



Research article

Impact of marketing expenditure and carbon emissions on dual-channel retailing

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Abstract: Businesses improve their operations by controlling carbon emissions and environmental impact. Emissions from a system can be controlled, which is more realistic in most industries rather than using a zero-emissions system. An eco-friendly technology helps to improve environmental impact and emissions compared with a traditional system without a green innovation technology. This research examines a carbon tax and utilization of green innovation in a dual-channel retailing with two independent channels. The dual-channel operations uses marketing to advertise products to their associated customers. The manufacturer and the retailer communicate through independent online and offline channels. The demand depends on green innovation, marketing expenditures, and the retail price of the respective channel. The model is solved for a centralized and a decentralized methods. The results demonstrate that the lack of green innovation in decentralized policy decrease the profits of the retailer and manufacturer by 7.27% and 7.16%, respectively, and under the centralized policy, it decreases by 6.98%. In the absence of the online channel, the retailer's and manufacturer's profits increase by 45.04% and 47.61%, respectively, under the decentralized policy for the offline channel.

Keywords: supply chain management; dual-channel retailing; retail management; transportation; marketing; carbon emissions

1. Introduction

The current global economic environment is undergoing extensive supply chain changes as environmental requirements [1], technological advances, and customers' needs transform the

industry [2]. Carbon emissions have become a major concern because supply chains, from procurement to distribution, generate substantial global greenhouse gas emissions through their operations [3]. Marketing directly impacts supply chain operations by generating product demand and determining how much consumers will spend [4]. Companies allocate significant resources to digital marketing to boost brand recognition, which, in turn, drives product demand [5]. The sustainable performance of the supply chain depends on green innovation because it improves the quality of products achieving environmental compliance. Government and organizations implement multiple strategies [6], including using renewable energy [7], reducing carbon emissions [8], establishing carbon pricing systems, and boosting energy efficiency from production to supply chain operations [9]. Modern supply chain management (SCM) [10] is undergoing its latest transformation through dual-channel retailing [11], which enables companies to sell their products through both online and brick-and-mortar stores [12].

Traditional models do not integrate multiple factors because they treat those factors as separate entities when they actually intersect at various points [13]. Dual but two independent channels are very much realistic even in the scenario of technologically advanced customers [14]. The concept of two independent channels for a single type of products with entirely two different price, quality, and marketing lies in the customers' psychology. For example, a standard Nike retail store and the Nike flagship store are two entirely different shopping channels. The Nike retail store is a everyday shopping outlet whereas the Nike flagship outlet showcases customized and exclusive products which are not available for everyday shopping. In the similar manner, customers who always prefer offline shopping, always choose brick-and-mortar stores so that they can touch and feel products. This market segment does not feel good when they choose products just by seeing the pictures and description. They buy products offline even though they pay a slightly higher price than the online channel. On the other hand, the market segment who prefers online shopping, always chooses online shopping irrespective of the price. Those customers choose comfort over pricing and hassle. This complex scenario is linked with modern supply chain operations.

Carbon emissions, with their links to marketing expenditure, green innovation, and dual-channel retailing, necessitate the development of comprehensive analytical models that measure their effects on supply chain operations [15]. Marketing expenditure generates more demand, which increases emissions, while green innovation reduces environmental impacts, and dual-channel retail operations either boost or reduce emissions depending on how efficiently they operate. The present study develops a comprehensive framework that explores the relationships across multiple dimensions, providing managers and policymakers with critical information to develop sustainable supply chain systems that will perform financially and environmentally in the future marketplace, which demands sustainability [16]. Among the three pillars of sustainability, the economic pillar (profit maximization) and the environmental pillar (consideration of emissions and green innovation) are discussed here [17]. However, the social pillar of sustainability [18] is not discussed in this study.

1.1. Research gaps

The research gaps of this study are as follows.

- Many studies have examined green innovation across different demand types. Most studies concentrate on linear demand functions. There has been less research on how carbon emissions

affect non-linear power demand in online and offline sales. This research addresses the issue using a game-theoretic approach.

- Many researchers have studied green innovation and marketing of products. However, few studies have examined the interrelationships among green innovation, independent dual-channel retailing, and marketing in competitive market settings.
- Increasing the supply chain's profit margin is important in retail marketing, and many authors have examined centralized and decentralized models. However, the perspective on profit margins in an independent dual-channel system that accounts for carbon tax is a less-explored area.

1.2. Contributions of the study

This study contributes the following points.

- Two independent retail channels for a single retailer sell two different products of the same type. As the customers for both channels are mutually exclusive, the demands for both channels is different.
- A nonlinear demand curve is used for the online and offline retail channels, where the demand of these two independent channels is different, while both marketing and product greenness are included as variables which affect the market demand.
- Carbon emissions have negative impacts on the environment and climate, challenging efforts to reduce them for environmental and climate protection and to promote the sustainable development of the social economy, underscoring the urgent need to reduce them. Both the supply chain players pay carbon taxes.
- Most significantly, marketing is vital in raising consumer awareness when two independent channels are involved for two different types of customers. The study demonstrates the importance of marketing as a key player in addressing the challenges of achieving coexistence between the economy and the environment.

1.3. Orientation

The remainder of this research has been partitioned into several sections. The relevant works are discussed in Section 2. In Section 3, the notation, assumptions, and the problems definitions are discussed. The mathematical model is explained in Section 4. In Section 5, the numerical analysis is explained. A detailed discussions is given in Section 6. Managerial insights are also discussed in Section 7. Finally, in Section 8, conclusions and future extensions are explained.

2. Literature review

Some literature highlights the existing research in this section. The contribution table of this study is shown in the Table 1.

2.1. Green innovation for clean technology

Green innovation is a mechanism to elevate the product to make it more environment-friendly than the traditional products. They are net providers of efficiency in terms of resource use, cost savings, and eco-efficient product development. Green innovation makes sense as a good investment for

developing sustainability in this age of SCM. Li and Zhang [19] examined the effects of green innovation and sustainability in urban areas. The study explained the positive impact on driving renewable energy consumption in urban areas. Yuan et al. [20] explained the government's attention to environmental growth. The study found a positive relationship between green innovation and governments environmental attention using panel data on firms. Sarkar et al. [21] explained green innovation in dual-channel retailing. However, the study avoided discussing the benefits of marketing expenditure. Cui [22] examined the impact of green innovation on financial capabilities in the supply chain. The study focused on the synergistic mechanism between supply chain finance and green innovation among A-share-listed companies in China. Ismail [23] worked on the influence of green absorptive capacity on agricultural products. The study established the mediating role of green innovation in the relationship between green absorptive capacity and precision agriculture technologies. Fan et al. [24] discussed the positive role of green innovation in the heavily polluted industry. However, the study failed to explain marketing expenditure and dual-channel retailing.

2.2. *Dual-channel retailing*

Dual-channel retailing integrates online and offline platforms to maximize market exposure and enhance customer convenience. Investigations suggest that such impacts relate to pricing strategies, channel cooperation, and competition across different products offered through traditional and digital channels. Bhattacharya et al. [25] studied a dual-channel retailing and transportation, focusing on customer care support, home delivery, and advertising. The study examined the effect of dual-channel retailing on the supply chain's profit. Sarkar et al. [26] worked on a two-echelon supply chain for a green product. Kar et al. [27] studied a global supply chain with reworking of defective items and a payment policy. The study used the benefits of imperfect production and reuse of defective products in the green supply chain. However, they did not consider marketing expenditure. Sana [28] described a novel approach to selling products in dual-channel retailing. The study examined the important role of online and offline selling prices, without accounting for marketing expenditure. Sarkar and Guchhait [29] explained the benefits of fourth party logistics management with remanufacturing for customized products. However, the study avoided discussing marketing expenditure. Datta et al. [30] discussed a retailing strategy to enhance customer satisfaction in dual-channel retailing. The research focused on customer satisfaction with dynamic pricing.

2.3. *Carbon emissions from the supply chain*

Carbon emissions are inevitable from supply chain operations. Almonares et al. [31] examined how environmental regulations, combined with carbon emissions disclosure requirements, affect corporate behavior in transportation companies operating in both developed nations and emerging markets. The study found that strict regulatory frameworks, combined with comprehensive emissions disclosure requirements, lead organizations to implement cleaner technology solutions, thereby enhancing environmental performance. Tong et al. [32] established a comprehensive system for evaluating the efficiency of marine carbon emissions by combining input-output analysis with marine carbon sink accounting across coastal regions of China. Wang et al. [33] assessed how integrated energy storage systems perform under different types of load and renewable generation uncertainties, using advanced optimization techniques. The study found that adaptive energy management strategies

improve storage efficiency, reduce energy waste, and enable sustainable energy system operation by reducing emissions. Jorgenson et al. [34] studied carbon emissions using advanced optimization techniques. The framework enables schools to improve their food environments through policy integration, yet its adoption faces obstacles due to insufficient staff resources and time constraints; strong institutional and political support, however, creates positive outcomes. Liu et al. [35] investigated the role of energy optimization techniques in shaping carbon emission reductions while enhancing systems' sustainability across integrated energy networks. The study showed that intelligent control systems, combined with multi-objective optimization, lead to energy-efficiency upgrades that reduce environmental damage. Bar et al. [36] created an SCM model that integrates product returns with controllable carbon emissions under a carbon policy. The authors found that smart energy utilization, together with optimal pricing and cycle time decisions, leads to higher profitability and lower emissions.

2.4. Transportation in supply chain operations

Research on transport has focused on understanding its significant role in improving supply chain efficiency, reducing production costs, and ensuring the timely delivery of goods. Zhang et al. [37] worked on public transportation for the low-carbon transition. The study examined the mechanisms behind the decoupling of carbon emissions from energy consumption and explored pathways to achieve synergy among the energy, transportation, and environmental systems. Lee et al. [38] introduced a sustainable transportation system. The study focused on the effects of environmental, social, and economic factors on transport. Ke et al. [39] investigated the impact of air transport on the supply chain. The study investigated the disruptions and risks to the transport industry. However, the study did not explain how to compare risks across multiple transportation modes. Manafi and Sayan [40] worked on multiple transportation modes in the supply chain. The proposed method was investigated to strengthen supply chain resilience, reduce costs, and improve customer responsiveness. Sarkar et al. [41] introduced electric vehicle production using outsourcing and end-of-life products. Lucchini et al. [42] investigated a new-generation electric vehicle for a sustainable environment based on a non-grain-oriented iron core and NdFeB magnets.

2.5. Sustainable SCM

Sustainable SCM is seen as the pillar of operational excellence and long-term business success with sustainability. Credle et al. [43] worked on development activities in the energy sector of the supply chain without considering sustainability issues. The study developed six energy systems and their inherent properties. Sarkar and Guchhait [44] introduced a novel approach to the value of data security in SCM. The study provided strong security by using radio frequency identification (RFID) in SCM. However, the study did not explain the effect of carbon emission. Olabode et al. [45] worked on food waste in the SCM. The research aimed to reduce food waste and loss in the supply chain through environmentally friendly strategies without mentioning recycling or sustainability issues. Sadeghi et al. [46] worked with empirical data in SCM. The research analyzed the effects of metaverse technology through text and design analysis but not anything related to technology waste management. Padiyar et al. [47] introduced the effect of deteriorated products in a fuzzy environment in the supply chain. The study explained the best way to minimize total cost after deterioration. However, the study did

not discuss all pillars of sustainability. Sarkar et al. [48] investigated an SCM for asymmetric power division among SCM players that was solved by game policy in a decentralized scenario. Falcone and Fiorentino [49] provided a circular bio-economy model for the fashion industry. They focused on nudge techniques to promote the development of a circular economy in the fashion industry. Giganti and Falcone [50] studied the advantages of the circular economy and energy transition in the supply chain. They focused on creating sustainable supply chains. Giganti et al. [51] evaluated the role of blockchain technologies in improving sustainability in the agri-food sector. They focused on enhancing transparency, traceability, and sustainable processes in agri-food supply chains. Rahman et al. [52] examined the interrelationships among urbanization, industrialization, the energy transition, and life expectancy in developing economies. In their study, they introduced the energy transition to measure its contribution to sustainable development and the well-being of the population. Tutore et al. [53] tested the effectiveness of experiential learning in the development of circular economy competencies. They focused on enhancing sustainability-related knowledge and skills through experiential learning.

Table 1. Authors' contribution table.

Author(s)	Carbon emissions	Dual-channel	Marketing expenditure	Transportation	Green product	Model type
Ghosh et al. [61]	×	×	√	×	×	Production
He et al. [62]	×	×	×	√	×	SCM
Singh et al. [63]	×	×	×	×	√	SCM
Chauhanetal et al. [64]	×	√	×	×	√	SSCM
Zhao et al. [65]	×	√	×	×	√	SCM
Golovko et al. [66]	×	×	√	×	×	Inventory
Guchhait and Sarkar [67]	×	×	×	√	×	SCM
Singh et al. [68]	×	×	√	×	√	SCM
Mahapatra et al. [69]	×	√	×	√	√	SCM
Barman and Sana [70]	√	√	×	√	√	SCM
Chauhan et al. [71]	√	×	×	√	√	SCM
Zhao and Zhang [72]	×	×	√	×	√	SCM
This paper	√	√	√	√	√	SCM

Note: SSCM, sustainable supply chain management; ×, not considered; SCM, supply chain management; √, considered.

2.6. Marketing expenditure in the supply chain

Marketing expenditure is being seen as a key strategy for growing sectors while promoting value creation. Adams et al. [54] explained innovation performance, the moderating effect, and the strategic orientation of marketing management. The paper examined the relationship between a firm's strategic orientation, marketing management in terms of marketing mix tactics, innovation performance, and a firm's strategic orientation in the marketing mix mode. Volkmar et al. [55] developed an artificial intelligence (AI) and machine learning framework to explore the drivers, barriers, and future developments in marketing management. The paper proposed identifying overarching themes and

dimensions, and ultimately research propositions, to stimulate further innovative research aimed at improving organizations' approach to AI. Wengler and Kolk [56] discussed applying multi-stage marketing in industrial markets, offering exploratory insights into its successful implementation, management, and adaptation in dynamic markets. Mero et al. [57] developed four managerial strategies and a conceptualization of influence marketing. The paper introduced consumer markets, product-centric social media promotions, and consumer attitudes and behaviors, offering limited insights into organizational processes and the unique features of influence marketing. Guenther et al. [58] proposed a way to improve partial least squares structural equation modeling in business marketing. The study provided a shortcut for identifying the next partial least squares structural equation modeling project. Cabanelas et al. [59] discussed the buying center concept in industrial marketing. This paper introduced new theories, permitting the polyhedral faces of buying centers to be investigated. Keränen et al. [60] investigated the changing market for a sustainable innovation. The study examined how the value network actors' intentions and activities influence how the market changes for sustainable innovation.

3. Problem definition, notation, and assumptions

This section explains the problem definition, notation, and assumptions of the study.

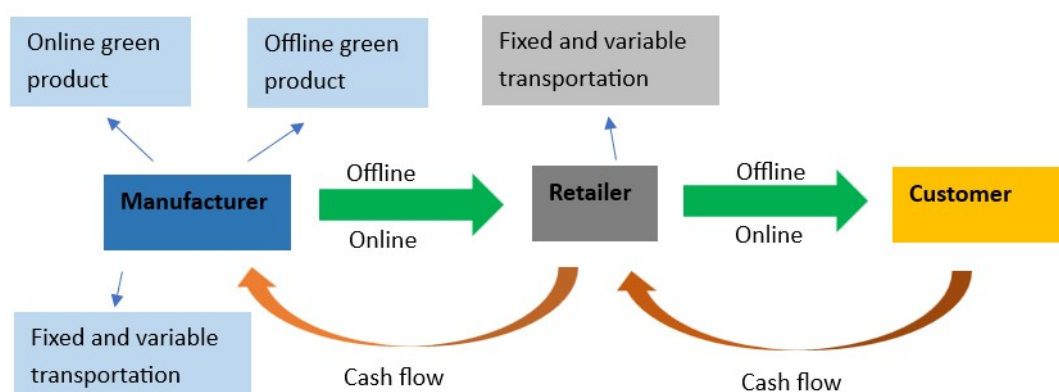


Figure 1. Pictorial presentation of the research problem.

3.1. Problem definition

Figure 1 gives a framework of the model. The supply chain has two independent channels because it focuses on customers who prefer either the online or offline channel, depending on their flexibility and circumstances. Marketing spending, its contribution to demand, and hence, its profitability within this duality of channels, even now, remains unexplored, especially for carbon emission taxes. As demand is lumpy, it does not follow a linear pattern. Market demand goes up sometimes, such as when a new electronic product is launched, and goes down when another version or an upgraded version of that product launches. Marketing expenditure can help increase demand up to a certain point. After that, it helps increase demand, but not to the same level as at the beginning. The Cobb-Douglas function represents the scenario better than other nonlinear functions. The elasticity of the shape parameter Z_1 is illustrated with a figure (Figure 2) that shows that an increasing value of Z_1 decreases the demand.

However, demand for marketing investment is increasing. Thus, the combination of both captures the reality that, even though the investment increases, the demand does not increase infinitely, and the shape parameter Z_1 balances it. The illustration explains the online demand pattern. The same applies to the offline scenario. Figure 2 shows how the demand changes with respect to the decision variable and parameters. The literature includes a wide range of studies that have already established the relationship and its possible outcomes [73].

