



Research article

Cyclic response of a reinforced concrete frame: Comparison of experimental results with different hysteretic models

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Abstract: An accurate hysteresis model is fundamental to well capture the non-linearity phenomena occurring in structural and non-structural elements in building structures, that are usually made of reinforced concrete or steel materials. In this sense, this paper aims to numerically estimate through simplified non-linear analyses, the cyclic response of a reinforced concrete frame using different hysteretic models present in the literature. A commercial Finite Element Method package is used to carry out most of the simulations using polygonal hysteretic models and a fiber model, and additionally, a MATLAB script is developed to use a smooth hysteresis model. The experimental data is based on the experiments carried out in the Laboratório Nacional de Engenharia Civil, Portugal. The numerical outcomes are further compared with the experimental result to evaluate the accuracy of the simplified analysis based on the lumped plasticity or plastic hinge method for the reinforced concrete bare frame. Results show that the tetralinear Takeda's model fits closely the experimental hysteresis loops. The fiber model can well capture the hysteresis behavior, though it requires knowledge and expertise on parameter calibration. Sivaselvan and Reinhorn's smooth hysteresis model was able to satisfactorily reproduce the actual non-linear cyclic behavior of the RC frame structure in a global way.

Keywords: cyclic response; reinforced concrete; non-linear analysis; hysteresis models; optimization

1. Introduction

Natural events, such as earthquakes, induce structural vibrations in building structures that can provoke permanent damage or even the collapse of the entire structural system. These negative consequences arise from the substantial floor's lateral deformation induced by moderate seismic events, being currently one of the main concerns in structural design. Hence, some level of damage is expected and may be purposely lumped at pre-determined locations, i.e., plastic hinges, to dissipate energy through inelastic deformation, preventing global collapse by holding the main structural elements, viz. the columns, during the seismic event, and thus not compromising the structure's stability.

Moderate to severe earthquakes can compromise the structural and seismic performance of structures, causing significant and unexpected damage to the structural and non-structural elements, which can further lead to the collapse of buildings or part of these as observed during recent events [1,2]. A large number of studies have been carried out by different investigators to assess the non-linear cyclic behavior of reinforced concrete framed structures under different conditions and to validate the respective results with experimental data [3–6]. Experimental tests in this context, allow for the development of numerical models that should be capable of replicating the real behavior of a structure under cyclic loading with reasonable accuracy. Hence, permitting further studies and leading to valuable conclusions about such behavior.

To assess the level of damage experienced by structural elements, e.g., beams and columns in moment-resisting frames, hysteresis models can be used to represent the corresponding constitutive relationships at the critical sections (usually at the extremities of the structural elements). Hysteresis is associated with the rate-independence effect and the memory nature of inelastic behavior in which the restoring force depends on the instantaneous deformation and the previous history of the deformation [7]. Thus, different hysteresis models are present in the literature to emulate the physical behavior of distinct structural or non-structural elements, materials, or structural systems, under cyclic loading. These models are broadly categorized into polygonal and smooth hysteresis models [8], and can be developed for a specific purpose or can be more versatile. Nowadays, versatile hysteresis models are often developed rather than for specific purposes, exhibiting degradation features, such as stiffness degradation, strength deterioration, and the pinching effect. The main purpose of this work is then to validate a set of simplified nonlinear models used to carry out a material nonlinear analysis.

2. Experimental results

The experimental campaign performed at LNEC [9] aimed to study the cyclic response of reinforced concrete (RC) bare frames and with masonry infill walls. Figure 1a presents a sketch of the general description of the RC bare frame, which is loaded at each column with a constant vertical load of 100 kN and is subjected to an increasing cyclical load/displacement pattern at the beam's level, obeying the law of displacements in Figure 1b.

The bare frame structure is made of concrete C20/25, longitudinal steel reinforcement of S400, and transverse reinforcement (stirrups) of S500. At the critical location (plastic hinges), near the end of each structural member, accurate concrete confinement is guaranteed by tightening the spacing between the stirrups as can be observed in Figure 2.

