median gap of approximately 0.003, although it remains susceptible to extreme negative outliers below -0.010 . In contrast, as the scaling multiplier increases through the “Neutral” ($c=1.0$) and “Highly similar” ($c=1.8$) scenarios, the median gap shifts toward zero or slightly negative values, and the interquartile ranges expand significantly. This confirms that the strategic benefit of integrating systematic carbon risk is most effective when carbon-driven volatility is low; when the carbon factor's volatility approaches or exceeds that of the broader market, the distribution of the performance gap becomes wider and less certain, reflecting a more complex risk–return trade-off.

The result echoes another finding (Alessandrini & Jondeau, 2021), which shows that the portfolio that maximizes its ESG quality while maintaining factor exposures within stated limits produces risk-adjusted performance at least as high as the standard MSCI benchmark. From a similar perspective, Madhavan et al. (2021) partitioned the ESG scores into components related to factors and idiosyncratic components and found strong positive relationships between fund alphas and factor ESG scores. Jin (2024) utilized the double-index model, applied it to US equity mutual funds, and showed that exposures to the systematic ESG risk significantly correlate with the sorted group's expected returns. Note that the gap increases as the scaling multiplier becomes smaller. This indicates that the loss from

ignoring the systematic carbon risk during portfolio construction becomes larger when returns on the carbon risk index are less volatile than those on the market risk index.

Figure 6 illustrates the risk-adjusted return (RAR) gap between the optimal and quasi-optimal portfolios across five sensitivity scenarios, where the gap is calculated by subtracting the quasi-optimal RAR from the optimal RAR.

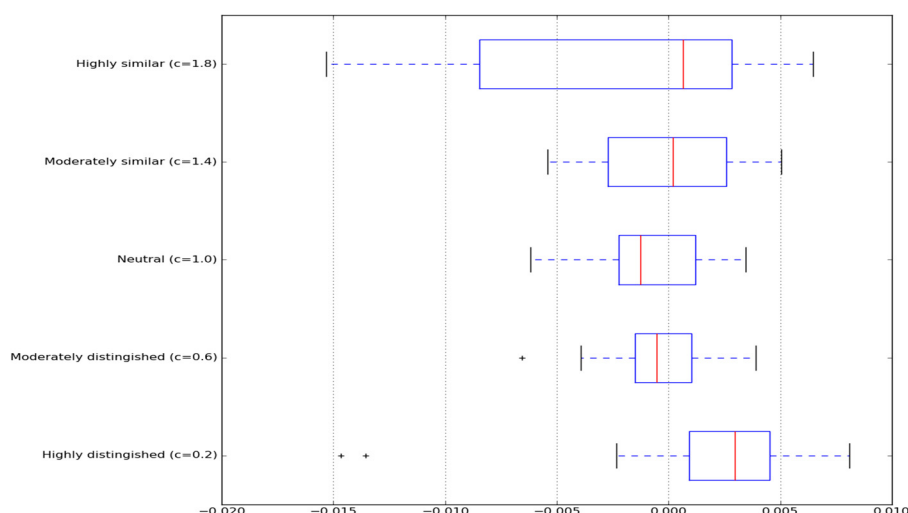


Figure 6. Performance comparison of optimized portfolios.

The simulation propagates uncertainty in the systematic carbon factor (α_s and σ_s^2) through the optimal portfolio using four stochastic process models, Geometric Brownian Motion (GBM), Ornstein-Uhlenbeck (OU), Jump-OU, and Cox-Ingersoll-Ross (CIR), each run over 2,000 paths at a weekly frequency across the full 2004–2024 panel. The results, summarized in Table 5, reveal a stable core performance profile with one significant exception.

Three of the four models, GBM, OU, and CIR, reproduce the deterministic baseline with near-exact fidelity. Portfolio return (+0.0429–0.0430), standard deviation (~ 0.0075), and Sharpe ratio (~ 5.93 – 6.00) are effectively invariant across these specifications, confirming that continuous carbon factor dynamics do not materially alter portfolio outcomes when the optimal portfolio weight structure is held fixed. The CIR model, whose volatility is proportional to the square root of the factor level, produces the strongest risk-adjusted performance (Sharpe 6.001; Risk-Adj. Return 0.0321) marginally, reflecting the non-negativity constraint it imposes on the carbon factor, a theoretically appropriate property for emissions-intensity series. These findings are consistent with the study by Ceci et al. (2024), which demonstrated that the CIR model and its advanced extensions can be suitable choices for portfolio modeling and investment.