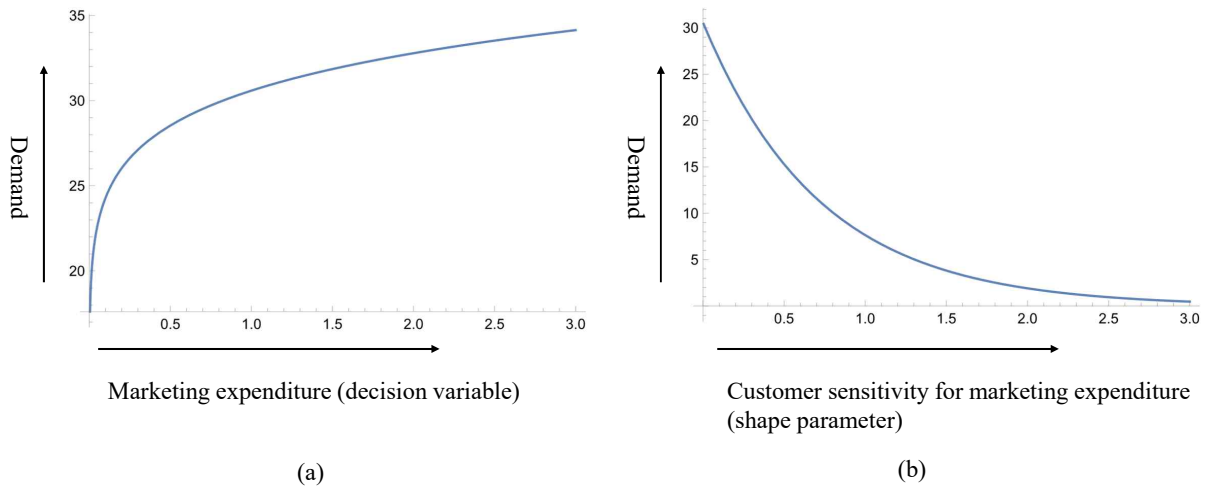


Figure 2. Demand pattern for marketing expenditure with respect to (a) investment and (b) the customer sensitivity parameter.

3.2. Notation

To formulate the model, some notation is used, which is given in Table 2. The conditions of λ_i, μ_i , and Z_i are used for the convergence of the demand function. Otherwise, the function will be divergent and will be no use for real-life applications.

3.3. Assumptions

The assumptions of the problem description are as follows.

- (1) A two-echelon SCM is considered, in which the manufacturer produces one single type of products but for two independent online and offline channels, and the retailer sells them in the market through its respective channels independent channels. As two market segments are different, the two channels are independent of each other and there is no interdependency.
- (2) Lumpy demand does not even follow any specific non-linear demand pattern. For a lumpy type demand, such as electronic gadgets or a new product launch, a power-type demand pattern is suitable for expressing non-linearity rather than using other non-linear demand patterns, such as a quadratic function. A Cobb-Douglas function is perfect for this situation, where the output depends on the inputs. The demand is influenced by selling price, marketing expenditure, and green innovation for the dual channel. The online and offline demand are $q_1(\vartheta_1, \kappa_1, \varsigma_1) = \phi_1 \vartheta_1^{-\mu_1} \kappa_1^{\lambda_1} \varsigma_1^{Z_1}$ and $q_2(\vartheta_2, \kappa_2, \varsigma_2) = \phi_2 \vartheta_2^{-\mu_2} \kappa_2^{\lambda_2} \varsigma_2^{Z_2}$, respectively [74, 75].

Table 2. Notation.

Index	
i	= 1,2; 1 = online channel, 2 = offline channel
Decision variables	
ϑ_i	Retail price (\$/unit)
ε_i	Wholesale price (\$/unit)
κ_i	Green innovation
ς_i	Marketing expenditure per unit of product (\$/unit)
Parameters	
ω_i	Retailer's profit margin (\$/unit)
λ_i	Customer sensitivity for the green innovation $0 < \lambda_i < 1$
μ_i	Customer sensitivity to the selling price $\mu_i > \lambda_i + 1$
Z_i	Customer sensitivity for marketing expenditure $0 < Z_i < 1$
P_i	Production cost of the product (\$/unit)
q_i	Demand (unit/time unit)
ϕ_i	Total population of potential customers $\phi_i > 0$
Y_i	Investment parameter on green innovation (\$)
v_i	Variable transportation cost of the manufacturer, including the maintenance policy (\$/unit)
v_i'	Variable carbon emissions tax of the manufacturer for transportation (\$/unit)
A_i	Fixed transportation cost of the manufacturer including maintenance policy (\$)
A_i'	Fixed carbon emissions tax of the manufacturer for transportation (\$)
τ_i	Variable transportation cost of the retailer including maintenance policy (\$/unit)
τ_i'	Variable carbon emissions tax of the retailer for transportation (\$/unit)
H_i	Fixed transportation cost of the retailer including maintenance policy (\$)
H_i'	Fixed carbon emissions tax of the retailer from transportation (\$)
Others	
π_M	Manufacturer's profit (\$)
π_R	Retailer's profit (\$)
π_{SC}	SCM's profit (\$)

- (3) The manufacturer invests $\sum_{i=1}^2 Y_i \kappa_i^2$ in green innovation for making products. The manufacturer has not considered the shortage, holding, and backordering costs.
- (4) The marginal profit of the retailer per unit product is $\omega_i = \vartheta_i - \varepsilon_i - \varsigma_i$. The retailer has not considered the holding and ordering costs.
- (5) For buying products from the manufacturer, the retailer spends the transportation cost $\sum_{i=1}^2 \tau_i q_i (\vartheta_i, \kappa_i) + H_i$ including maintenance policies for loading, unloading, inspection, and

documentation. The manufacturer spends the transportation cost $\sum_{i=1}^2 v_i q_i(\vartheta_i, \kappa_i)$ including some maintenance policies, such as manufacturer collection of raw materials for buying materials.

- (6) Both the manufacturer and the retailer spend a fixed (A'_i and H'_i) and variable (v'_i and τ'_i) carbon emissions tax during transportation. The model does not explicitly quantify the physical amount of carbon emissions in kg or tonnes from the system. Instead, it considers the taxes associated with carbon emissions from transportation as a part of the total system cost. That is, both of the SCM players pay some carbon taxes depending on the transported quantity. It is well-known that overloaded trucks emit more carbon than loaded or empty trucks. Therefore, the objective is to minimize the total cost considering the emissions cost rather than directly minimizing the quantity of emissions or emphasizing any emissions reduction policy.

4. Mathematical model

This section discusses the mathematical model for both decentralized and centralized policies. The total market demand equation is given in Eq (4.1).

$$q(\vartheta, \kappa, \varsigma) = q_1(\vartheta_1, \kappa_1, \varsigma_1) + q_2(\vartheta_2, \kappa_2, \varsigma_2) = \sum_{i=1}^2 \phi_i \vartheta_i^{-\mu_i} \kappa_i^{\lambda_i} \varsigma_i^{Z_i} \quad (4.1)$$

The first term ($i = 1$) of Eq (4.1) indicates the independent online demand and the second term ($i = 2$) indicates the independent offline demand, i.e., both types of demand are independent of each other. There is no cross-price elasticity or other interdependency. The manufacturer supplies all products to the retailer to satisfy the demand from both independent channels. The manufacturer's profit is given by Eq (4.2).

$$\pi_M = \sum_{i=1}^2 \left((\varepsilon_i - P_i - v_i - v'_i) q_i - Y_i \kappa_i^2 - A_i - A'_i \right) \quad (4.2)$$

The first term of Eq (4.2) is the wholesale price followed by the production cost, variable transportation and the related carbon emissions taxes, green innovation cost, and fixed transportation and the related carbon emissions taxes. The retailer's profit is given by Eq (4.3).

$$\pi_R = \sum_{i=1}^2 \left((\vartheta_i - \varepsilon_i - \varsigma_i - \tau_i - \tau'_i) q_i - H_i - H'_i \right) \quad (4.3)$$

The first term of Eq (4.3) is selling price, followed by the wholesale price, marketing expenditure, variable transportation and the related carbon emissions taxes, and fixed transportation and the related carbon emissions taxes. The total profit of the supply chain is given by Eq (4.4).

$$\pi_{sc} = \sum_{i=1}^2 \left((\vartheta_i - P_i - \varsigma_i - v_i - v'_i - \tau_i - \tau'_i) q_i - Y_i \kappa_i^2 - A_i - A'_i - H_i - H'_i \right) \quad (4.4)$$

The first term of Eq (4.4) is selling price, followed by production cost, marketing expenditure, variable transportation and the related carbon emissions taxes, green innovation cost, and fixed transportation and the related carbon emissions taxes.

4.1. Decentralized decision-making model

In this subsection, the decentralized policy is discussed; that is, both the manufacturer and retailer makes their decision individually. To find individual decisions, a backward Stackelberg game strategy is used, where one supply chain player acts as a leader and other one as follower. In this Stackelberg game, the manufacturer acts as a leader and the retailer acts as a follower. According to the backward Stackelberg game strategy, the leader gives a chance to the follower to make the decision first because when the leader makes decisions, those are the final decisions. The follower never has a chance to make any decision. First, the retailer makes its own decision, and the manufacturer makes the final decision. Solutions are given in the next section. By putting Eq (4.1) into Eq (4.3), Eq (4.5) is obtained.

$$\pi_R(m) = \sum_{i=1}^2 \left((\omega_i - \tau_i - \tau'_i) \phi_i(\omega_i + \varepsilon_i + \varsigma_i)^{-\mu_i} M_i^{\gamma_i} \kappa_i^{\lambda_i} - H_i - H'_i \right) \quad (4.5)$$

The profit margin ω_i is used instead of the selling price ϑ_i .

Now, the profit equation of the manufacturer is obtained from Eqs (4.1) and (4.2), which is as follows:

$$\pi_M(\varepsilon_i, \kappa_i) = \sum_{i=1}^2 \left((\varepsilon_i - P_i - v_i + v'_i) \phi_i(\omega_i + \varepsilon_i + \varsigma_i)^{-\mu_i} \kappa_i^{\lambda_i} M^{Z_i} - Y_i \kappa_i^2 - A_1 - A'_1 \right). \quad (4.6)$$

Finally, the maximum profit of manufacturer and retailer are obtained in Eqs (4.7) and (4.8), respectively.

$$\begin{aligned} \pi_M^* = & \sum_{i=1}^2 \left(\left(\frac{(P_i + \tau_i + \tau'_i + v_i + v'_i)}{(\mu_i - 1 - Z_i)} \right) \left(\frac{(\mu_i - Z_i)(P_i + v_i + v'_i + \tau_i + \tau'_i)\mu_i}{(\mu_i - 1 - Z_i)^2} \right)^{-\mu_i} \left(\frac{Z_i(\mu_i - Z_i)(P_i + v_i + v'_i + \tau_i + \tau'_i)}{(\mu_i - 1 - Z_i)^2} \right)^{Z_i} \right. \\ & \exp \left(\left(\ln 2 - Z_i \ln \left(\frac{Z_i(\mu_i - Z_i)(P_i + v_i + v'_i + \tau_i + \tau'_i)}{(\mu_i - 1 - Z_i)^2} \right) + \mu_i \ln \left(\frac{(\mu_i - Z_i)(P_i + v_i + v'_i + \tau_i + \tau'_i)\mu_i}{(\mu_i - 1 - Z_i)^2} \right) \right. \right. \\ & \left. \left. + \ln \left(\frac{Y_i(\mu_i - 1 - Z_i)}{\lambda_i \phi_i(P_i + \tau_i + \tau'_i + v_i + v'_i)} \right) \right) / (\lambda_i - 2) \right) - A_i - A'_i - Y_i \times \exp \left(\left(2 \ln 2 - 2Z_i \right. \right. \\ & \left. \left. \ln \left(\frac{Z_i(\mu_i - Z_i)(P_i + v_i + v'_i + \tau_i + \tau'_i)}{(\mu_i - 1 - Z_i)^2} \right) + 2\mu_i \ln \left(\frac{(\mu_i - Z_i)(P_i + v_i + v'_i + \tau_i + \tau'_i)\mu_i}{(\mu_i - 1 - Z_i)^2} \right) \right. \right. \\ & \left. \left. + 2 \ln \left(\frac{Y_i(\mu_i - 1 - Z_i)}{\lambda_i \phi_i(P_i + \tau_i + \tau'_i + v_i + v'_i)} \right) \right) / (\lambda_i - 2) \right) \end{aligned} \quad (4.7)$$

$$\begin{aligned} \pi_R^* = & \sum_{i=1}^2 \left[\left(\frac{(P_i + \tau_i + \tau'_i + v_i + v'_i)(\mu_i - Z_i)}{(\mu_i - 1 - Z_i)^2} \right) \left(\frac{(\mu_i - Z_i)(P_i + v_i + v'_i + \tau_i + \tau'_i)\mu_i}{(\mu_i - 1 - Z_i)^2} \right)^{-\mu_i} \exp \left(\left(\ln 2 - Z_i \right. \right. \right. \\ & \left. \left. \ln \left(\frac{Z_i(\mu_i - Z_i)(P_i + v_i + v'_i + \tau_i + \tau'_i)}{(\mu_i - 1 - Z_i)^2} \right) + \mu_i \ln \left(\frac{(\mu_i - Z_i)(P_i + v_i + v'_i + \tau_i + \tau'_i)\mu_i}{(\mu_i - 1 - Z_i)^2} \right) \right. \right. \\ & \left. \left. + \ln \left(\frac{Y_i(\mu_i - 1 - Z_i)}{\lambda_i \phi_i(P_i + \tau_i + \tau'_i + v_i + v'_i)} \right) \right) / (\lambda_i - 2) \right) \left(\frac{Z_i(\mu_i - Z_i)(P_i + v_i + v'_i + \tau_i + \tau'_i)}{(\mu_i - 1 - Z_i)^2} \right)^{Z_i} - H_i - H'_i \right] \end{aligned} \quad (4.8)$$

4.2. Centralized decision-making model

According to the centralized decision-making scenario, the manufacturer and retailer make decisions collaboratively. The profit of the supply chain is

$$\pi_{SC}(\vartheta_1, \vartheta_2, \kappa_1, \kappa_2, \varsigma_1, \varsigma_2) = \sum_{i=1}^2 \left[(\vartheta_i - P_i - \varsigma_i - v_i + v'_i - \tau_i + \tau'_i) q_i - Y_i \kappa_i^2 - A_i - A'_i - H_i - H' \right]. \quad (4.9)$$

The first term of Eq (4.9) is selling price, followed by production cost, marketing expenditure, variable transportation and the related emissions costs, green innovation cost, and fixed transportation and the related emissions costs. Finally, the maximum profit of SCM is obtained in Eq (4.10).

$$\begin{aligned} \pi_{SC}^* = & \sum_{i=1}^2 \left[\frac{(P_i + \tau_i + \tau'_i + v_i + v'_i) \phi_i}{\mu_i - 1 - Z_i} \left(\frac{(P_i + v_i + v'_i + \tau_i + \tau'_i) \mu_i}{(\mu_i - 1 - Z_i)} \right)^{-\mu_i} \right. \\ & \left. \left(e^{\frac{\ln(2) + \ln \left(\frac{Y_i(\mu_i - 1 - Z_i)}{\lambda_i \phi_i (P_i + v_i + v'_i + \tau_i + \tau'_i)} \right) - Z_i \ln \left(\frac{(P_i + v_i + v'_i + \tau_i + \tau'_i) Z_i}{\mu_i - 1 - Z_i} \right) + \mu_i \ln \left(\frac{(P_i + v_i + v'_i + \tau_i + \tau'_i) \mu_i}{Z_i - \mu_i + 1} \right)}{\lambda_i - 2}} \right)^{\lambda_i} \left(\frac{Z_i (P_i + \tau_i + \tau'_i + v_i + v'_i)}{\mu_i - Z_i - 1} \right)^{Z_i} \right. \\ & \left. - A_i - A'_i - H_i - H' - Y_i \left(e^{\frac{\ln(2) + \ln \left(\frac{Y_i(\mu_i - 1 - Z_i)}{\lambda_i \phi_i (P_i + v_i + v'_i + \tau_i + \tau'_i)} \right) - Z_i \ln \left(\frac{(P_i + v_i + v'_i + \tau_i + \tau'_i) Z_i}{\mu_i - 1 - Z_i} \right) + \mu_i \ln \left(\frac{(P_i + v_i + v'_i + \tau_i + \tau'_i) \mu_i}{Z_i - \mu_i + 1} \right)}{\lambda_i - 2}} \right)^2 \right] \end{aligned} \quad (4.10)$$

5. Solution methodology

The solution methodology is explained for centralized and decentralized policy in this section.

5.1. Decentralized decision-making model

Stackelberg game policy is used to solve the decentralized problem, where the manufacturer acts as a leader and the retailer acts as a follower. The retailer makes the first decision. Based on the retailer's decision, the manufacturer subsequently makes decisions.

5.1.1. Retailer's decision

Equations (5.1) and (5.2) are obtained by differentiating of the Eq (4.5) with respect to ω_i , and ς_i (see Appendix for detailed calculations).

$$\omega_i(\varepsilon_i, \varsigma_i) = \frac{\varsigma_i + \varepsilon_i + \mu_i(\tau_i + \tau'_i)}{\mu_i - 1} \quad (5.1)$$

$$\varsigma_i(\varepsilon_i) = \frac{Z_i \varepsilon_i + (\tau_i + \tau'_i) Z_i}{\mu_i - 1 - Z_i} \quad (5.2)$$

Using Eqs (5.1) and (5.2), Eq (5.3) is obtained for the retailer's updated profit margin.

$$\omega_i(\varepsilon_i) = \frac{\varepsilon_i - (\tau_i + \tau'_i) Z_i + \mu_i(\tau_i + \tau'_i)}{\mu_i - 1 - Z_i} \quad (5.3)$$

The sufficient condition for global optimality states that a function reaches a global optimum at a unique point if the principal minors of its Hessian matrix change sign alternately as the optimal decision variables vary. That is, global optimality is guaranteed when the principal minors of the Hessian matrix have an alternating sign pattern. The global profitability of Eq (4.5) at the unique solutions above is established by Theorem 1 with the help of Lemmas 1, 2, and 3.