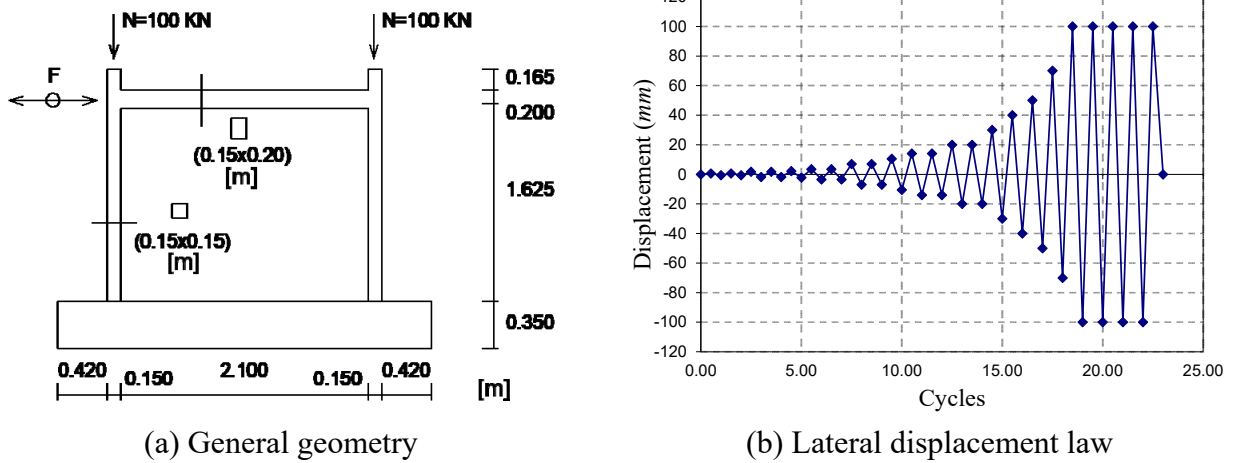


Figure 1. Experimental frame and loading pattern [10].

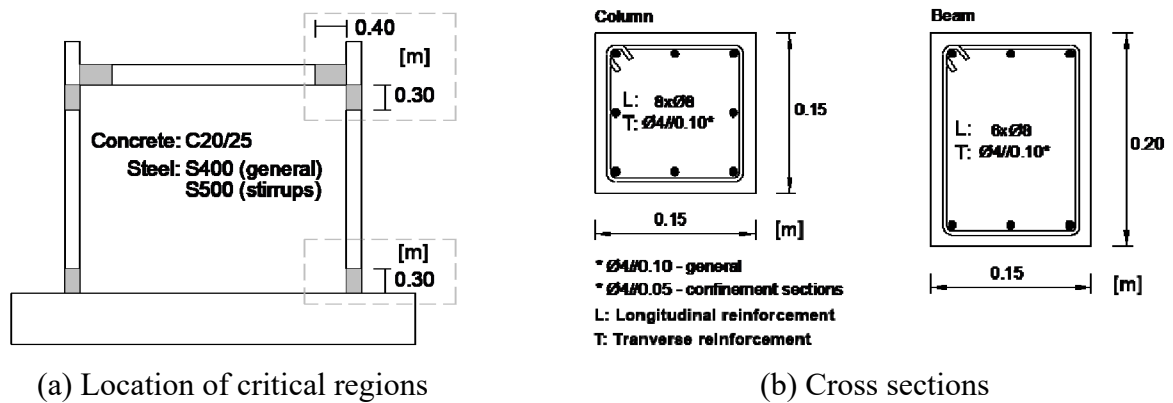


Figure 2. RC frame confinement of the critical regions [10].

Figure 3 shows the cyclic response of the RC bare frame. A smooth evolution is observed with a resulting maximum before the complete concrete cracking at the top and bottom ends of the columns. Subsequently, a soft stiffness decrease occurs without collapse, although with substantial damage and inelastic hinge spread in the columns.

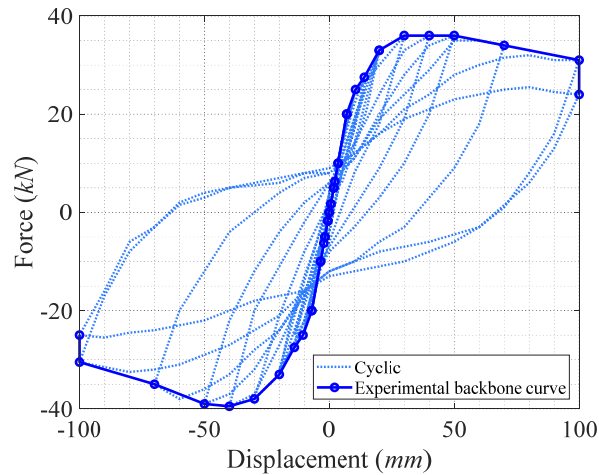


Figure 3. Experimental cyclic response and corresponding backbone curve of the RC frame structure.

3. Numerical results and discussion

The performance of different hysteresis models will be assessed, attempting to emulate the cyclic behavior of the RC frame structure, whose experimental result is presented in Figure 3.

The inelastic behavior of the frame structure is generally manifested by the formation of plastic hinges at the extremities of the elements as represented in Figure 4. This type of behavior that admits the concentration of plasticity at specific locations, allowed the creation of several simplified methodologies that enable the performance of simpler static or dynamic non-linear analysis [11].