The Jump-OU model is the clear outlier. Incorporating compound Poisson regulatory shocks ($\lambda = 2$ events per year) generates a meaningfully lower Sharpe ratio of 4.727 (versus the deterministic baseline of 5.927) and compresses the risk-adjusted return from 0.0319 to 0.0235. Furthermore, portfolio volatility rises to 0.0096, 28% above the baseline, while CVaR improves slightly to -0.0224 from -0.0274 , indicating that tail losses are paradoxically cushioned even as mid-distribution risk increases. Most diagnostically, Jump-OU produces a markedly elevated excess kurtosis of 0.469

(standard deviation 0.504), confirming that discrete carbon shocks fatten the return distribution's tails in a manner not captured by diffusion-only models. The key modeling finding is as follows: Deterministic and continuous-diffusion models systematically understate the Sharpe ratio degradation that episodic regulatory carbon events can inflict.

Taken together, the simulation confirms that the optimal portfolio from the double-index model is broadly resilient to uncertainty in the carbon factor's continuous dynamics but remains materially exposed to discrete regulatory jump events. The widening post-2015 calibration parameters suggest that this jump exposure is likely to intensify over the forward horizon, reinforcing the case for explicit gamma-loading management within the two-factor framework.

Table 5. Portfolio metrics by the systematic carbon risk factor model.

Metric	Det. Double	GBM	OU	Jump-OU	CIR
Portfolio Return	+0.0429 (0.0036)	+0.0430 (0.0036)	+0.0429 (0.0036)	+0.0429 (0.0036)	+0.0429 (0.0036)
Std Deviation	+0.0075 (0.0015)	+0.0075 (0.0015)	+0.0075 (0.0015)	+0.0096 (0.0023)	+0.0074 (0.0016)
Sharpe Ratio	+5.9265 (1.2578)	+5.9725 (1.2581)	+5.9263 (1.2628)	+4.7274 (1.2265)	+6.0009 (1.3194)
CVaR 5%	−0.0274 (0.0050)	−0.0275 (0.0049)	−0.0275 (0.0050)	−0.0224 (0.0062)	−0.0274 (0.0051)
Risk-Adj. Return	+0.0319 (0.0082)	+0.0321 (0.0082)	+0.0319 (0.0080)	+0.0235 (0.0155)	+0.0321 (0.0080)
Skewness	0.0000 (0.0000)	−0.0215 (0.0463)	+0.0267 (0.0566)	−0.0164 (0.0814)	−0.0240 (0.0628)
Excess Kurtosis	0.0000 (0.0000)	+0.0030 (0.1117)	−0.0037 (0.1008)	+0.4692 (0.5039)	+0.0157 (0.1261)

Table 5 Simulation results from the geometric Brownian motion (GBM), Ornstein–Uhlenbeck (OU), jump-OU, and Cox–Ingersoll–Ross (CIR) models, with each model simulating 2,000 paths on a weekly basis using the full panel data from 2004 to 2024. All values are means over 21 annual observations (2004–2024) with cross-sectional standard deviations in parentheses. Det. Double = deterministic baseline from the double-index model.

4.3. Non-diversifiability of systematic carbon risk

To assess whether carbon risk constitutes a non-diversifiable, priced systematic factor, I estimate cross-sectional regressions of portfolio expected returns on two risk loadings: the market beta (β) and the carbon risk sensitivity (γ). The dependent variable is the one-step-ahead expected excess return derived from a two-factor model, and all standard errors are HAC-adjusted (Newey–West, 3 lags) to account for heteroscedasticity and serial correlation. Regressions are run over three sample periods, the full sample (2004–2023, $N = 800$), the first sub-period (2004–2013, $N = 400$), and the second sub-period (2014–2023, $N = 400$), and separately for the contemporaneous (current) and one-period-ahead (future) expected return specifications, yielding six sets of results in total.

As shown in Table 6, across all six specifications, the coefficient on market beta is positive, economically sizable, and highly statistically significant. Estimates range from 0.0179 to 0.0253, with z-statistics uniformly exceeding 7 and p-values of effectively zero. This confirms that the market factor commands a robust and stable risk premium throughout the sample, consistent with standard asset-pricing theory. The coefficient rises noticeably from the first sub-period (≈ 0.018 – 0.019) to the second (≈ 0.025), suggesting that the market risk premium intensifies in the post-2014 period.