Theorem 1. The Hessian matrix of order 4 of Eq (4.5) is calculated because ς_i and ω_i are the retailer's decision variables. Hence, the principal minors of the Hessian matrix are $|H_{1 \times 1}| < 0$, $|H_{2 \times 2}| > 0$, $|H_{3 \times 3}| < 0$, and $|H_{4 \times 4}| > 0$.

Proof. The proof of the theorem is shown in the Appendix.

Lemma 1. The value of first-order principal minor becomes negative when $\omega_1 + 2(\varsigma_1 + \omega_1) + (\tau_1 + \tau'_1)(1 + \mu_1) > \omega_1\mu_1$.

Proof. The first-order derivative with respect to ω_1 is $H_{1 \times 1} = \frac{\partial^2 \pi_R}{\partial \omega_1^2} = -\phi_1\mu_1\kappa_1^{\lambda_1}\varsigma_1^{Z_1}[\omega_1 + 2(\varsigma_1 + \omega_1) - \omega_1\mu_1 + (\tau_1 + \tau'_1)(1 + \mu_1)](\omega_1 + \varepsilon_1 + \varsigma_1)^{-\mu_1-2}$. For a global maximum, $H_{1 \times 1} < 0$ which implies $-\phi_1\mu_1\kappa_1^{\lambda_1}\varsigma_1^{Z_1}[\omega_1 + 2(\varsigma_1 + \omega_1) - \omega_1\mu_1 + (\tau_1 + \tau'_1)(1 + \mu_1)](\omega_1 + \varepsilon_1 + \varsigma_1)^{-\mu_1-2} < 0$, i.e., $\phi_1\mu_1\kappa_1^{\lambda_1}\varsigma_1^{Z_1}[\omega_1 + 2(\varsigma_1 + \omega_1) - \omega_1\mu_1 + (\tau_1 + \tau'_1)(1 + \mu_1)](\omega_1 + \varepsilon_1 + \varsigma_1)^{-\mu_1-2} > 0$. When $\phi_1\mu_1\kappa_1^{\lambda_1}\varsigma_1^{Z_1}[\omega_1 + 2(\varsigma_1 + \omega_1) - \omega_1\mu_1 + (\tau_1 + \tau'_1)(1 + \mu_1)] > 0$ as $(\omega_1 + \varepsilon_1 + \varsigma_1)^{-\mu_1-2} > 0$ i.e., $\omega_1 + 2(\varsigma_1 + \omega_1) - \omega_1\mu_1 + (\tau_1 + \tau'_1)(1 + \mu_1) > 0$ as $\phi_1\mu_1\kappa_1^{\lambda_1}\varsigma_1^{Z_1} > 0$, i.e., $\omega_1 + 2(\varsigma_1 + \omega_1) + (\tau_1 + \tau'_1)(1 + \mu_1) > \omega_1\mu_1$. Thus, the lemma holds if $\omega_1 + 2(\varsigma_1 + \omega_1) + (\tau_1 + \tau'_1)(1 + \mu_1) > \omega_1\mu_1$. Hence, the proof is completed.

Lemma 2. The value of the second-order principal minor becomes positive when $2\varsigma_2 + 3\omega_2 + (\tau_2 + \tau'_2)(1 + \mu_2) < \omega_2\mu_2$.

Proof. The second-order derivative is $H_{2 \times 2} = -\phi_2\mu_2\kappa_2^{\lambda_2}\varsigma_2^{Z_2}(2\varsigma_2 + 3\omega_2 - \omega_2\mu_2 + (\tau_2 + \tau'_2)(1 + \mu_2))(\omega_2 + \varepsilon_2\varsigma_2)^{-\mu_2-2}$. For a global maximum, $H_{2 \times 2} > 0$, which implies $-\phi_2\mu_2\kappa_2^{\lambda_2}\varsigma_2^{Z_2}(2\varsigma_2 + 3\omega_2 - \omega_2\mu_2 + (\tau_2 + \tau'_2)(1 + \mu_2))(\omega_2 + \varepsilon_2\varsigma_2)^{-\mu_2-2} > 0$, i.e., $\phi_2\mu_2\kappa_2^{\lambda_2}\varsigma_2^{Z_2}(2\varsigma_2 + 3\omega_2 - \omega_2\mu_2 + (\tau_2 + \tau'_2)(1 + \mu_2))(\omega_2 + \varepsilon_2\varsigma_2)^{-\mu_2-2} < 0$. This implies that $2\varsigma_2 + 3\omega_2 - \omega_2\mu_2 + (\tau_2 + \tau'_2)(1 + \mu_2) < 0$, as $\phi_2\mu_2\kappa_2^{\lambda_2}\varsigma_2^{Z_2} > 0$ and $(\omega_2 + \varepsilon_2\varsigma_2)^{-\mu_2-2} > 0$. Thus, the lemma holds if $2\varsigma_2 + 3\omega_2 + (\tau_2 + \tau'_2)(1 + \mu_2) < \omega_2\mu_2$. Hence, the proof is completed.

Lemma 3. The value of the third-order principal minor becomes negative when the following condition holds:

$\varsigma_1\mu_1(\omega_1 + 2(\varsigma_1 + \omega_1) + (\tau_1 + \tau'_1)(1 + \mu_1)) + \varsigma_1^2(Z_1 - \mu_1) + (\tau_1 + \tau'_1)\mu_1((\omega_1 + \varepsilon_1)Z_1 + \varsigma_1(Z_1 - 1 - \mu_1)) + \varsigma_1\omega_1(\mu_1^2 + 2Z_1 - \mu_1Z_1) > \varsigma_1\mu_1^2\omega_1 + (\omega_1 + \varepsilon_1)(\omega_1(\mu_1 - 1) - \varepsilon_1)Z_1 + \varsigma_1\varepsilon_1(\mu_1 - 2Z_1)$.

Proof. The third-order derivative is $H_{3 \times 3} = -\phi_1\kappa_1^{\lambda_1}[\mu_1\varsigma_1^{Z_1}(\omega_1 + 2(\varsigma_1 + \omega_1) - \omega_1\mu_1 + (\tau_1 + \tau'_1)(1 + \mu_1))(\omega_1 + \varepsilon_1 + \varsigma_1)^{-\mu_1-2} + \varsigma_1^{Z_1-1}(\omega_1 + \varsigma_1 + \varepsilon_1)^{-\mu_1-2}(-((\omega_1 + \varepsilon_1)(\omega_1(\mu_1 - 1) - \varepsilon_1)Z_1) + \varsigma_1^2(Z_1 - \mu_1) + (\tau_1 + \tau'_1)\mu_1((\omega_1 + \varepsilon_1)Z_1 + \varsigma_1(Z_1 - 1 - \mu_1)) + \varsigma_1(-\varepsilon_1(\mu_1 - 2Z_1) + \omega_1(\mu_1^2 + 2Z_1 - \mu_1Z_1)))]$. For a global maximum, $H_{3 \times 3} < 0$ which implies either $\phi_1\kappa_1^{\lambda_1} > 0$, or $\mu_1\varsigma_1^{Z_1}(\omega_1 + 2(\varsigma_1 + \omega_1) - \omega_1\mu_1 + (\tau_1 + \tau'_1)(1 + \mu_1))(\omega_1 + \varepsilon_1 + \varsigma_1)^{-\mu_1-2} + \varsigma_1^{Z_1-1}(\omega_1 + \varsigma_1 + \varepsilon_1)^{-\mu_1-2}(-((\omega_1 + \varepsilon_1)(\omega_1(\mu_1 - 1) - \varepsilon_1)Z_1) + \varsigma_1^2(Z_1 - \mu_1) + (\tau_1 + \tau'_1)\mu_1((\omega_1 + \varepsilon_1)Z_1 + \varsigma_1(Z_1 - 1 - \mu_1)) + \varsigma_1(-\varepsilon_1(\mu_1 - 2Z_1) + \omega_1(\mu_1^2 + 2Z_1 - \mu_1Z_1))) > 0$. As

$\phi_1 \kappa_1^{\lambda_1} > 0$, is positive, from the second condition, $\varsigma_1 \mu_1 (\omega_1 + 2(\varsigma_1 + \omega_1) + (\tau_1 + \tau'_1)(1 + \mu_1)) + \varsigma_1^2 (Z_1 - \mu_1) + (\tau_1 + \tau'_1) \mu_1 ((\omega_1 + \varepsilon_1) Z_1 + \varsigma_1 (Z_1 - 1 - \mu_1)) + \varsigma_1 \omega_1 (\mu_1^2 + 2Z_1 - \mu_1 Z_1) > \varsigma_1 \mu_1^2 \omega_1 + (\omega_1 + \varepsilon_1)(\omega_1 (\mu_1 - 1) - \varepsilon_1) Z_1 + \varsigma_1 \varepsilon_1 (\mu_1 - 2Z_1)$. Hence, the proof is completed.

5.1.2. Manufacturer's decision

Putting the value of $\omega_i(\varepsilon_i)$ and $\varsigma_i(\varepsilon_i)$ from Eqs (5.2) and (5.3) in Eq (4.6) and differentiate with respect to ε_i , and $\varkappa_i$, Eqs (5.4) and (5.5) are obtained (see Appendix for details).

$$\varepsilon_i^* = \frac{\tau_i + \tau'_i + (P_i + v_i + v'_i)(\mu_i - Z_i)}{\mu_i - 1 - \gamma} \quad (5.4)$$

$$\varkappa_i^* = \exp \left(\frac{\ln 2 - \mu_i \ln \left(\frac{\mu_i(\tau_i + \tau'_i + \varepsilon_i)}{\mu_i - 1} \right) + Z_i \ln \left(\frac{(\tau_i + \tau'_i + \varepsilon_i) Z_i}{(\mu_i - 1 - Z_i)} \right) + \ln \left(\frac{Y_i}{\lambda_i \phi_i(\varepsilon_i - v_i - v'_i - P_i)} \right)}{\lambda_i - 2} \right) x_i^{\lambda_i} \varsigma_i^{Z_i} \quad (5.5)$$

Substituting the value ε_i^* in Eqs (5.2) and (5.3), the optimal values ς_i^* and ω_i^* are obtained via Eqs (5.6) and (5.7).

$$\varsigma_i^* = \frac{Z_i(\mu_i - Z_i)(P_i + v_i + v'_i + \tau_i + \tau'_i)}{(\mu_i - 1 - Z_i)^2} \quad (5.6)$$

$$\omega_i^* = \frac{\tau_i + \tau'_i + (P_i + v_i + v'_i - \tau_i - \tau'_i - (\tau_i + \tau'_i) Z_i + (\tau_i + \tau'_i) \mu_i)(\mu_i - Z_i)}{(1 - \mu_i + Z_i)^2} \quad (5.7)$$

Therefore, the optimal value of the dependent variable, i.e., retail price, of the retailer is

$$\vartheta_i^* = \frac{(\mu_i - Z_i)(P_i + v_i + v'_i + \tau_i + \tau'_i) \mu_i}{(\mu_i - 1 - Z_i)^2}. \quad (5.8)$$

Putting the value of ε_i^* , ϑ_i^* , and ς_i^* in Eq (5.5), Eq (5.9) is obtained as

$$\begin{aligned} \varkappa_i^* = & \exp \left(\left(\ln 2 - Z_i \ln \left(\frac{Z_i(\mu_i - Z_i)(P_i + v_i + v'_i + \tau_i + \tau'_i)}{(\mu_i - 1 - Z_i)^2} \right) + \mu_i \ln \left(\frac{(\mu_i - Z_i)(P_i + v_i + v'_i + \tau_i + \tau'_i) \mu_i}{(\mu_i - 1 - Z_i)^2} \right) \right. \right. \\ & \left. \left. + \ln \left(\frac{Y_i(\mu_i - 1 - Z_i)}{\lambda_i \phi_i(P_i + \tau_i + \tau'_i + v_i + v'_i)} \right) \right) / (\lambda_i - 2) \right) \end{aligned} \quad (5.9)$$

The global profitability of Eq (4.6) at the unique solutions above is established by Theorem 2 along with Lemmas 4, 5, and 6.

Theorem 2. The Hessian matrix for Eq (4.6) of order 4 is calculated because ε and $\varkappa$ are the retailer's decision variables. Hence, the principal minors of the Hessian matrix are $|H_{1 \times 1}| < 0$, $|H_{2 \times 2}| > 0$, $|H_{3 \times 3}| < 0$, and $|H_{4 \times 4}| > 0$.

Proof. The proof is shown in Appendix.

Lemma 4. The value of the first-order principal minor becomes negative when $2(\tau_1 + \tau'_1) + (v_1 + v'_1)(1 + \mu_1) + \varepsilon_1(1 + Z_1) + P_1(1 + \mu_1 - Z_1) > \varepsilon_1 \mu_1 + (v_1 + v'_1) Z_1$.

Proof. The value of the first-order principal minor is $H_{1 \times 1} = -\phi_1(\mu_1 - Z_1)\kappa_1^{\lambda_1} \frac{1}{((\tau_1 + \tau'_1) + \varepsilon_1)^2} \left(\frac{((\tau_1 + \tau'_1) + \varepsilon_1)\mu_1}{\mu_1 - Z_1 - 1} \right)^{-\mu_1} \left(\frac{((\tau_1 + \tau'_1) + \varepsilon_1)Z_1}{\mu_1 - Z_1 - 1} \right)^{Z_1} \left[2(\tau_1 + \tau'_1) + (v_1 + v'_1) + \varepsilon_1 + (v_1 + v'_1)\mu_1 - \varepsilon_1\mu_1 + P_1(1 + \mu_1 - Z_1) - (v_1 + v'_1)Z_1 + \varepsilon_1Z_1 \right]$. For a global maximum, the value should be $H_{1 \times 1} < 0$, i.e., $2(\tau_1 + \tau'_1) + (v_1 + v'_1) + \varepsilon_1 + (v_1 + v'_1)\mu_1 - \varepsilon_1\mu_1 + P_1(1 + \mu_1 - Z_1) - (v_1 + v'_1)Z_1 + \varepsilon_1Z_1 > 0$, as $\phi_1 > 0$, $(\mu_1 - Z_1) > 0$, $\kappa_1^{\lambda_1} > 0$, $\frac{1}{((\tau_1 + \tau'_1) + \varepsilon_1)^2} > 0$, $\left(\frac{((\tau_1 + \tau'_1) + \varepsilon_1)\mu_1}{\mu_1 - Z_1 - 1} \right)^{-\mu_1} > 0$, and $\left(\frac{((\tau_1 + \tau'_1) + \varepsilon_1)Z_1}{\mu_1 - Z_1 - 1} \right)^{Z_1} > 0$. This implies that $2(\tau_1 + \tau'_1) + (v_1 + v'_1) + \varepsilon_1 + (v_1 + v'_1)\mu_1 + P_1(1 + \mu_1 - Z_1) + \varepsilon_1Z_1 > \varepsilon_1\mu_1 + (v_1 + v'_1)Z_1$, i.e., $2(\tau_1 + \tau'_1) + (v_1 + v'_1)(1 + \mu_1) + \varepsilon_1(1 + Z_1) + P_1(1 + \mu_1 - Z_1) > \varepsilon_1\mu_1 + (v_1 + v'_1)Z_1$. Hence, the proof is completed.

Lemma 5. The value of the second-order principal minor becomes positive when $2(\tau_2 + \tau'_2) + (v_2 + v'_2)(1 + \mu_2) + \varepsilon_2(1 + Z_2) + P_2(1 + \mu_2 - Z_2) > \varepsilon_2\mu_2 + (v_2 + v'_2)Z_2$.

Proof. The value of the second-order principal minor is $H_{2 \times 2} > 0$. To satisfy this, $H_{2 \times 2} = -\phi_2(\mu_2 - Z_2)\kappa_2^{\lambda_2} \frac{1}{((\tau_2 + \tau'_2) + \varepsilon_2)^2} \left(\frac{((\tau_2 + \tau'_2) + \varepsilon_2)\mu_2}{\mu_2 - Z_2 - 1} \right)^{-\mu_2} \left(\frac{((\tau_2 + \tau'_2) + \varepsilon_2)Z_2}{\mu_2 - Z_2 - 1} \right)^{Z_2} \left[2(\tau_2 + \tau'_2) + (v_2 + v'_2) + \varepsilon_2 + (v_2 + v'_2)\mu_2 - \varepsilon_2\mu_2 + P_2(1 + \mu_2 - Z_2) - (v_2 + v'_2)Z_2 + \varepsilon_2Z_2 \right] > 0$, which implies $-\phi_2(\mu_2 - Z_2)\kappa_2^{\lambda_2} \frac{1}{((\tau_2 + \tau'_2) + \varepsilon_2)^2} \left(\frac{((\tau_2 + \tau'_2) + \varepsilon_2)\mu_2}{\mu_2 - Z_2 - 1} \right)^{-\mu_2} \left(\frac{((\tau_2 + \tau'_2) + \varepsilon_2)Z_2}{\mu_2 - Z_2 - 1} \right)^{Z_2} \left[2(\tau_2 + \tau'_2) + (v_2 + v'_2) + \varepsilon_2 + (v_2 + v'_2)\mu_2 - \varepsilon_2\mu_2 + P_2(1 + \mu_2 - Z_2) - (v_2 + v'_2)Z_2 + \varepsilon_2Z_2 \right] < 0$. This implies that $2(\tau_2 + \tau'_2) + (v_2 + v'_2) + \varepsilon_2 + (v_2 + v'_2)\mu_2 - \varepsilon_2\mu_2 + P_2(1 + \mu_2 - Z_2) - (v_2 + v'_2)Z_2 + \varepsilon_2Z_2 > 0$, as $\phi_2 > 0$, $(\mu_2 - Z_2) > 0$, $\kappa_2^{\lambda_2} > 0$, $\frac{1}{((\tau_2 + \tau'_2) + \varepsilon_2)^2} > 0$, $\left(\frac{((\tau_2 + \tau'_2) + \varepsilon_2)\mu_2}{\mu_2 - Z_2 - 1} \right)^{-\mu_2} > 0$, and $\left(\frac{((\tau_2 + \tau'_2) + \varepsilon_2)Z_2}{\mu_2 - Z_2 - 1} \right)^{Z_2} > 0$. Thus, $2(\tau_2 + \tau'_2) + (v_2 + v'_2) + \varepsilon_2 + (v_2 + v'_2)\mu_2 + P_2(1 + \mu_2 - Z_2) + \varepsilon_2Z_2 > \varepsilon_2\mu_2 + (v_2 + v'_2)Z_2$, i.e., $2(\tau_2 + \tau'_2) + (v_2 + v'_2)(1 + \mu_2) + \varepsilon_2(1 + Z_2) + P_2(1 + \mu_2 - Z_2) > \varepsilon_2\mu_2 + (v_2 + v'_2)Z_2$. Hence, the proof is completed.