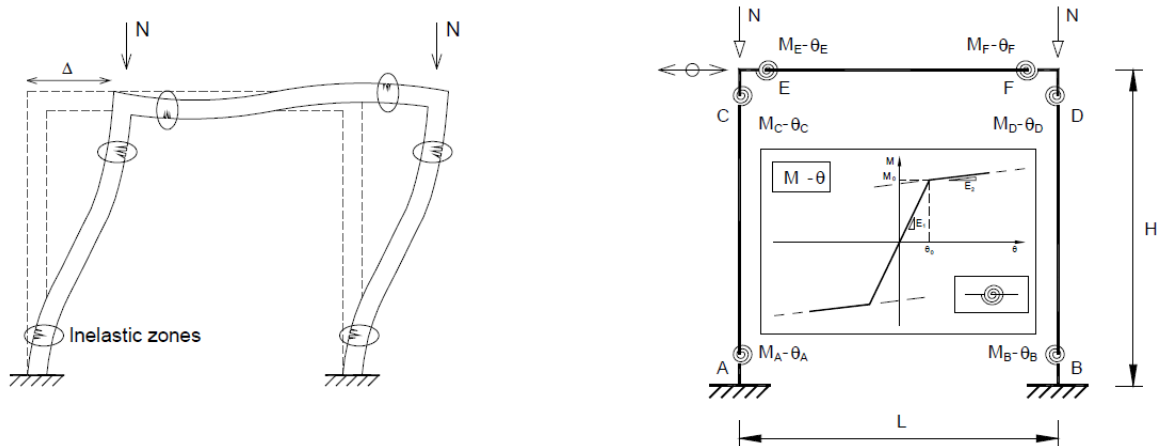


Figure 4. Critical zones of plastic hinges formation [10].

3.1. Simplified non-linear analysis of bare frames

To begin the mentioned simplified procedure, the definition of the constitutive relations in terms of moment-curvature of the columns and beam's sections must be derived, presented in Figure 5.

These will be important in the definition of the different initial levels of the structure's stiffness during cyclic loading.

This study considers two different methods to obtain the non-linear cyclic behavior of RC frame structures [12], viz., Plastic Hinge Model (PHM) and Fiber Model (FM). The key difference between the two models lies in the way the constitutive laws are defined and used. In the hinge models, an envelope curve is defined to represent the global behavior of the complete cross-section in terms of moment-curvature; the FM requires the establishment of constitutive laws associated with the axial deformation of each of the materials (or fibers) that comprise the section. The modeling of the plastic hinges in the PHM is carried out by including a zero-length element linking two adjacent beam elements, in which a constitutive law must be defined for each degree of freedom considered. A third way to analyze the non-linear cyclic behavior of the RC frame structure may be the use of a versatile hysteresis model, capable of modeling the respective behavior of the structural system in a global or macro way. This can be achieved by the proper optimization and calibration of the parameters defining the hysteresis model.

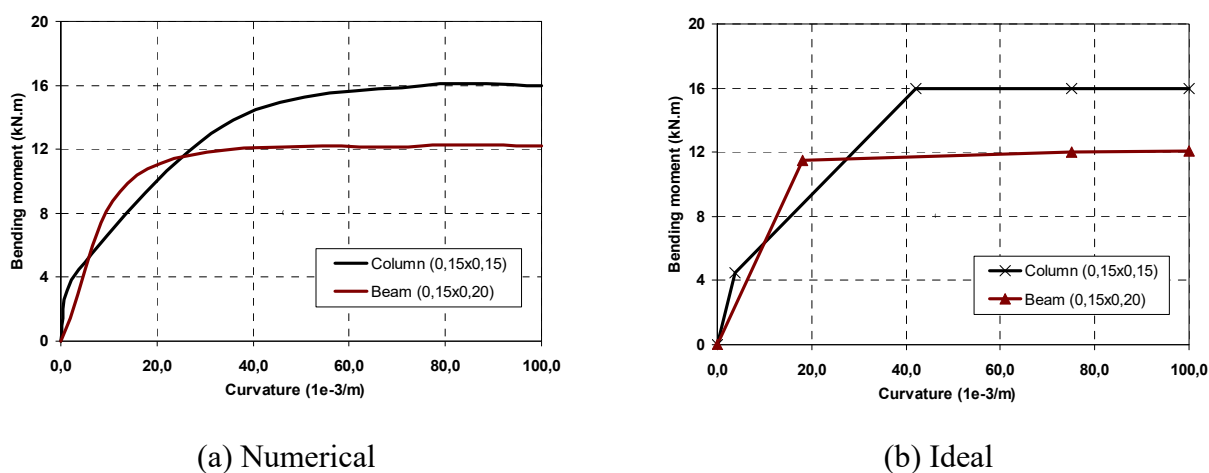


Figure 5. Moment-curvature relationships for the columns and beam [10].