The coefficient on carbon risk sensitivity (γ) exhibits a statistically significant pattern, but one

that changes meaningfully over time and across specifications, and this variation is informative. In the full-sample regressions (curr0, futu0), γ is small and statistically indistinguishable from zero (coefficient ≈ 0.0018 – 0.0017 ; p-values of 0.164 and 0.162), suggesting that when the 2004–2023 period is pooled, the average compensation for carbon risk is negligible. However, this aggregate insignificance masks important structural heterogeneity revealed in the sub-period estimates. In the first sub-period (2004–2013), γ is positive, economically meaningful (≈ 0.0057 – 0.0064), and statistically significant at conventional levels ($z = 3.705$ and 4.060 ; $p < 0.001$). This indicates that, during this period, portfolios with higher sensitivity to carbon risk earn a positive premium, investors require additional compensation for bearing carbon exposure, consistent with carbon risk being perceived as an undiversifiable systematic risk even in this earlier era. In the second sub-period (2014–2023), γ reverses sign and becomes negative and statistically significant (≈ -0.0029 to -0.0034 ; $z = -3.284$ and -3.474 ; $p < 0.001$). This sign reversal is substantively important: It implies that, in the more recent period, portfolios with greater carbon sensitivity are associated with lower expected returns. One interpretation consistent with asset-pricing theory is a carbon risk repricing dynamic; as climate awareness, regulatory risk, and investor ESG preferences intensify after 2015 (e.g., following the Paris Agreement), high-carbon assets may carry a negative premium, reflecting investors' willingness to accept lower returns on greener portfolios (i.e., a green preference effect), or alternatively, that high-carbon firms become systematically re-rated downward in ways that generate capital losses for high- γ portfolios.

The central finding from these results is that carbon risk is not idiosyncratic; it cannot be diversified away within a conventional portfolio framework. The consistent statistical significance of γ in both sub-periods (even as its sign changes) demonstrates that cross-sectional variation in carbon risk loadings explains a meaningful component of the cross-section of expected returns, above and beyond the market factor. If carbon risk were purely firm-specific and diversifiable, it would carry no risk premium and γ would be zero in expectation. The rejection of $\gamma = 0$ in both sub-periods, at the 1% significance level, constitutes direct evidence that carbon risk is priced by the market; that is, it is systematic in nature. The full-sample insignificance of γ , paradoxically, further reinforces this conclusion: The positive premium in the first sub-period and the negative premium in the second largely offset each other in aggregate, producing a near-zero average. This cancellation reflects a genuine structural shift in how carbon risk is priced rather than the absence of pricing altogether, and underscores the importance of examining sub-period dynamics when evaluating the systematic character of emerging risk factors.

Table 6. Regression results for non-diversifiability of systematic carbon risk.

Specification		Current expected return			Future expected return		
Sample		P0	P1	P2	P0	P1	P2
β	Coefficient	0.0208	0.0179	0.0253	0.0211	0.0188	0.0246
	z-statistic	12.383	7.747	20.767	13.66	8.544	21.132
	p-value	0.000	0.000	0.000	0.000	0.000	0.000
γ	Coefficient	0.0018	0.0064	−0.0034	0.0017	0.0057	−0.0029
	z-statistic	1.393	4.06	−3.474	1.399	3.705	−3.284
	p-value	0.164	0.000	0.001	0.162	0.000	0.001
GoF	Adj. R ²	0.46	0.429	0.563	0.473	0.445	0.557
	F-statistic	133.1	139.5	286.1	145.3	118.8	296.2

Table 6. The results of empirical analyses showing that systematic carbon risk cannot be eliminated through portfolio diversification. HAC standard errors (Newey–West, 3 lags). β = systematic market risk sensitivity; γ = systematic carbon risk sensitivity. GoF = Goodness of fit. No intercept; Adj. R^2 is uncentered. P0 = Full sample, 2004–2023 (N=800); P1 = First half sample, 2004–2013 (N=400); P2 = Second half sample, 2014–2023 (N=400).

4.4. Considerations in the context of foreign direct investment

The primary methodological challenge involves bridging the gap between the liquid assumptions of Modern Portfolio Theory and the sticky reality of FDI. The framework of this analysis assumes that MNCs act like portfolio managers seeking to optimize their global business portfolios across dimensions. However, many countries impose trade restrictions, making complete trade freedom unattainable, and country-specific investments must also consider strategic factors, tariffs, and other considerations. The double-index model focuses on the global systematic risk that is affected by trade restrictions rather than diversifiable risk, acknowledging that perfect trade freedom is an unattainable benchmark.

To quantitatively address the diversifiable friction between theoretical portfolio optimization and the geopolitical realities of FDI, the double-index model can be modified by incorporating friction constraints that account for market imperfections. For example, the double-index model can incorporate exit barriers and sunk costs by treating the derived optimal weights (X_i^*) as strategic long-term targets rather than liquid positions. By factoring in the higher rebalancing costs associated with physical assets, the model can analytically demonstrate that the actual achievable portfolio sits below the theoretically efficient frontier, thereby incorporating FDI-specific operational constraints directly into the performance expectations. This constraint acts as a penalty for jurisdictions with low trade freedom or high capital controls, effectively reducing the weight of restricted markets regardless of their theoretical risk-return profile. Another possibility is integrating an extended gravity model into the expected return estimation. This adjusts the strategic value of locations based on trade barriers (tariffs) and geopolitical alignment (e.g., friendshoring index), ensuring the model prioritizes strategic resilience over simple dispersion reduction.