Lemma 6. The value of the third-order principal minor becomes negative when $(\tau_1 + \tau'_1 + \varepsilon_1)[(\tau_1 + \tau'_1) + (P_1 + v_1 + v'_1)(\mu_1 - Z_1) + \varepsilon_1(1 - \mu_1 + Z_1)] < x_1[2(\tau_1 + \tau'_1) + (v_1 + v'_1)(1 + \mu_1 - Z_1) + \varepsilon_1(1 - \mu_1 + Z_1) + P_1(1 + \mu_1 - Z_1)]$.

Proof. The value of the third-order principal minor is $H_{3 \times 3} < 0$. To satisfy this condition, $-\phi_1(\mu_1 - Z_1)\kappa_1^{\lambda_1} \frac{1}{((\tau_1 + \tau'_1) + \varepsilon_1)^2} \left(\frac{((\tau_1 + \tau'_1) + \varepsilon_1)\mu_1}{\mu_1 - Z_1 - 1} \right)^{-\mu_1} \left(\frac{((\tau_1 + \tau'_1) + \varepsilon_1)Z_1}{\mu_1 - Z_1 - 1} \right)^{Z_1} [2(\tau_1 + \tau'_1) + (v_1 + v'_1) + \varepsilon_1 + (v_1 + v'_1)\mu_1 - \varepsilon_1\mu_1 + P_1(1 + \mu_1 - Z_1) - (v_1 + v'_1)Z_1 + \varepsilon_1Z_1] + \phi_1\kappa_1^{\lambda_1-1} \frac{1}{(\tau_1 + \tau'_1) + \varepsilon_1} \left(\frac{((\tau_1 + \tau'_1) + \varepsilon_1)\mu_1}{\mu_1 - Z_1 - 1} \right)^{-\mu_1} \left(\frac{((\tau_1 + \tau'_1) + \varepsilon_1)Z_1}{\mu_1 - Z_1 - 1} \right)^{Z_1} [(\tau_1 + \tau'_1) + (P_1 + v_1 + v'_1)(\mu_1 - Z_1) + \varepsilon_1(1 - \mu_1 + Z_1)] < 0$. As $\phi_1 > 0$, $\kappa_1^{\lambda_1} > 0$, $\frac{1}{(\tau_1 + \tau'_1) + \varepsilon_1} > 0$, $\left(\frac{((\tau_1 + \tau'_1) + \varepsilon_1)\mu_1}{\mu_1 - Z_1 - 1} \right)^{-\mu_1} > 0$, and $\left(\frac{((\tau_1 + \tau'_1) + \varepsilon_1)Z_1}{\mu_1 - Z_1 - 1} \right)^{Z_1} > 0$, and thus, $(\tau_1 + \tau'_1 + \varepsilon_1)[(\tau_1 + \tau'_1) + (P_1 + v_1 + v'_1)(\mu_1 - Z_1) + \varepsilon_1(1 - \mu_1 + Z_1)] < x_1[2(\tau_1 + \tau'_1) + (v_1 + v'_1)(1 + \mu_1 - Z_1) + \varepsilon_1(1 - \mu_1 + Z_1) + P_1(1 + \mu_1 - Z_1)]$. Hence, the proof is completed.

5.2. Centralized policy

In this policy, the manufacturer and the retailer make a decision collaboratively.

The equations (5.10)–(5.12) are obtained by differentiating of the Eq (4.9) with respect to ϑ_i , λ_i , and ς_i (Appendix).

$$\vartheta_i^* = \frac{\mu_i(P_i + \varsigma_i + v_i + v'_i + \tau_i + \tau'_i)}{\mu_i - 1} \quad (5.10)$$

$$\varsigma_i^* = \frac{(\vartheta_i - \tau_i - \tau'_i - v_i - v'_i - P_i)Z_i}{1 + Z_i} \quad (5.11)$$

$$\kappa_i^* = e^{\frac{\ln(2) + \ln\left(\frac{Y_i}{\vartheta_i - P_i - \varsigma_i - v_i - v'_i - \tau_i - \tau'_i}\right) - Z_i \ln \varsigma_i + \mu_i \ln \vartheta_i}{\lambda_i - 2}} \quad (5.12)$$

Equations (5.13)–(5.15) are obtained after simplifying equations (5.10)–(5.12).

$$\varsigma_i^* = \frac{(P_i + v_i + v'_i + \tau_i + \tau'_i)Z_i}{\mu_i - 1 - Z_i} \quad (5.13)$$

$$\vartheta_i^* = \frac{(P_i + v_i + v'_i + \tau_i + \tau'_i)\mu_i}{Z_i - \mu_i + 1} \quad (5.14)$$

$$\kappa_i^* = e^{\frac{\ln(2) + \ln\left(\frac{Y_i(\mu_i - 1 - Z_i)}{\lambda_i \vartheta_i (P_i + v_i + v'_i + \tau_i + \tau'_i)}\right) - Z_i \ln\left(\frac{(P_i + v_i + v'_i + \tau_i + \tau'_i)Z_i}{\mu_i - 1 - Z_i}\right) + \mu_i \ln\left(\frac{(P_i + v_i + v'_i + \tau_i + \tau'_i)\mu_i}{Z_i - \mu_i + 1}\right)}{\lambda_i - 2}} \quad (5.15)$$

The global profitability of Eq (4.9) at the unique solutions above is established by Theorem 3 along with Lemmas 7, 8, 9, and 10.

Theorem 3. The Hessian matrix of order 6 for Eq (4.9) is calculated with respect to ς_i , ϑ_i , and κ_i are the retailer's decision variables. Hence, the principal minors of the Hessian matrix are $|H_{1 \times 1}| < 0$, $|H_{2 \times 2}| > 0$, $|H_{3 \times 3}| < 0$, $|H_{4 \times 4}| > 0$, $|H_{5 \times 5}| < 0$, and $|H_{6 \times 6}| > 0$.

Proof The proof is shown in the Appendix.

Lemma 7. The value of the first-order principal minor becomes negative when $\vartheta_1 + (v_1 + v'_1 + P_1 + \varsigma_1)(1 + \mu_1) + (\tau_1 + \tau'_1)\mu_1 < \vartheta_1\mu_1$.

Proof. The value of the first-order principal minor is $H_{1 \times 1} < 0$. To satisfy this condition, $-\phi_1\mu_1\kappa_1^{\lambda_1}M_1^{Z_1}\vartheta_1^{-\mu_1-2}\left[\vartheta_1 + P_1 + \varsigma_1 + (v_1 + v'_1) + P_1\mu_1 + (\tau_1 + \tau'_1)\mu_1 + \varsigma_1\mu_1 - \vartheta_1\mu_1 + (v_1 + v'_1)\mu_1\right] < 0$, which implies $\vartheta_1 + P_1 + \varsigma_1 + (v_1 + v'_1) + P_1\mu_1 + (\tau_1 + \tau'_1)\mu_1 + \varsigma_1\mu_1 - \vartheta_1\mu_1 + (v_1 + v'_1)\mu_1 < 0$, as $\phi_1\mu_1\kappa_1^{\lambda_1}M_1^{Z_1}\vartheta_1^{-\mu_1-2} > 0$. Thus, $\vartheta_1 + P_1 + \varsigma_1 + (v_1 + v'_1) + P_1\mu_1 + (\tau_1 + \tau'_1)\mu_1 + \varsigma_1\mu_1 + (v_1 + v'_1)\mu_1 < \vartheta_1\mu_1$, i.e., $\vartheta_1 + (v_1 + v'_1 + P_1 + \varsigma_1)(1 + \mu_1) + (\tau_1 + \tau'_1)\mu_1 < \vartheta_1\mu_1$. Hence, the proof.

Lemma 8. The value of the second-order principal minor becomes positive when $\vartheta_2 + (P_2 + \varsigma_2)(1 + \mu_2 + v_2 + v'_2) + (\tau_2 + \tau'_2)\mu_2 < \vartheta_2\mu_2$.

Proof. The value of the second-order principal minor is $H_{2 \times 2} > 0$. To satisfy this, $-\phi_2\mu_2\kappa_2^{\lambda_2}M_2^{Z_2}\vartheta_2^{-\mu_2-2}\left[\vartheta_2 + P_2 + \varsigma_2 + (v_2 + v'_2) + P_2\mu_2 + (\tau_2 + \tau'_2)\mu_2 + \varsigma_2\mu_2 - \vartheta_2\mu_2 + (v_2 + v'_2)\mu_2\right] < 0$, i.e., $\vartheta_2 + P_2 + \varsigma_2 + (v_2 + v'_2) + P_2\mu_2 + (\tau_2 + \tau'_2)\mu_2 + \varsigma_2\mu_2 - \vartheta_2\mu_2 + (v_2 + v'_2)\mu_2 < 0$, as $\phi_2\mu_2\kappa_2^{\lambda_2}M_2^{Z_2}\vartheta_2^{-\mu_2-2} > 0$. That is, $\vartheta_2 + (P_2 + \varsigma_2)(1 + \mu_2 + v_2 + v'_2) + (\tau_2 + \tau'_2)\mu_2 < \vartheta_2\mu_2$. Hence, the proof.

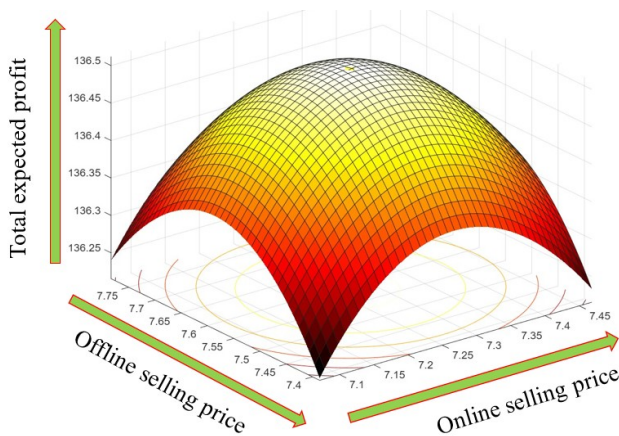
is completed.

Lemma 9. The value of the third-order principal minor becomes negative when $\vartheta_1[(\vartheta_1 + \mu_1(P_1 + \varsigma_1 + \tau_1 + \tau'_1 + v_1 + v'_1))Z_1 + \vartheta_1\mu_1] < \mu_1\varsigma_1[(P_1 + v_1 + v'_1 + \varsigma_1)(1 + \mu_1) + (\tau_1 + \tau'_1 - \vartheta_1)\mu_1]$.

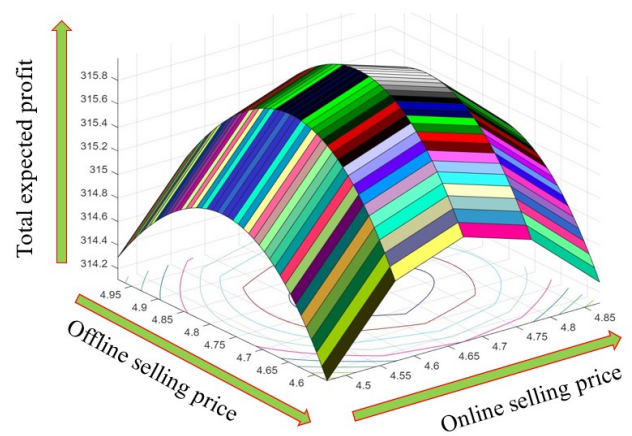
Proof. The value of the third-order principal minor is $H_{3 \times 3} < 0$. For satisfying the condition, $-\phi_1\mu_1\kappa_1^{\lambda_1}\varsigma_1^{Z_1}\vartheta_1^{-\mu_1-2}[\vartheta_1 + P_1 + \varsigma_1 + (v_1 + v'_1) + P_1\mu_1 + (\tau_1 + \tau'_1)\mu_1 + \varsigma_1\mu_1 - \vartheta_1\mu_1 + (v_1 + v'_1)\mu_1] + \phi_1\kappa_1^{\lambda_1}\varsigma_1^{Z_1-1}[(\vartheta_1 + \mu_1(P_1 + \varsigma_1 + (\tau_1 + \tau'_1) + (v_1 + v'_1)))Z_1 + (\varsigma_1 + \vartheta_1)\mu_1]\vartheta_1^{-\mu_1-1} < 0$. As $\phi_1\kappa_1^{\lambda_1}\varsigma_1^{Z_1}\vartheta_1^{-\mu_1-1} > 0$, $\vartheta_1[(\vartheta_1 + \mu_1(P_1 + \varsigma_1 + \tau_1 + \tau'_1 + v_1 + v'_1))Z_1 + \vartheta_1\mu_1] < \mu_1\varsigma_1[(P_1 + v_1 + v'_1 + \varsigma_1)(1 + \mu_1) + (\tau_1 + \tau'_1 - \vartheta_1)\mu_1]$. Hence, the proof is complete.

Lemma 10. The value of the fourth-order principal minor becomes positive when $(\vartheta_1 + \mu_1(P_1 + \varsigma_1 + (\tau_1 + \tau'_1) + (v_1 + v'_1)))Z_1 + (\varsigma_1 + \vartheta_1)\mu_1 + \vartheta_1x_1\mu_1^2 + \lambda_1\vartheta_1^2 < \mu_1x_1(\vartheta_1 + (P_1 + \varsigma_1 + v_1 + v'_1)(1 + \mu_1) + (\tau_1 + \tau'_1)\mu_1) + \lambda_1\vartheta_1((\vartheta_1 + P_1 + \tau_1 + \tau'_1 + \varsigma_1 + v_1 + v'_1)\mu_1 - \vartheta_1)$.

Proof. The value of the fourth-order principal minor is $H_{4 \times 4} > 0$. For satisfying the condition, $-\phi_1\mu_1\kappa_1^{\lambda_1}\varsigma_1^{Z_1}\vartheta_1^{-\mu_1-2}[\vartheta_1 + P_1 + \varsigma_1 + (v_1 + v'_1) + P_1\mu_1 + (\tau_1 + \tau'_1)\mu_1 + \varsigma_1\mu_1 - \vartheta_1\mu_1 + (v_1 + v'_1)\mu_1] + \phi_1\kappa_1^{\lambda_1}\varsigma_1^{Z_1-1}[(\vartheta_1 + \mu_1(P_1 + \varsigma_1 + (\tau_1 + \tau'_1) + (v_1 + v'_1)))Z_1 + (\varsigma_1 + \vartheta_1)\mu_1]\vartheta_1^{-\mu_1-1} - \phi_1\lambda_1\kappa_1^{\lambda_1-1}\varsigma_1^{Z_1}[(\vartheta_1 + P_1 + (\tau_1 + \tau'_1) + \varsigma_1 + (v_1 + v'_1))\mu_1 - \vartheta_1]\vartheta_1^{-\mu_1-1} < 0$. As $\phi_1\kappa_1^{\lambda_1}\varsigma_1^{Z_1-1}\vartheta_1^{-\mu_1-1} > 0$, $(\vartheta_1 + \mu_1(P_1 + \varsigma_1 + (\tau_1 + \tau'_1) + (v_1 + v'_1)))Z_1 + (\varsigma_1 + \vartheta_1)\mu_1 + \vartheta_1x_1\mu_1^2 + \lambda_1\vartheta_1^2 < \mu_1x_1(\vartheta_1 + (P_1 + \varsigma_1 + v_1 + v'_1)(1 + \mu_1) + (\tau_1 + \tau'_1)\mu_1) + \lambda_1\vartheta_1((\vartheta_1 + P_1 + \tau_1 + \tau'_1 + \varsigma_1 + v_1 + v'_1)\mu_1 - \vartheta_1)$. Hence, the proof is completed.



(a) Total profit of the retailer with respect to offline and online selling prices for the global concavity of the profit (decentralized policy).



(b) Total profit of SCM with respect to offline and online selling prices for the global concavity of the profit (centralized policy)

Figure 3. Three-dimensional concavity curve with respect to decision variables for comparing several cases

6. Numerical experiments

A numerical example, a few special discussions, and sensitivity analysis are discussed in this section.

6.1. Numerical example

Numerical example, a few special discussions, and sensitivity analysis are discussed in this section. Parameter values are taken from Davoudi et al. [76]. Due to convergence, some of the data have been modified as follows: $Y_1 = \$2$, $Y_2 = \$4$, $\phi_1 = 9600$, $\phi_2 = 9500$, $P_1 = 0.6$ (\$/unit), $P_2 = 0.6$ (\$/unit), $\lambda_1 = 0.1$, $\lambda_2 = 0.2$, $\mu_1 = 2.9$, $\mu_2 = 2.8$, $\nu_1 = 0.7$ (\$/unit), $\nu'_1 = 0.8$ (\$/unit), $\nu_2 = 0.6$ (\$/unit), $\nu'_2 = 0.8$ (\$/unit), $H_1 = \$3$, $H'_1 = \$2.3$, $H_2 = \$2.3$, $H'_2 = \$2.2$, $\tau_1 = 0.4$ (\$/unit), $\tau'_1 = 0.4$ (\$/unit), $\tau_2 = 0.5$ (\$/unit), $\tau'_2 = 0.4$ (\$/unit), $A_1 = \$1.1$, $A'_1 = \$1$, $A_2 = \$1.3$, $A'_2 = \$1.1$, $Z_1 = 0.1$, and $Z_2 = 0.1$. The optimal results are obtained by MATLAB R2023b. Optimum decision variables and profits are shown in the Table 3. The maximum profit is obtained with the centralized policy. The convey graph of the result are shown in Figures 3a,3b. Robustness of the parameters is given in Table 6.

Table 3. Optimum values and profit for the problem.

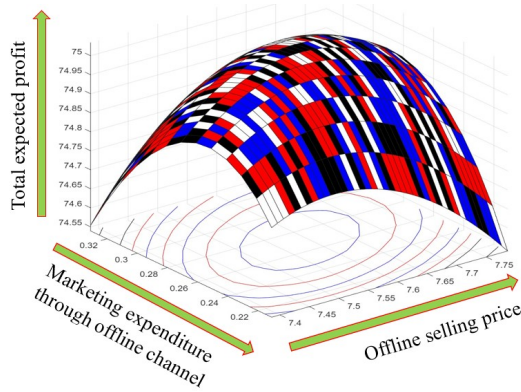
Decision variables	Decentralized policy	Centralized policy
κ_1^* (\$)	1.04	1.99
κ_2^* (\$)	1.12	2.24
ϑ_1^* (\$/unit)	7.27	4.67
ϑ_2^* (\$/unit)	7.59	4.78
ς_1^* (\$/unit)	0.25	0.16
ς_2^* (\$/unit)	0.27	0.17
ε_1^* (\$/unit)	3.71	–
ε_2^* (\$/unit)	3.7	–
Profits		
π_M^* (\$)	81.24	–
π_R^* (\$)	136.31	–
π_{sc}^* (\$)	–	315.78

–: Not specified

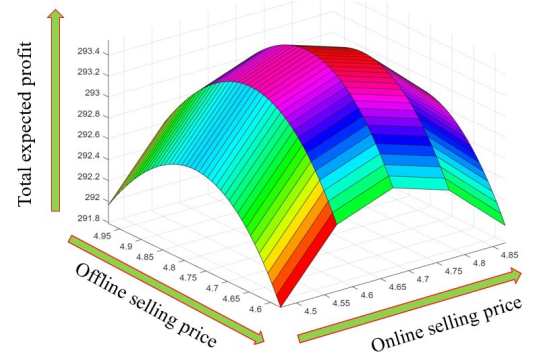
The results show that the total profit in the centralized case (\$315.98) is higher than that in the decentralized case with the Stackelberg game policy. Meanwhile, under the decentralized policy, the retailer's profit (\$136.51) exceeds the manufacturer's (\$81.24). The retail price of products is higher in the offline channel than in the online channel under both centralized and decentralized policies. A similar pattern is followed by green innovation and marketing expenditures. The result follows the general pattern of optimization and is not robust.

6.2. Special cases

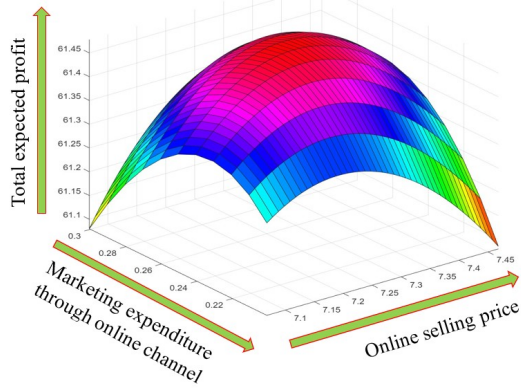
A few special cases are explained in this subsection. Graphical representations of the results and the special cases are presented in Figure 5a.



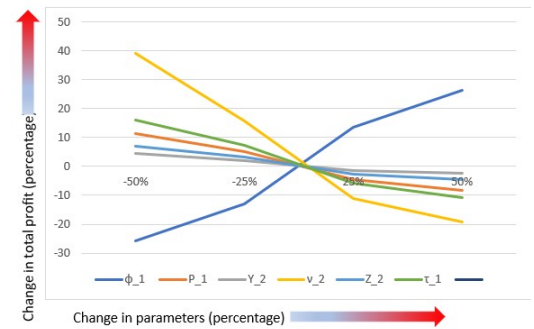
(a) Profit of the retailer with respect to marketing expenditure through the offline channel under the decentralized policy (Special Case I)



(b) Total profit of SCM with respect to online and offline channels under the centralized policy (Special Case II)



(c) Profit with respect to marketing expenditure and retail price with respect to online channel under the decentralized policy (Special Case III)



(d) Change in the manufacturers' profit in the decentralized policy for ϕ_1 , P_1 , μ_1 , Y_2 , v_2 , Z_2 , and τ_1

Figure 4. Three-dimensional concavity curve with respect to the decision variables and graphical presentation of the parameters with the change in profit.

6.2.1. Special Case I: Offline channel only

This special case is considered without an online channel, i.e., the retailer has only the offline channel. The demand of the offline channel is $q(\vartheta, \kappa, \varsigma) = q_2(\vartheta_2, \kappa_2, \varsigma_2)$. The optimum results are obtained on the basis of this demand and the relevant profit. The maximum values of profit and the different decision variables are $\pi_M^* = \$42.56$, $\pi_R^* = \$75.03$, $\kappa_2^* = \$1.12$, $\vartheta_2^* = \$7.59/\text{unit}$, $\varsigma_2^* = \$0.27$, and $\varepsilon_2^* = \$3.71/\text{unit}$ under the decentralized policy and $\pi_{sc}^* = \$173.40$, $\kappa_2^* = \$2.24$, $\vartheta_2^* = \$4.78/\text{unit}$, and $\varsigma_2^* = \$0.17$ under the centralized policy. However, without the online channel, the profits of the retailer, manufacturer, and SCM decrease by 45.04%, and 47.61% under the decentralized policy, and by 45.12% under the centralized policy. Thus, the use of online channels is highly beneficial for the manufacturer, the retailer, and SCM. The concave curve of the retailer for the special case is shown in Figure 4a.

6.2.2. Special Case II: Without investing in green innovation

The special cases examines the scenario without green innovation. In that case, the demand becomes $q(\vartheta, \varsigma) = q_1(\vartheta_1, \varsigma_1) + q_2(\vartheta_2, \varsigma_2) = \sum_{i=1}^2 \phi_i \vartheta_i^{-\mu_i} \varsigma_i^{Z_i}$. The associated profit functions are developed on the basis of revised demand. The optimal values of the profit functions and different decision variables are $\pi_{sc}^* = \$293.54$, $\vartheta_1^* = \$4.67/\text{unit}$, $\vartheta_2^* = \$4.78/\text{unit}$, and $\varsigma_1^* = \$0.16$, $\varsigma_2^* = \$0.17$ under the centralized policy and $\pi_M^* = \$75.42$, $\pi_R^* = \$126.58$, $\vartheta_1^* = \$7.27/\text{unit}$, $\vartheta_2^* = \$7.59/\text{unit}$, $\varsigma_1^* = \$0.25$, $\varsigma_2^* = \$0.27$, $\varepsilon_1^* = \$3.71/\text{unit}$, and $\varepsilon_2^* = \$3.71/\text{unit}$ under the decentralized policy, respectively. Without green innovation, the profits of the retailer, the manufacturer, and SCM decrease by 7.27% and 7.16% under the decentralized policy, and by 6.98% under the centralized policy, respectively. Thus, the application of green innovation benefits the manufacturer, the retailer, and the supply chain. The concave curve of the SCM for this special case is shown in Figure 4b.

6.2.3. Special Case III: Online channel only

This special case is experimented without an offline channel, i.e., the retailer has only the online channel. The associated demand in this scenario is $q(\vartheta, \kappa, \varsigma) = q_1(\vartheta_1, \kappa_1, \varsigma_1)$. The results are obtained on the basis of this demand from the associated profit functions. The optimal values of the profit functions and different decision variables are $\pi_M^* = \$38.68$, $\pi_R^* = \$61.48$, $\kappa_1^* = \$1.04$, $\vartheta_1^* = \$7.27/\text{unit}$, $\varsigma_1^* = \$0.25$, and $\varepsilon_1^* = \$3.71/\text{unit}$ under the decentralized policy, and $\pi_{sc}^* = \$142.58$, $\kappa_1^* = \$1.98$, $\vartheta_1^* = \$4.67/\text{unit}$, and $\varsigma_1^* = \$0.16$ under the centralized policy. Without an offline channel, the profits of the retailer, the manufacturer, and SCM decrease by 54.96%, and 52.39% under the decentralized policy, and by 54.88% under the centralized policy. Thus, the use of the offline channel is highly beneficial for manufacturers, retailers, and the supply chain. The concave curve for the retailer's profit function of this special case is shown in Figure 4c.

6.3. Discussions and comparison

The results show that effective marketing substantially increases acceptance of products.

6.3.1. Robustness analysis

A robustness analysis is explained in Tables 4 and 5 for the decision variables and Table 6 for the parameters to check the robustness of the decision variables and parameters.

- The robustness analysis of the decision variables is shown in Tables 4 and 5. Bold font in these two tables indicates the global maximum profit at the optimum value of the decision variables. It is seen that the profit is the global maximum for the obtained values of the decision variables. Other profits are less than the optimum profit for any increasing or decreasing value of the decision variables. However, the robustness is higher for wholesale price (ε_i) and green innovation (κ_i). Wholesale price (ε_i) does not have an effect under centralized decision-making.
- Table 6 gives the robustness of the parameters. Customer sensitivity (μ_i) is the most robust among all others parameters, followed by the variable for the carbon emissions tax (ν'_i, τ'_i), transportation costs (τ_i, ν_i), and customer sensitivity (λ_i).

Table 4. Robustness analysis of decision variables for the decentralized and centralized policies.

Parameters	Parameter's value	MS		CP	
		Retailer $\Pi_R^*(\$)$	Manufacturer $\Pi_M^*(\$)$	Parameter's value	Supply chain $\Pi_{SC}^*(\$)$
ϑ_1	8.77	131.48	63.22	6.17	293.88
	8.27	133.82	67.85	5.67	303.71
	7.77	135.58	73.70	5.17	311.96
	7.27	136.31	81.24	4.67	315.78
	6.77	135.26	72.1	4.17	309.09
	6.27	131.18	66.25	3.67	278.31
	5.77	121.94	62.23	3.17	191.38
ϑ_2	9.09	131.34	61.43	6.28	290.72
	8.59	133.77	66.60	5.78	302.09
	8.09	135.57	73.06	5.28	311.49
	7.59	136.31	81.24	4.78	315.78
	7.09	135.28	71.74	4.28	308.46
	6.59	131.33	65.49	3.78	275.52
	6.09	122.6	59.85	3.28	185.06
$\varkappa_1$	1.94	125.62	78.66	2.89	313.02
	1.64	131.43	80.04	2.59	314.52
	1.34	135.03	80.92	2.29	315.46
	1.04	136.31	81.24	1.99	315.78
	0.74	134.07	80.86	1.69	315.43
	0.44	130.78	79.44	1.39	314.26
	0.14	124.04	75.46	1.09	312.07
$\varkappa_2$	2.02	116.26	76.22	3.14	310.44
	1.72	128.43	78.92	2.84	313.35
	1.42	132.18	80.63	2.54	315.16
	1.12	136.31	81.24	2.24	315.78
	0.82	131.50	80.53	1.94	315.11
	0.52	124.99	78.03	1.64	312.96
	0.22	114.16	72.1	1.34	309.07
ς_1	0.37	135.65	78.95	0.28	312.39
	0.33	136	79.84	0.24	314.12
	0.29	136.23	80.88	0.2	315.32
	0.25	136.31	81.24	0.16	315.78
	0.21	136.21	80.5	0.12	315.15
	0.17	135.84	79.62	0.08	312.63
	0.13	135.09	78.53	0.4	305.89

Note: **Bold** font indicates the optimal values.

Table 5. Robustness analysis of decision variables for the decentralized and centralized policies.

Parameters	Parameter's value	MS		CP	
		Retailer $\Pi_R^*(\$)$	Manufacturer $\Pi_M^*(\$)$	Parameter's value	Supply chain $\Pi_{SC}^*(\$)$
ς_2	0.39	135.63	79.10	0.29	311.89
	0.35	135.99	79.75	0.25	313.88
	0.31	136.22	80.83	0.21	315.25
	0.27	136.31	81.24	0.17	315.78
	0.23	136.21	80.45	0.13	315.08
	0.19	135.84	79.52	0.09	312.32
	0.15	135.12	78.4	0.05	305.35
ε_1	5.21	96.34	44.58	–	–
	4.71	109.67	57.84	–	–
	4.21	122.99	69.56	–	–
	3.71	136.31	81.24	–	–
	3.21	121.75	67.92	–	–
	2.71	106.96	54.59	–	–
	1.21	94.85	41.27	–	–
ε_2	5.2	92.39	39.21	–	–
	4.7	107.03	53.12	–	–
	4.2	121.75	67.88	–	–
	3.7	136.31	81.24	–	–
	3.2	120.55	66.6	–	–
	2.7	104.96	51.95	–	–
	1.2	87.37	37.32	–	–

Note: **Bold** font indicates optimal values. –: Not applicable.

6.3.2. Discussions

A few observations are listed below based the results obtained.

- The results show that the dual-channel retailing strategy yields higher profits than a single-channel strategy. For the offline channel, the supply chain's profit decreases by 45.12% under both the centralized and decentralized policies compared with the original profit. However, among the offline and online channels, the offline channel is more profitable than the online channel for both the centralized and decentralized policies. This result is obtained from special cases I and III. That is, if the retailer of the supply chain manager has to choose either market segment, it should be the offline channel, i.e., the brick-and-mortar policy.
- As the marketing expenditure is a decision variable, the effect of it is tested through robustness analysis. This shows that it is the least robust decision variable among others. Green innovation investment has the least impact on profit. The economic effects of marketing expenditure and green innovation investment are investigated in this study. However, any direct environmental effect of these two parameters are not modelled in this study.

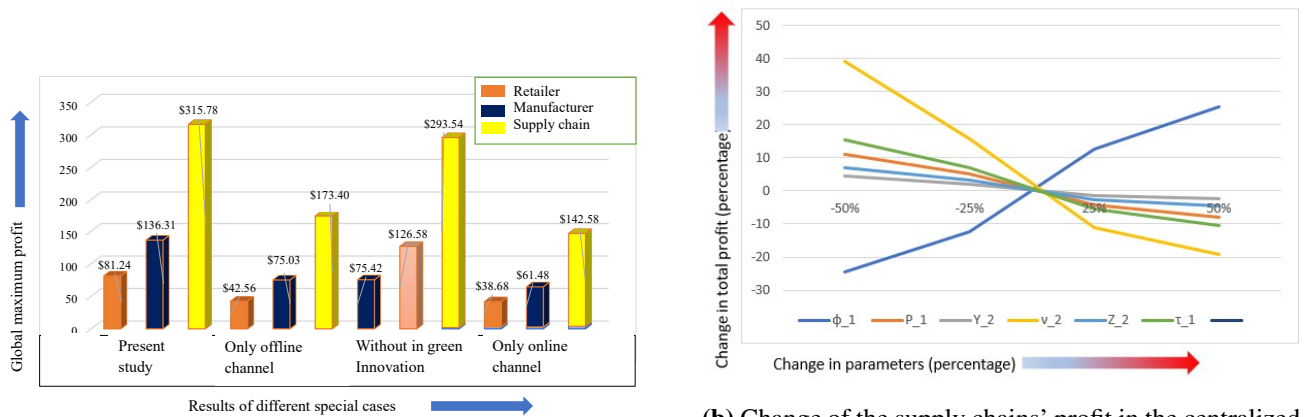
Table 6. Robustness analysis of parameters for the decentralized and centralized policies.

MS		CP		MS		CP			
Parameters	VP	Retailer	Manufacturer	SC	Parameters	VP	Retailer	Manufacturer	SC
		$\Pi_R^*(\%)$	$\Pi_M^*(\%)$	$\Pi_{SC}^*(\%)$			$\Pi_R^*(\%)$	$\Pi_M^*(\%)$	$\Pi_{SC}^*(\%)$
μ_1	4.4	-44.13	-41.38	-42.53	μ_2	4.3	-54.53	-53.71	-51.9
	3.9	-38.59	-36.93	-37.28		3.8	-47.36	-46.98	-45.95
	3.4	-26.12	-25.71	-25.88		3.3	-32.92	-33.81	-32.43
	2.4	57.47	66.85	62.98		2.3	86.11	93.81	84.24
	1.9	186.86	276.96	235.71		1.8	289.27	422.37	336.3
	1.4	420.86	1135.47	852.26		1.3	572.68	1457.57	1397.61
λ_1	0.19	4.09	5.51	4.32	λ_2	0.32	7.62	9.71	8.27
	0.16	2.46	3.01	2.61		0.28	4.73	5.99	5.01
	0.13	1.07	1.59	1.17		0.24	2.16	2.54	2.27
	0.07	-0.81	-1.05	-0.87		0.16	-1.72	-2.03	-1.82
	0.04	-1.33	-1.64	-1.39		0.12	-2.98	-3.75	-3.19
	0.01	-1.23	-1.56	-1.28		0.08	-3.87	-4.98	-4.08
v_1	1.3	-14.38	-14.17	-14.23	τ_2	1.1	-16.91	-17.27	-17.05
	1.1	-10.41	-10.27	-10.31		0.9	-12.21	-12.57	-12.35
	0.9	-5.67	-5.6	-5.63		0.7	-6.69	-6.85	-6.75
	0.5	6.94	6.86	6.89		0.3	8.19	8.37	8.24
	0.3	15.59	15.35	15.42		0.1	18.32	18.77	18.45
	0.1	26.63	26.03	26.19		-0.1	30.99	31.87	31.33
τ_1'	1.3	-19.83	-18.37	-19.03	v_2'	1.7	-22.69	-23.32	-22.8
	1	-14.79	-15.96	-14.23		1.4	-16.99	-17.27	-17.05
	0.7	-8.34	-7.97	-8.08		1.1	-9.65	-9.79	-9.68
	0.1	11.28	10.71	10.91		0.5	13	13.22	13.06
	-0.2	27.01	25.38	26.19		0.2	30.77	31.71	31.33
	-0.5	49.47	47.82	48.53		-0.1	57.02	59.02	58.03

Note: VP: Parameter's value; SC: Supply chain.