Despite the assumption of a globally priced carbon risk, the double-index model acknowledges regional regulatory divergence by treating carbon risk as a fragmented systematic factor. Rather than assuming a uniform price, the model can use the systematic carbon risk factor (γ) to capture the varying sensitivities of different jurisdictions to global transition pressures. This enables the framework to identify carbon betas even in regions with incomplete or illiquid markets, acknowledging that while carbon risk is globally pervasive, its impact is mediated through localized regulatory maturity and information asymmetry.

To enhance scenario realism, the framework can transition from a purely historical panel analysis to a forward-looking stress-testing approach. The double-index model is uniquely suited for this because it can be used to simulate how abrupt policy shocks or technological disruptions (such as a sudden breakthrough in low-carbon steel) decouple specific jurisdictions from the global systematic carbon index. This enables MNCs to move beyond static mean-variance results and evaluate portfolio resilience under diverse green swan scenarios where historical correlations may converge or break down.

Furthermore, the model addresses the issue of non-Gaussianity and tail risk by replacing standard variance with metrics like Conditional Value-at-Risk (CVaR) within the optimization workflow. By explicitly accounting for the skewness and kurtosis identified in carbon risk returns, the double-index model captures the unique risk profile of the green transition more effectively than traditional Gaussian models. This technical refinement ensures that the optimality of the portfolio is grounded in a rigorous assessment of the extreme downside risks associated with sudden carbon price hikes or global regulatory shifts.

In conclusion, the double-index model provides a balanced solution that maintains the mathematical rigor of factor analysis while respecting the strategic and geopolitical complexities of global manufacturing. By integrating behavioral regularizers, such as Shannon Entropy, to mitigate concentration risk in green bubbles, and utilizing a two-layer grouping criterion to minimize estimation errors, the framework offers a robust methodological workflow. This ensures that the identification of global systematic carbon risks leads to a pragmatic and resilient capital allocation strategy for multinational corporations operating in a carbon-constrained world.

5. Conclusions

As global pressure to reduce emissions increases, multinational corporations are increasingly viewed as portfolio managers who must navigate an optimal frontier where international growth potential is balanced against non-diversifiable systematic carbon risk. The primary importance of this research lies in its capacity to identify and isolate global systematic carbon risk (a factor driven by globally synchronized policies like the Paris Agreement and CBAM), which cannot be neutralized through simple geographic expansion. By utilizing a double-index model and a two-layer grouping procedure, the framework successfully disentangles the specific carbon beta (γ) from traditional market risk (β), enabling MNCs to treat climate transition sensitivity as a distinct and manageable dimension of their global operations.

The empirical results validate this approach, demonstrating that isolating systematic factors is essential for strategic capital reallocation. The optimal portfolio, which integrates these isolated risks, proactively shifts weights toward low-risk jurisdictions, concentrating roughly half of the total portfolio weight in the three groups exhibiting the lowest systematic risk (K00–K02). Furthermore, the analysis reveals that the effectiveness of this optimization is highly period-dependent; following the 2015 Paris Agreement, the optimal portfolio achieves a statistically significant reduction in risk and an improvement in risk-adjusted returns by shifting toward a defensive concentration in low-carbon jurisdictions. These performance advantages become statistically insignificant during the 2020–2024 period, because global systemic shocks, such as the COVID-19 pandemic, overshadow carbon-specific risk factors. Sensitivity analysis further confirms that the strategic value of accounting for systematic carbon risk is most pronounced when carbon-driven volatility is low, as the losses from ignoring this factor increase when the carbon risk index is less volatile than the market.

In summary, the double-index framework provides a rigorous, analytically grounded solution for multinational corporations to construct resilient business portfolios in an increasingly carbon-constrained world. By isolating the systematic carbon risk premium, the model enables firms to hedge against non-linear transition shocks and green swan events effectively. Researchers should continue to

address the complex realities of foreign direct investment, such as exit barriers, sunk costs, and multi-period probabilistic frameworks, and validate the findings of this analysis using firm-level data from multinational corporations with varying carbon intensities. Despite these limitations, this study contributes to quantifying the vulnerability of multinational corporations in carbon-intensive industries to the global transition shock and presents a key methodological workflow for formulating long-term capital allocation strategies.

Use of AI tools declaration

The author declares he has not used Artificial Intelligence (AI) tools in the creation of this article.

Conflict of interest

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