- Carbon tax policy is used in this study as both of the SCM players pay carbon tax for transportation. However, governments can encourage other carbon policies such as carbon pricing mechanisms, subsidies, and tax credits for eco-friendly technologies. Investing in green logistics, including transportation infrastructure and the use of digital technologies, helps minimize transportation emissions while keeping the environmental impact low. If green innovation is avoided, the profit decreases by 6.98% and 7.24%.
- If transportation costs and emissions costs for the retailer or the manufacturer increase, retail prices rise too much, and profits decrease under both the centralized and decentralized policies. Thus, to get the best result, the value of the transportation parameter is taken as $v_1 + v_1' = 3.5$ (\$/unit), $v_2 + v_2' = 3.4$ (\$/unit), $H_1 + H_1' = \$5.3$, $H_2 + H_2' = \$4.3$, $\tau_1 + \tau_1' = 0.8$ (\$/unit), $\tau_2 + \tau_2' = 0.9$ (\$/unit), $A_1 + A_1' = \$2.1$, and $A_2 + A_2' = \$2.4$.
- Note that if the value of P_i or μ_i increases, then retail prices increase, and the players' and the

supply chain's profits decrease. Thus, for a suitable result, the values of μ_i and P_i are taken to be $\phi_1 = 9600$, $\phi_2 = 9500$, $\mu_1 = 2.5$, and $\mu_2 = 2.4$.



(a) Graphical presentation of result and special cases

(b) Change of the supply chains' profit in the centralized policy for $\phi_1, P_1, \mu_1, Y_2, v_2, Z_2$, and τ_1

Figure 5. Graphical illustrations of the special cases results and parameters with the change in profit.

6.3.3. Comparison with existing literature

To highlight the contribution of this research, a theoretical comparison with extant research is briefly outlined. No study has discussed marketing expenditure.

- The proposed model is developed on the basis of green innovation, marketing expenditure, carbon emissions, transportation, and dual-channel retailing. Two independent retail channels are examined to determine the optimal policy between the centralized and decentralized approaches. This study is compared theoretically with the studies by Davoudi et al. [76] and Gautam et al. [77], and the present model provides a more comprehensive representation of a sustainable supply chain by simultaneously considering economic and environmental factors.
- Davoudi et al. [76] mainly focused on optimal decision-making under a supply chain framework, but they did not consider the combined influence of marketing expenditure and carbon emissions tax in an independent dual-channel retailing environment. In contrast, the proposed model shows that marketing increases market demand, while green innovation decreases the environmental impacts. Consequently, the model achieves a better balance between profitability and environmental sustainability.
- Similarly, Gautam et al. [77] investigated sustainable supply chains with green innovation, but their study did not analyze independent online and offline retail channels and transportation-related carbon emissions. Not only that, they considered only a centralized policy for finding their results. The proposed model fills this gap by considering both retail channels together and optimizing pricing, green innovation, and marketing decisions under centralized and decentralized policies with a Stackelberg game.

Table 7. Sensitivity analysis of the principal parameters for the decentralized and centralized policies.

Parameters	Changes (in%)	MS		CP
		Retailer Π_R^* (%)	Manufacturer Π_M^* (%)	Supply chain Π_{SC}^* (%)
ϕ_1	−50%	−26	−25.33	−24.58
	−25%	−13.12	−12.78	−12.4
	+25%	+13.29	+12.95	+12.57
	+50%	+26.72	+26.04	+25.27
P_1	−50%	+11.54	+11.24	+10.91
	−25%	+5.31	+5.18	+5.02
	+25%	−4.57	−4.46	−4.32
	+50%	−8.54	−8.32	−8.08
μ_1	−50%	+399.06	+1036.59	+739.05
	−25%	+103.3	+136.69	+118.51
	+25%	−32.88	−33.77	−32.11
	+50%	−43.97	−43.81	−42.14
Y_2	−50%	+4.43	+4.65	+4.56
	−25%	+1.8	+1.89	+1.85
	+25%	−1.36	−1.42	−1.4
	+50%	−2.44	−2.56	−2.51
v_2'	−50%	+37.91	+39.81	+39.05
	−25%	+15.22	+15.98	+15.67
	+25%	−10.72	−11.25	−11.04
	+50%	−18.56	−19.49	−19.11
Z_2	−50%	+7.24	+6.91	+7.02
	−25%	+3.22	+3.05	+3.11
	+25%	−2.66	−2.49	−2.55
	+50%	−4.89	−4.54	−4.66
τ_1'	−50%	+16.3	+15.88	+15.41
	−25%	+7.28	+7.09	+6.88
	+25%	−5.96	−5.81	−5.63
	+50%	−10.9	−10.62	−10.31

6.4. Sensitivity analysis

A sensitivity analysis of some important parameters is presented here. The effects of the parameters on the profit are shown for changes of +50%, −50%, +25%, and −25% in Table 7. The graphical representation of the sensitivity analysis is shown in Figure 4d and Figure 5b.

- (1) Customer sensitivity to the retailing price for the online channel (μ_1) is the most sensitive parameter, and the changes are inversely proportional to the total profit. For a −50% change in μ_1 , the profits of all players and the supply chain increase much more than the positive changes. The parameter's effect has the greatest impact on the manufacturer's profit, followed by the

retailer's and the SCM's profits.

- (2) The carbon emissions tax of the manufacturer for the offline channel (ν'_2) is the next most sensitive parameter. Carbon emissions tax and profit are inversely proportional. Negative percentage changes, e.g., -50% and -25% , have a greater effect on profit than positive percentage changes. As a similar pattern, the effect on the manufacturer's profit is more sensitive than the retailer's.
- (3) Total population of potential customers for the online channel (ϕ_1) is the third most important sensitive parameter. Here, ϕ_1 is proportionally related to the profits of the retailer, the manufacturer, and the SCM under the centralized and decentralized policies. Positive and negative changes are almost similar. However, the sensitivity of potential customers affects the manufacturer's profit more than the retailer's and the SCM's.
- (4) The carbon emissions tax of the retailer for the online channel (τ'_1) is the fourth most sensitive parameter. Carbon emissions tax and profit are inversely proportional. Profits sensitivity to positive changes is greater than that to negative changes. As a similar pattern to the effects above, the effect on the manufacturer's profit is more than the retailer's.
- (5) Production cost for the online channel is the next most sensitive parameter. Negative percentage changes in the parameter exceed the positive changes. The relation between the production cost parameter and total profit is inversely proportional. The effect of production costs is greater on the retailer's profit than on the manufacturer's and the SCM's.
- (6) Customer sensitivity for marketing expenditure in the offline channel (Z_2) is the second least sensitive parameter. Z_2 is inversely proportional to profit, and the negative percentage changes in the parameter exceed the positive changes.
- (7) Green innovation cost for the offline channel (Y_2) is the least sensitive parameter. It is inversely related to the profits of the retailer, the manufacturer, and the SCM.

7. Managerial insights

This section provides managerial insights derived from the theoretical and numerical experiments.

- (1) The manufacturer needs more strategic decision-making policies [78] for the decentralized business policy, especially for the game strategy. These insights are supported by the parameter sensitivity analysis. The manufacturer's profit is most affected by parameter changes, both increases and decreases in the parameter values.
- (2) Industrial managers need to adopt low-carbon technologies and optimize logistics to increase efficiency and reduce emissions. Therefore, industrial managers can utilize this study to stay ahead in the competitive market.
- (3) Effective independent dual-channel retailing improves market coverage while ensuring sustainable supply chain operations through efficient inventory allocation and transport planning. Profitability is likely to be improved through coordinated decisions across both the online and offline channels. Thus, industrial managers should focus on mutually exclusive dual-channel retailing.
- (4) Green innovation provides innovative ideas for creating eco-friendly production systems. This innovation leads to increased the production efficiency as well as the product quality in terms of environmental impact. Strategic investments in green innovation enhance brand reputation,

improve long-term profitability, and attract customer loyalty to environmentally less impactful products. Hence, the industry managers should invest in green innovation for economic and environmental benefits.

8. Conclusions

Emissions control, along with green innovation, enables firms to develop products aligned with environmentally conscious consumer preferences. Two independent dual-channel systems enabled firms to market and sell products to distinct market segments through two sales channels. Although marketing costs were a primary driver of this relationship, well-structured marketing initiatives stimulated demand for products and promoted long-term sustainable brand equity. The results indicated that marketing was the primary driver of demand, whereas carbon innovation played a central role in developing a sustainable competitive advantage. The integration of dual-channel systems with effective marketing strategies was aimed at maximizing market share, minimizing overall system costs, and enhancing profitability. This research found that carbon emission reduction targets, green innovation, and dual-channel marketing strategies were interdependent factors that contributed to achieving firms' sustainability. The research found that the absence of dual-channel systems resulted in a profit loss of 45.04% for retailers and 47.61% for manufacturers, and a similar 45.12% loss in a centralized system. This research showed that the lack of green innovation causes the profits of retailers and manufacturers to decline by 7.27% and 7.16%, respectively, under a decentralized policy, and by 6.98% under a centralized policy.

The paper mainly focused on customers which are mutually exclusive, and thus the two independent retail channels were discussed without examining any co-dependency. The co-dependency among two channel in a similar market environment would be an interesting future extension of the paper. The paper can be expanded to include remanufacturing [79]. Advertisement is another important factor across industries. The study can be extended to include advertising [80]. This study did not focused widely on manufacturing. Thus, the study can be extended to flexible production, cybersecurity, material reuse, and advanced payment policies [81]. Carbon emissions tax was considered in this study, but the study did not explain emissions reduction policies such as cap and trade and subsidies. The study can be extended using emissions reduction policies. Emissions from the system were not measured, which can be an immediate extension. Even through the study considered green innovation, he did not consider circular economy to reuse the waste for a secondary supply chain [82]. Extension of the study using the circular economy will be effective [83].

Use of AI tools declaration

The authors declare they have not used artificial intelligence (AI) tools in the creation of this article.

Conflict of interest

The authors declare no conflicts of interest in this paper.

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Appendix

The appendices explain the decentralized and centralized policies.

All derivatives of this appendix are given here.

$$\begin{aligned}\frac{\partial}{\partial \omega_i}(\pi_R) &= \phi_i \zeta_i^{Z_i} \kappa_i^{\lambda_i} (\omega_i + \zeta_i + \varepsilon_i)^{-\mu_i-1} (\omega_i + \zeta_i + \varepsilon_i + \mu_i((\tau_i + \tau'_i) - \omega_i)) \\ \frac{\partial}{\partial \zeta_i}(\pi_R) &= \phi_i((\tau_i + \tau'_i) - \omega_i) \zeta_i^{Z_i-1} \kappa_i^{\lambda_i} (\omega_i + \zeta_i + \varepsilon_i)^{-\mu_i-1} ((\omega_i + \varepsilon_i)Z_i + \zeta_i(Z_i - \mu_i)) \\ \frac{\partial^2}{\partial \omega_i^2}(\pi_R) &= -\phi_i \mu_i \kappa_i^{\lambda_i} M_i^{Z_i} (\omega_i + 2(\zeta_i + \omega_i) - \omega_i \mu_i + (\tau_i + \tau'_i)(1 + \mu_i)) (\omega_i + \varepsilon_i + \zeta_i)^{-\mu_i-2} \\ \frac{\partial^2}{\partial \omega_i \partial \zeta_i}(\pi_R) &= \frac{\partial^2}{\partial \zeta_i \partial \omega_i}(\pi_R) = -\phi_i \kappa_i^{\lambda_i} M_i^{Z_i-1} (\omega_i + \zeta_i + \varepsilon_i)^{-\mu_i-2} (-(\omega_i + \varepsilon_i)(\omega_i(\mu_i - 1) - \varepsilon_i)Z_i + \zeta_i^2(Z_i - \mu_i) + \\ &(\tau_i + \tau'_i)\mu_i((\omega_i + \varepsilon_i)Z_i + \zeta_i(Z_i - 1 - \mu_i)) + \zeta_i(-\varepsilon_i(\mu_i - 2Z_i) + \omega_i(\mu_i^2 + 2Z_i - \mu_i Z_i))) \\ \frac{\partial^2}{\partial \zeta_i^2}(\pi_R) &= -\phi_i((\tau_i + \tau'_i) - \omega_i) M_i^{Z_i-2} (\omega_i + \zeta_i + \varepsilon_i)^{-\mu_i-2} (-2\zeta_i Z_i (\omega_i + \varepsilon_i)(1 + \mu_i - Z_i) + (\omega_i + \varepsilon_i)^2(Z_i - 1)Z_i + \\ &M_i^2(\mu_i + \mu_i^2 - 2\mu_i Z_i + (Z_i - 1)Z_i)) \kappa_i^{\lambda_i}, \quad \frac{\partial^2}{\partial \omega_1 \partial \zeta_2}(\pi_R) = \frac{\partial^2}{\partial \zeta_2 \partial \omega_1}(\pi_R) = 0, \quad \frac{\partial^2}{\partial \omega_2 \partial \zeta_1}(\pi_R) = \frac{\partial^2}{\partial \zeta_1 \partial \omega_2}(\pi_R) = 0, \\ \frac{\partial^2}{\partial \omega_1 \partial \omega_2}(\pi_R) &= \frac{\partial^2}{\partial \omega_2 \partial \omega_1}(\pi_R) = 0, \quad \frac{\partial^2}{\partial \zeta_1 \partial \zeta_2}(\pi_R) = \frac{\partial^2}{\partial \zeta_2 \partial \zeta_1}(\pi_R) = 0.\end{aligned}$$

$$\text{The Hessian matrix } (H) \text{ of this appendix is } [H_{A1}] = \begin{bmatrix} \frac{\partial^2}{\partial \omega_1^2} \pi_R & \frac{\partial^2}{\partial \omega_1 \partial \omega_2} \pi_R & \frac{\partial^2}{\partial \omega_1 \partial \zeta_1} \pi_R & \frac{\partial^2}{\partial \omega_1 \partial \zeta_2} \pi_R \\ \frac{\partial^2}{\partial \omega_2 \partial \omega_1} \pi_R & \frac{\partial^2}{\partial \omega_2^2} \pi_R & \frac{\partial^2}{\partial \omega_2 \partial \zeta_1} \pi_R & \frac{\partial^2}{\partial \omega_2 \partial \zeta_2} \pi_R \\ \frac{\partial^2}{\partial \zeta_1 \partial \omega_1} \pi_R & \frac{\partial^2}{\partial \zeta_1 \partial \omega_2} \pi_R & \frac{\partial^2}{\partial \zeta_1^2} \pi_R & \frac{\partial^2}{\partial \zeta_1 \partial \zeta_2} \pi_R \\ \frac{\partial^2}{\partial \zeta_2 \partial \omega_1} \pi_R & \frac{\partial^2}{\partial \zeta_2 \partial \omega_2} \pi_R & \frac{\partial^2}{\partial \zeta_2 \partial \zeta_1} \pi_R & \frac{\partial^2}{\partial \zeta_2^2} \pi_R \end{bmatrix}. \text{ For}$$

first-order principal minor of Hessian matrix of 1×1 is $|H_{1 \times 1}| = \frac{\partial^2}{\partial \omega_1^2} \pi_R = -\phi_1 \mu_1 \kappa_1^{\lambda_1} M_1^{Z_1} (\omega_1 + 2(\zeta_1 + \omega_1) - \omega_1 \mu_1 + (\tau_1 + \tau'_1)(1 + \mu_1)) (\omega_1 + \varepsilon_1 + \zeta_1)^{-\mu_1-2} < 0$ [Lemma 1].

For second-order principal minor of Hessian matrix of 2×2 is $|H_{2 \times 2}| = \begin{vmatrix} \frac{\partial^2}{\partial \omega_1^2} \pi_R & \frac{\partial^2}{\partial \omega_1 \partial \omega_2} \pi_R \\ \frac{\partial^2}{\partial \omega_2 \partial \omega_1} \pi_R & \frac{\partial^2}{\partial \omega_2^2} \pi_R \end{vmatrix}.$

$\frac{\partial^2}{\partial \omega_1^2} \pi_R \frac{\partial^2}{\partial \omega_2^2} \pi_R > 0$, as $\frac{\partial^2}{\partial \omega_1^2} \pi_R < 0$ [Lemma 2] and as $|H_{1 \times 1}| < 0$. For third-order principal minor of Hessian

$$\text{matrix of } 3 \times 3 \text{ } |H_{3 \times 3}| = \begin{vmatrix} \frac{\partial^2}{\partial \omega_1^2} \pi_R & \frac{\partial^2}{\partial \omega_1 \partial \omega_2} \pi_R & \frac{\partial^2}{\partial \omega_1 \partial \zeta_1} \pi_R \\ \frac{\partial^2}{\partial \omega_2 \partial \omega_1} \pi_R & \frac{\partial^2}{\partial \omega_2^2} \pi_R & \frac{\partial^2}{\partial \omega_2 \partial \zeta_1} \pi_R \\ \frac{\partial^2}{\partial \zeta_1 \partial \omega_1} \pi_R & \frac{\partial^2}{\partial \zeta_1 \partial \omega_2} \pi_R & \frac{\partial^2}{\partial \zeta_1^2} \pi_R \end{vmatrix} = (L_1 = \frac{\partial^2}{\partial \omega_1^2} \pi_R - \frac{\partial^2}{\partial \omega_2 \partial \omega_1} \pi_R + \frac{\partial^2}{\partial \zeta_1 \partial \omega_1} \pi_R) |H_{22}| < 0,$$

as $L_1 = \frac{\partial^2}{\partial \omega_1^2} \pi_R - \frac{\partial^2}{\partial \omega_2 \partial \omega_1} \pi_R + \frac{\partial^2}{\partial \varsigma_1 \partial \omega_1} \pi_R < 0$ [Lemma 3] and as $|H_{22}| > 0$. For fourth-order principal minor

$$\text{of Hessian matrix of } 4 \times 4, |H_{44}| = \begin{vmatrix} \frac{\partial^2}{\partial \omega_1^2} \pi_R & \frac{\partial^2}{\partial \omega_1 \partial \omega_2} \pi_R & \frac{\partial^2}{\partial \omega_1 \partial \varsigma_1} \pi_R & \frac{\partial^2}{\partial \omega_1 \partial \varsigma_2} \pi_R \\ \frac{\partial^2}{\partial \omega_2 \partial \omega_1} \pi_R & \frac{\partial^2}{\partial \omega_2^2} \pi_R & \frac{\partial^2}{\partial \omega_2 \partial \varsigma_1} \pi_R & \frac{\partial^2}{\partial \omega_2 \partial \varsigma_2} \pi_R \\ \frac{\partial^2}{\partial \varsigma_1 \partial \omega_1} \pi_R & \frac{\partial^2}{\partial \varsigma_1 \partial \omega_2} \pi_R & \frac{\partial^2}{\partial \varsigma_1^2} \pi_R & \frac{\partial^2}{\partial \varsigma_1 \partial \varsigma_2} \pi_R \\ \frac{\partial^2}{\partial \varsigma_2 \partial \omega_1} \pi_R & \frac{\partial^2}{\partial \varsigma_2 \partial \omega_2} \pi_R & \frac{\partial^2}{\partial \varsigma_2 \partial \varsigma_1} \pi_R & \frac{\partial^2}{\partial \varsigma_2^2} \pi_R \end{vmatrix} = (L_1 - \frac{\partial^2}{\partial \varsigma_2 \partial \omega_1} \pi_R)$$

$|H_{33}| > 0$. As $|H_{33}| < 0$ and $L_1 - \frac{\partial^2}{\partial \varsigma_2 \partial \omega_1} \pi_R = L_1 < 0$.

All derivatives of this appendix are given here.

$$\begin{aligned} \frac{\partial}{\partial \varepsilon_i} (\pi_M) &= \phi_i \kappa_i^{\lambda_i} \frac{1}{((\tau_i + \tau'_i) + \varepsilon_i)} \left(\frac{((\tau_i + \tau'_i) + \varepsilon_i) \mu_i}{\mu_i - Z_i - 1} \right)^{-\mu_i} \left(\frac{((\tau_i + \tau'_i) + \varepsilon_i) Z_i}{\mu_i - Z_i - 1} \right)^{Z_i} ((\tau_i + \tau'_i) + (P_i + (v_i + v'_i))(\mu_i - Z_i) + \varepsilon_i(1 - \mu_i + Z_i)) \\ \frac{\partial}{\partial \kappa_i} (\pi_M) &= -2Y_i \kappa_i + \phi_i \lambda_i (\varepsilon_i - P_i - (v_i + v'_i)) \kappa_i^{\lambda_i-1} \left(\frac{((\tau_i + \tau'_i) + \varepsilon_i) \mu_i}{\mu_i - Z_i - 1} \right)^{-\mu_i} \left(\frac{((\tau_i + \tau'_i) + \varepsilon_i) Z_i}{\mu_i - Z_i - 1} \right)^{Z_i}, \\ \frac{\partial^2}{\partial \varepsilon_i^2} (\pi_M) &= -\phi_i (\mu_i - Z_i) \kappa_i^{\lambda_i} \frac{1}{((\tau_i + \tau'_i) + \varepsilon_i)^2} \left(\frac{((\tau_i + \tau'_i) + \varepsilon_i) \mu_i}{\mu_i - Z_i - 1} \right)^{-\mu_i} \left(\frac{((\tau_i + \tau'_i) + \varepsilon_i) Z_i}{\mu_i - Z_i - 1} \right)^{Z_i} (2(\tau_i + \tau'_i) \\ &+ (v_i + v'_i) + \varepsilon_i + (v_i + v'_i) \mu_i - \varepsilon_i \mu_i + P_i(1 + \mu_i - Z_i) - (v_i + v'_i) Z_i + \varepsilon_i Z_i) \\ \frac{\partial^2}{\partial \varepsilon_i \partial \kappa_i} (\pi_M) &= \frac{\partial^2}{\partial \kappa_i \partial \varepsilon_i} (\pi_M) = \phi_i \kappa_i^{\lambda_i-1} \frac{1}{((\tau_i + \tau'_i) + \varepsilon_i)} \left(\frac{((\tau_i + \tau'_i) + \varepsilon_i) \mu_i}{\mu_i - Z_i - 1} \right)^{-\mu_i} \left(\frac{((\tau_i + \tau'_i) + \varepsilon_i) Z_i}{\mu_i - Z_i - 1} \right)^{Z_i} ((\tau_i + \tau'_i) \\ &+ (P_i + (v_i + v'_i))(\mu_i - Z_i) + \varepsilon_i(1 - \mu_i + Z_i)) \\ \frac{\partial^2}{\partial \kappa_i^2} (\pi_M) &= -\left(2Y_i + \phi_i \lambda_i (\lambda_i - 1) \kappa_i^{\lambda_i-2} (P_i + (v_i + v'_i) - \varepsilon_i) \right) \left(\frac{((\tau_i + \tau'_i) + \varepsilon_i) \mu_i}{\mu_i - Z_i - 1} \right)^{-\mu_i} \left(\frac{((\tau_i + \tau'_i) + \varepsilon_i) Z_i}{\mu_i - Z_i - 1} \right)^{Z_i}, \\ \frac{\partial^2}{\partial \varepsilon_1 \partial \kappa_2} (\pi_M) &= \frac{\partial^2}{\partial \kappa_2 \partial \varepsilon_1} (\pi_M) = 0 \quad \frac{\partial^2}{\partial \varepsilon_2 \partial \kappa_1} (\pi_M) = \frac{\partial^2}{\partial \kappa_1 \partial \varepsilon_2} (\pi_M) = 0, \quad \frac{\partial^2}{\partial \varepsilon_1 \partial \varepsilon_2} (\pi_M) = \frac{\partial^2}{\partial \varepsilon_2 \partial \varepsilon_1} (\pi_M) = 0, \\ \frac{\partial^2}{\partial \kappa_1 \partial \kappa_2} (\pi_M) &= \frac{\partial^2}{\partial \kappa_2 \partial \kappa_1} (\pi_M) = 0. \end{aligned}$$

$$\text{The } H \text{ of this appendix is } [H_{A2}] = \begin{bmatrix} \frac{\partial^2}{\partial \varepsilon_1^2} \pi_M & \frac{\partial^2}{\partial \varepsilon_1 \partial \varepsilon_2} \pi_M & \frac{\partial^2}{\partial \varepsilon_1 \partial \kappa_1} \pi_M & \frac{\partial^2}{\partial \varepsilon_1 \partial \kappa_2} \pi_M \\ \frac{\partial^2}{\partial \varepsilon_2 \partial \varepsilon_1} \pi_M & \frac{\partial^2}{\partial \varepsilon_2^2} \pi_M & \frac{\partial^2}{\partial \varepsilon_2 \partial \kappa_1} \pi_M & \frac{\partial^2}{\partial \varepsilon_2 \partial \kappa_2} \pi_M \\ \frac{\partial^2}{\partial \kappa_1 \partial \varepsilon_1} \pi_M & \frac{\partial^2}{\partial \kappa_1 \partial \varepsilon_2} \pi_M & \frac{\partial^2}{\partial \kappa_1^2} \pi_M & \frac{\partial^2}{\partial \kappa_1 \partial \kappa_2} \pi_M \\ \frac{\partial^2}{\partial \kappa_2 \partial \varepsilon_1} \pi_M & \frac{\partial^2}{\partial \kappa_2 \partial \varepsilon_2} \pi_M & \frac{\partial^2}{\partial \kappa_2 \partial \kappa_1} \pi_M & \frac{\partial^2}{\partial \kappa_2^2} \pi_M \end{bmatrix}. \text{ For the first-order}$$

principal minor of Hessian matrix of 1×1 is $|H_{11}| = \left| \frac{\partial^2}{\partial \varepsilon_1^2} \pi_M \right| = -\phi_1 (\mu_1 - Z_1) \kappa_1^{\lambda_1} \frac{1}{((\tau_1 + \tau'_1) + \varepsilon_1)^2} \left(\frac{((\tau_1 + \tau'_1) + \varepsilon_1) \mu_1}{\mu_1 - Z_1 - 1} \right)^{-\mu_1} \left(\frac{((\tau_1 + \tau'_1) + \varepsilon_1) Z_1}{\mu_1 - Z_1 - 1} \right)^{Z_1} (2(\tau_1 + \tau'_1) + (v_1 + v'_1) + \varepsilon_1 + (v_1 + v'_1) \mu_1 - \varepsilon_1 \mu_1 + P_1(1 + \mu_1 - Z_1) - (v_1 + v'_1) Z_1 + \varepsilon_1 Z_1) < 0$ [See Lemma 4].

For the second-order principal minor of Hessian matrix of 2×2 $|H_{22}| = \begin{vmatrix} \frac{\partial^2}{\partial \varepsilon_1^2} \pi_M & \frac{\partial^2}{\partial \varepsilon_1 \partial \varepsilon_2} \pi_M \\ \frac{\partial^2}{\partial \varepsilon_2 \partial \varepsilon_1} \pi_M & \frac{\partial^2}{\partial \varepsilon_2^2} \pi_M \end{vmatrix} = (\frac{\partial^2}{\partial \varepsilon_2^2} \pi_M) |H_{11}|$ as $(\frac{\partial^2}{\partial \varepsilon_2^2} \pi_M) < 0$ [Lemma 5] and as $|H_{11}| < 0$. For the third-order principal minor of

$$\text{Hessian matrix of } 3 \times 3 \text{ is } |H_{33}| = \begin{vmatrix} \frac{\partial^2}{\partial \varepsilon_1^2} \pi_M & \frac{\partial^2}{\partial \varepsilon_1 \partial \varepsilon_2} \pi_M & \frac{\partial^2}{\partial \varepsilon_1 \partial \kappa_1} \pi_M \\ \frac{\partial^2}{\partial \varepsilon_2 \partial \varepsilon_1} \pi_M & \frac{\partial^2}{\partial \varepsilon_2^2} \pi_M & \frac{\partial^2}{\partial \varepsilon_2 \partial \kappa_1} \pi_M \\ \frac{\partial^2}{\partial \kappa_1 \partial \varepsilon_1} \pi_M & \frac{\partial^2}{\partial \kappa_1 \partial \varepsilon_2} \pi_M & \frac{\partial^2}{\partial \kappa_1^2} \pi_M \end{vmatrix} = \left(\frac{\partial^2}{\partial \varepsilon_1^2} \pi_M - \frac{\partial^2}{\partial \varepsilon_2 \partial \varepsilon_1} \pi_M + \frac{\partial^2}{\partial \kappa_1 \partial \varepsilon_1} \pi_M \right)$$

$|H_{22}| < 0$. As $(L_1 = \frac{\partial^2}{\partial \varepsilon_1^2} \pi_M - \frac{\partial^2}{\partial \varepsilon_2 \partial \varepsilon_1} \pi_M + \frac{\partial^2}{\partial \kappa_1 \partial \varepsilon_1} \pi_M) < 0$ [Lemma 6] and $[H_{22}] > 0$. For the fourth-order

principal minor of Hessian matrix of 4×4 is $|H_{44}| = \begin{vmatrix} \frac{\partial^2}{\partial \varepsilon_1^2} \pi_M & \frac{\partial^2}{\partial \varepsilon_1 \partial \varepsilon_2} \pi_M & \frac{\partial^2}{\partial \varepsilon_1 \partial \kappa_1} \pi_M & \frac{\partial^2}{\partial \varepsilon_1 \partial \kappa_2} \pi_M \\ \frac{\partial^2}{\partial \varepsilon_2 \partial \varepsilon_1} \pi_M & \frac{\partial^2}{\partial \varepsilon_2^2} \pi_M & \frac{\partial^2}{\partial \varepsilon_2 \partial \kappa_1} \pi_M & \frac{\partial^2}{\partial \varepsilon_2 \partial \kappa_2} \pi_M \\ \frac{\partial^2}{\partial \kappa_1 \partial \varepsilon_1} \pi_M & \frac{\partial^2}{\partial \kappa_1 \partial \varepsilon_2} \pi_M & \frac{\partial^2}{\partial \kappa_1^2} \pi_M & \frac{\partial^2}{\partial \kappa_1 \partial \kappa_2} \pi_M \\ \frac{\partial^2}{\partial \kappa_2 \partial \varepsilon_1} \pi_M & \frac{\partial^2}{\partial \kappa_2 \partial \varepsilon_2} \pi_M & \frac{\partial^2}{\partial \kappa_2 \partial \kappa_1} \pi_M & \frac{\partial^2}{\partial \kappa_2^2} \pi_M \end{vmatrix} =$

$(L_1 - \frac{\partial^2}{\partial \kappa_2 \partial \varepsilon_1} \pi_M |H_{33}|) > 0$ As $|H_{33}| < 0$ and $L_1 - \frac{\partial^2}{\partial \kappa_2 \partial \varepsilon_1} \pi_M = L_1 < 0$.

$$\begin{aligned} \frac{\partial}{\partial \vartheta_i} (\pi_{SC}) &= \phi_i S_i^{Z_i} \vartheta_i^{-\mu_i-1} \kappa_i^{\lambda_i} ((P_i + (\tau_i + \tau'_i) + \varsigma_i + (v_i + v'_i) - \vartheta_i) \mu_i + \vartheta_i) \\ \frac{\partial}{\partial \varsigma_i} (\pi_{SC}) &= -\phi_i S_i^{Z_i-1} \vartheta_i^{-\mu_i} \kappa_i^{\lambda_i} ((P_i + (\tau_i + \tau'_i) + (v_i + v'_i) - \vartheta_i + \varsigma_i) Z_i + \varsigma_i) \\ \frac{\partial}{\partial \kappa_i} (\pi_{SC}) &= \phi_i \lambda_i S_i^{Z_i} \vartheta_i^{-\mu_i} \kappa_i^{\lambda_i-1} (\vartheta_i - P_i - (\tau_i + \tau'_i) - \varsigma_i - (v_i + v'_i)) - 2Y_i \kappa_i \\ \frac{\partial^2}{\partial \vartheta_i^2} (\pi_{SC}) &= -\phi_i \mu_i \kappa_i^{\lambda_i} M_i^{Z_i} \vartheta_i^{-\mu_i-2} (\vartheta_i + P_i + \varsigma_i + (v_i + v'_i) + P_i \mu_i + (\tau_i + \tau'_i) \mu_i + \varsigma_i \mu_i - \vartheta_i \mu_i + (v_i + v'_i) \mu_i) \\ \frac{\partial^2}{\partial \varsigma_i^2} (\pi_{SC}) &= -\phi_i Z_i \kappa_i^{\lambda_i} M_i^{Z_i-2} \vartheta_i^{-\mu_i} (\vartheta_i - P_i - (\tau_i + \tau'_i) + \varsigma_i - (v_i + v'_i) + Z_i((\tau_i + \tau'_i) + \varsigma_i + P_i + (v_i + v'_i) - \vartheta_i)) \\ \frac{\partial^2}{\partial \kappa_i^2} (\pi_{SC}) &= -(Y_i \kappa_i + \phi_i \lambda_i (\lambda_i - 1) M_i^{Z_i} \kappa_i^{\lambda_i-2} (\vartheta_i)^{-\mu_i} (P_i + (\tau_i + \tau'_i) + \varsigma_i + (v_i + v'_i) - \vartheta_i)) \\ \frac{\partial^2}{\partial \vartheta_i \partial \kappa_i} (\pi_{SC}) &= \frac{\partial^2}{\partial \kappa_i \partial \vartheta_i} (\pi_{SC}) = -\phi_i \lambda_i \kappa_i^{\lambda_i-1} M_i^{Z_i} ((\vartheta_i + P_i + (\tau_i + \tau'_i) + \varsigma_i + (v_i + v'_i)) \mu_i - \vartheta_i) \vartheta_i^{-\mu_i-1} \\ \frac{\partial^2}{\partial \varsigma_i \partial \kappa_i} (\pi_{SC}) &= \frac{\partial^2}{\partial \kappa_i \partial \varsigma_i} (\pi_{SC}) = -\phi_i \lambda_i \kappa_i^{\lambda_i-1} M_i^{Z_i-1} ((-\vartheta_i + P_i + (\tau_i + \tau'_i) + \varsigma_i + (v_i + v'_i)) Z_i + \varsigma_i) \vartheta_i^{-\mu_i} \\ \frac{\partial^2}{\partial \varsigma_i \partial \vartheta_i} (\pi_{SC}) &= \frac{\partial^2}{\partial \vartheta_i \partial \varsigma_i} (\pi_{SC}) = \phi_i \kappa_i^{\lambda_i} M_i^{Z_i-1} ((\vartheta_i + \mu_i (P_i + \varsigma_i + (\tau_i + \tau'_i) + (v_i + v'_i))) Z_i + (\varsigma_i + \vartheta_i) \mu_i) \vartheta_i^{-\mu_i-1} \\ \frac{\partial^2}{\partial \vartheta_1 \partial \kappa_2} (\pi_{SC}) &= \frac{\partial^2}{\partial \kappa_2 \partial \vartheta_1} (\pi_{SC}) = \frac{\partial^2}{\partial \vartheta_2 \partial \kappa_1} (\pi_{SC}) = \frac{\partial^2}{\partial \kappa_1 \partial \vartheta_2} (\pi_{SC}) = \frac{\partial^2}{\partial \vartheta_1 \partial \varsigma_2} (\pi_{SC}) = \frac{\partial^2}{\partial \varsigma_2 \partial \vartheta_1} (\pi_{SC}) = \\ \frac{\partial^2}{\partial \vartheta_2 \partial \varsigma_1} (\pi_{SC}) &= \frac{\partial^2}{\partial \varsigma_1 \partial \vartheta_2} (\pi_{SC}) = \frac{\partial^2}{\partial \vartheta_1 \partial \vartheta_2} (\pi_{SC}) = \frac{\partial^2}{\partial \vartheta_2 \partial \vartheta_1} (\pi_{SC}) = \frac{\partial^2}{\partial \varsigma_1 \partial \varsigma_2} (\pi_{SC}) = \frac{\partial^2}{\partial \varsigma_2 \partial \varsigma_1} (\pi_{SC}) = \\ \frac{\partial^2}{\partial \varsigma_1 \partial \kappa_2} (\pi_{SC}) &= \frac{\partial^2}{\partial \kappa_2 \partial \varsigma_1} (\pi_{SC}) = \frac{\partial^2}{\partial \varsigma_2 \partial \kappa_1} (\pi_{SC}) = \frac{\partial^2}{\partial \kappa_1 \partial \varsigma_2} (\pi_{SC}) = 0 \end{aligned}$$

The H of this appendix is $|H_B| = \begin{vmatrix} \frac{\partial^2}{\partial \vartheta_1^2} \pi_{SC} & \frac{\partial^2}{\partial \vartheta_1 \partial \vartheta_2} \pi_{SC} & \frac{\partial^2}{\partial \vartheta_1 \partial \varsigma_1} \pi_{SC} & \frac{\partial^2}{\partial \vartheta_1 \partial \varsigma_2} \pi_{SC} & \frac{\partial^2}{\partial \vartheta_1 \partial \kappa_1} \pi_{SC} & \frac{\partial^2}{\partial \vartheta_1 \partial \kappa_2} \pi_{SC} \\ \frac{\partial^2}{\partial \vartheta_2 \partial \vartheta_1} \pi_{SC} & \frac{\partial^2}{\partial \vartheta_2^2} \pi_{SC} & \frac{\partial^2}{\partial \vartheta_2 \partial \varsigma_1} \pi_{SC} & \frac{\partial^2}{\partial \vartheta_2 \partial \varsigma_2} \pi_{SC} & \frac{\partial^2}{\partial \vartheta_2 \partial \kappa_1} \pi_{SC} & \frac{\partial^2}{\partial \vartheta_2 \partial \kappa_2} \pi_{SC} \\ \frac{\partial^2}{\partial \varsigma_1 \partial \vartheta_1} \pi_{SC} & \frac{\partial^2}{\partial \varsigma_1 \partial \vartheta_2} \pi_{SC} & \frac{\partial^2}{\partial \varsigma_1^2} \pi_{SC} & \frac{\partial^2}{\partial \varsigma_1 \partial \varsigma_2} \pi_{SC} & \frac{\partial^2}{\partial \varsigma_1 \partial \kappa_1} \pi_{SC} & \frac{\partial^2}{\partial \varsigma_1 \partial \kappa_2} \pi_{SC} \\ \frac{\partial^2}{\partial \varsigma_2 \partial \vartheta_1} \pi_{SC} & \frac{\partial^2}{\partial \varsigma_2 \partial \vartheta_2} \pi_{SC} & \frac{\partial^2}{\partial \varsigma_2 \partial \varsigma_1} \pi_{SC} & \frac{\partial^2}{\partial \varsigma_2^2} \pi_{SC} & \frac{\partial^2}{\partial \varsigma_2 \partial \kappa_1} \pi_{SC} & \frac{\partial^2}{\partial \varsigma_2 \partial \kappa_2} \pi_{SC} \\ \frac{\partial^2}{\partial \kappa_1 \partial \vartheta_1} \pi_{SC} & \frac{\partial^2}{\partial \kappa_1 \partial \vartheta_2} \pi_{SC} & \frac{\partial^2}{\partial \kappa_1 \partial \varsigma_1} \pi_{SC} & \frac{\partial^2}{\partial \kappa_1 \partial \varsigma_2} \pi_{SC} & \frac{\partial^2}{\partial \kappa_1^2} \pi_{SC} & \frac{\partial^2}{\partial \kappa_1 \partial \kappa_2} \pi_{SC} \\ \frac{\partial^2}{\partial \kappa_2 \partial \vartheta_1} \pi_{SC} & \frac{\partial^2}{\partial \kappa_2 \partial \vartheta_2} \pi_{SC} & \frac{\partial^2}{\partial \kappa_2 \partial \varsigma_1} \pi_{SC} & \frac{\partial^2}{\partial \kappa_2 \partial \varsigma_2} \pi_{SC} & \frac{\partial^2}{\partial \kappa_2 \partial \kappa_1} \pi_{SC} & \frac{\partial^2}{\partial \kappa_2^2} \pi_{SC} \end{vmatrix}.$

For the first-order principal minor of Hessian matrix of 1×1 is $|H_{11}| = |\frac{\partial^2}{\partial \vartheta_1^2} \pi_{SC}| = -\phi_1 \mu_1 \kappa_1^{\lambda_1} M_1^{Z_1} \vartheta_1^{-\mu_1-2} (\vartheta_1 + P_1 + \varsigma_1 + (v_1 + v'_1) + P_1 \mu_1 + (\tau_1 + \tau'_1) \mu_1 + \varsigma_1 \mu_1 - \vartheta_1 \mu_1 + (v_1 + v'_1) \mu_1) < 0$ [See Lemma 7]. For the second-order principal minor of Hessian matrix of 2×2

$|H_{22}| = \begin{vmatrix} \frac{\partial^2}{\partial \vartheta_1^2} \pi_{SC} & \frac{\partial^2}{\partial \vartheta_1 \partial \vartheta_2} \pi_{SC} \\ \frac{\partial^2}{\partial \vartheta_2 \partial \vartheta_1} \pi_{SC} & \frac{\partial^2}{\partial \vartheta_2^2} \pi_{SC} \end{vmatrix} = (\frac{\partial^2}{\partial \vartheta_1^2} \pi_{SC})(\frac{\partial^2}{\partial \vartheta_2^2} \pi_{SC}) > 0$. As $\frac{\partial^2}{\partial \vartheta_2^2} \pi_{SC} < 0$ [Lemma 8] and $[H_{11}] < 0$. For

the third-order principal minor of Hessian matrix of 3×3 is $|H_{33}| = \begin{vmatrix} \frac{\partial^2}{\partial \vartheta_1^2} \pi_{SC} & \frac{\partial^2}{\partial \vartheta_1 \partial \vartheta_2} \pi_{SC} & \frac{\partial^2}{\partial \vartheta_1 \partial \varsigma_1} \pi_{SC} \\ \frac{\partial^2}{\partial \vartheta_2 \partial \vartheta_1} \pi_{SC} & \frac{\partial^2}{\partial \vartheta_2^2} \pi_{SC} & \frac{\partial^2}{\partial \vartheta_2 \partial \varsigma_1} \pi_{SC} \\ \frac{\partial^2}{\partial \varsigma_1 \partial \vartheta_1} \pi_{SC} & \frac{\partial^2}{\partial \varsigma_1 \partial \vartheta_2} \pi_{SC} & \frac{\partial^2}{\partial \varsigma_1^2} \pi_{SC} \end{vmatrix}$

$= (\frac{\partial^2}{\partial \vartheta_1^2} \pi_{sc} - \frac{\partial^2}{\partial \vartheta_2 \partial \vartheta_1} \pi_{sc} + \frac{\partial^2}{\partial \varsigma_1 \partial \vartheta_1} \pi_{sc}) |H_{22}| < 0$. As $(L_1 = \frac{\partial^2}{\partial \vartheta_1^2} \pi_{sc} - \frac{\partial^2}{\partial \vartheta_2 \partial \vartheta_1} \pi_{sc} + \frac{\partial^2}{\partial \varsigma_1 \partial \vartheta_1} \pi_{sc}) < 0$ [Lemma 9] and $[H_{22}] > 0$. For the fourth-order principal minor of Hessian matrix of 4×4 is

$$|H_{44}| = \begin{vmatrix} \frac{\partial^2}{\partial \theta_1^2} \pi_{SC} & \frac{\partial^2}{\partial \theta_1 \partial \theta_2} \pi_{SC} & \frac{\partial^2}{\partial \theta_1 \partial \varsigma_1} \pi_{SC} & \frac{\partial^2}{\partial \theta_1 \partial \varsigma_2} \pi_{SC} \\ \frac{\partial^2}{\partial \theta_2 \partial \theta_1} \pi_{SC} & \frac{\partial^2}{\partial \theta_2^2} \pi_{SC} & \frac{\partial^2}{\partial \theta_2 \partial \varsigma_1} \pi_{SC} & \frac{\partial^2}{\partial \theta_2 \partial \varsigma_2} \pi_{SC} \\ \frac{\partial^2}{\partial \varsigma_1 \partial \theta_1} \pi_{SC} & \frac{\partial^2}{\partial \varsigma_1 \partial \theta_2} \pi_{SC} & \frac{\partial^2}{\partial \varsigma_1^2} \pi_{SC} & \frac{\partial^2}{\partial \varsigma_1 \partial \varsigma_2} \pi_{SC} \\ \frac{\partial^2}{\partial \varsigma_2 \partial \theta_1} \pi_{SC} & \frac{\partial^2}{\partial \varsigma_2 \partial \theta_2} \pi_{SC} & \frac{\partial^2}{\partial \varsigma_2 \partial \varsigma_1} \pi_{SC} & \frac{\partial^2}{\partial \varsigma_2^2} \pi_{SC} \end{vmatrix} = L_1 - \frac{\partial^2}{\partial \varsigma_2 \partial \theta_1} \pi_{SC} [H_{33}] > 0. \text{ As } |H_{33}| < 0 \text{ and}$$

$L_2 = L_1 - \frac{\partial^2}{\partial \varsigma_2 \partial \theta_1} \pi_{SC} = L_1 < 0$. For the fifth-order principal minor of Hessian matrix of 5×5 is

$$|H_{55}| = \begin{vmatrix} \frac{\partial^2}{\partial \theta_1^2} \pi_{SC} & \frac{\partial^2}{\partial \theta_1 \partial \theta_2} \pi_{SC} & \frac{\partial^2}{\partial \theta_1 \partial \varsigma_1} \pi_{SC} & \frac{\partial^2}{\partial \theta_1 \partial \varsigma_2} \pi_{SC} & \frac{\partial^2}{\partial \theta_1 \partial \kappa_1} \pi_{SC} \\ \frac{\partial^2}{\partial \theta_2 \partial \theta_1} \pi_{SC} & \frac{\partial^2}{\partial \theta_2^2} \pi_{SC} & \frac{\partial^2}{\partial \theta_2 \partial \varsigma_1} \pi_{SC} & \frac{\partial^2}{\partial \theta_2 \partial \varsigma_2} \pi_{SC} & \frac{\partial^2}{\partial \theta_2 \partial \kappa_1} \pi_{SC} \\ \frac{\partial^2}{\partial \varsigma_1 \partial \theta_1} \pi_{SC} & \frac{\partial^2}{\partial \varsigma_1 \partial \theta_2} \pi_{SC} & \frac{\partial^2}{\partial \varsigma_1^2} \pi_{SC} & \frac{\partial^2}{\partial \varsigma_1 \partial \varsigma_2} \pi_{SC} & \frac{\partial^2}{\partial \varsigma_1 \partial \kappa_1} \pi_{SC} \\ \frac{\partial^2}{\partial \varsigma_2 \partial \theta_1} \pi_{SC} & \frac{\partial^2}{\partial \varsigma_2 \partial \theta_2} \pi_{SC} & \frac{\partial^2}{\partial \varsigma_2 \partial \varsigma_1} \pi_{SC} & \frac{\partial^2}{\partial \varsigma_2^2} \pi_{SC} & \frac{\partial^2}{\partial \varsigma_2 \partial \kappa_1} \pi_{SC} \\ \frac{\partial^2}{\partial \kappa_1 \partial \theta_1} \pi_{SC} & \frac{\partial^2}{\partial \kappa_1 \partial \theta_2} \pi_{SC} & \frac{\partial^2}{\partial \kappa_1 \partial \varsigma_1} \pi_{SC} & \frac{\partial^2}{\partial \kappa_1 \partial \varsigma_2} \pi_{SC} & \frac{\partial^2}{\partial \kappa_1^2} \pi_{SC} \end{vmatrix} = (L_2 - \frac{\partial^2}{\partial \kappa_1 \partial \theta_1} \pi_{SC}) [H_{44}] < 0. \text{ As}$$

$(L_3 = L_2 - \frac{\partial^2}{\partial \kappa_1 \partial \theta_1} \pi_{SC}) < 0$ [Lemma 10] and $[H_{44}] > 0$. For the sixth-order principal minor of Hessian

matrix of 6×6 is $|H_{66}| =$

$$\begin{vmatrix} \frac{\partial^2}{\partial \theta_1^2} \pi_{SC} & \frac{\partial^2}{\partial \theta_1 \partial \theta_2} \pi_{SC} & \frac{\partial^2}{\partial \theta_1 \partial \varsigma_1} \pi_{SC} & \frac{\partial^2}{\partial \theta_1 \partial \varsigma_2} \pi_{SC} & \frac{\partial^2}{\partial \theta_1 \partial \kappa_1} \pi_{SC} & \frac{\partial^2}{\partial \theta_1 \partial \kappa_2} \pi_{SC} \\ \frac{\partial^2}{\partial \theta_2 \partial \theta_1} \pi_{SC} & \frac{\partial^2}{\partial \theta_2^2} \pi_{SC} & \frac{\partial^2}{\partial \theta_2 \partial \varsigma_1} \pi_{SC} & \frac{\partial^2}{\partial \theta_2 \partial \varsigma_2} \pi_{SC} & \frac{\partial^2}{\partial \theta_2 \partial \kappa_1} \pi_{SC} & \frac{\partial^2}{\partial \theta_2 \partial \kappa_2} \pi_{SC} \\ \frac{\partial^2}{\partial \varsigma_1 \partial \theta_1} \pi_{SC} & \frac{\partial^2}{\partial \varsigma_1 \partial \theta_2} \pi_{SC} & \frac{\partial^2}{\partial \varsigma_1^2} \pi_{SC} & \frac{\partial^2}{\partial \varsigma_1 \partial \varsigma_2} \pi_{SC} & \frac{\partial^2}{\partial \varsigma_1 \partial \kappa_1} \pi_{SC} & \frac{\partial^2}{\partial \varsigma_1 \partial \kappa_2} \pi_{SC} \\ \frac{\partial^2}{\partial \varsigma_2 \partial \theta_1} \pi_{SC} & \frac{\partial^2}{\partial \varsigma_2 \partial \theta_2} \pi_{SC} & \frac{\partial^2}{\partial \varsigma_2 \partial \varsigma_1} \pi_{SC} & \frac{\partial^2}{\partial \varsigma_2^2} \pi_{SC} & \frac{\partial^2}{\partial \varsigma_2 \partial \kappa_1} \pi_{SC} & \frac{\partial^2}{\partial \varsigma_2 \partial \kappa_2} \pi_{SC} \\ \frac{\partial^2}{\partial \kappa_1 \partial \theta_1} \pi_{SC} & \frac{\partial^2}{\partial \kappa_1 \partial \theta_2} \pi_{SC} & \frac{\partial^2}{\partial \kappa_1 \partial \varsigma_1} \pi_{SC} & \frac{\partial^2}{\partial \kappa_1 \partial \varsigma_2} \pi_{SC} & \frac{\partial^2}{\partial \kappa_1^2} \pi_{SC} & \frac{\partial^2}{\partial \kappa_1 \partial \kappa_2} \pi_{SC} \\ \frac{\partial^2}{\partial \kappa_2 \partial \theta_1} \pi_{SC} & \frac{\partial^2}{\partial \kappa_2 \partial \theta_2} \pi_{SC} & \frac{\partial^2}{\partial \kappa_2 \partial \varsigma_1} \pi_{SC} & \frac{\partial^2}{\partial \kappa_2 \partial \varsigma_2} \pi_{SC} & \frac{\partial^2}{\partial \kappa_2 \partial \kappa_1} \pi_{SC} & \frac{\partial^2}{\partial \kappa_2^2} \pi_{SC} \end{vmatrix} =$$

$L_3 - \frac{\partial^2}{\partial \kappa_2 \partial \theta_1} \pi_{SC} |H_{55}| > 0$ As $|H_{55}| < 0$ and $L_3 - \frac{\partial^2}{\partial \kappa_2 \partial \theta_1} \pi_{SC} = L_3 < 0$.



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