Hence, a Fiber model (FM) and four hysteresis models, i.e., three polygonal hysteresis models (Clough's model [13], trilinear Takeda's model [14], and tetralinear Takeda's model [15])—used as zero-length elements, and a smooth hysteresis model (Sivaselvan and Reinhorn's model [8])—used to analyze the global non-linear cyclic behavior; will be used and compared to the experimental response to assess the capability to model the non-linear cyclic behavior of RC bare frame structures.

The polygonal hysteresis models and the fiber model were carried out using the software Midas-Civil [15], and the smooth hysteresis model was developed using a MATLAB [16] script.

Table 1 presents the initial and reduced bending stiffness values of the different backbone curves of the polygonal hysteresis models, to be introduced in the next sections using the mentioned software.

Table 1. Bending stiffness levels according to the type of backbone curve admitted.

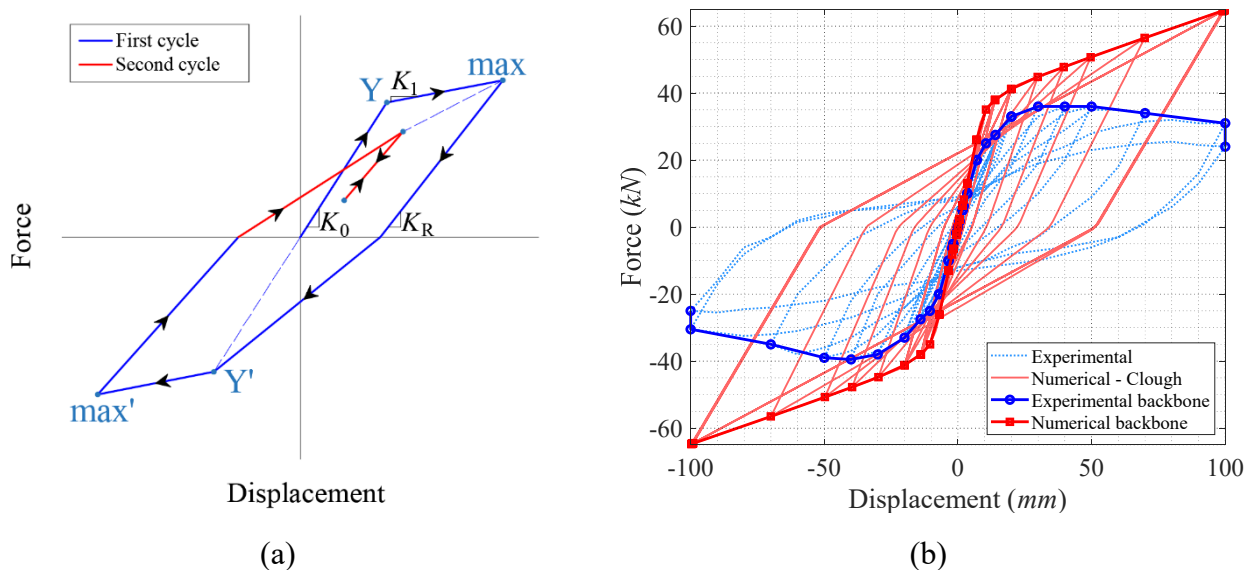
Structural element	Backbone curve	Bending stiffness (kN/m ²)			
		K_0	K_1	K_2	K_3
Columns	<i>Bilinear</i>	1380	6.9 (0.5%)		
	<i>Trilinear</i>		276 (20%)	6.9 (0.5%)	
	<i>Tetralinear</i>				-6.9 (-0.5%)
Beam	<i>Bilinear</i>	610	30.5 (5%)		
	<i>Trilinear</i>				
	<i>Tetralinear</i>				

The subsequent subsections present the models' outline or the representation of the hysteresis laws in generalized force-displacement relations, and the corresponding final iteration result that best fits the experimental results.

3.2. Clough's hysteresis model

Clough's model [13], one of the first hysteresis models, is based on the simplified constitutive law with a bilinear backbone curve as represented in Figure 6a and a formula that translates the stiffness reduction on unloading (K_R), comprising an unloading stiffness parameter. The fitting to the experimental results is based on the values in Table 1 and on the variation of the parameter related to the unloading stiffness reduction (the value adopted for this parameter was 0.5).

This model does not represent the real cyclic behavior of the RC frame structure, since a substantial increase in the strength capacity is verified with the increase in lateral displacement, which is not consistent with the experimental results as proved by Figure 6b. This is mostly due to the bilinear backbone curve, which in reality is not associated with RC behavior due to concrete cracking.

**Figure 6.** Clough's model. (a) Hinge model, (b) comparison of results.

3.3. Trilinear Takeda's hysteresis model

A trilinear backbone curve is more adequate to represent RC cyclic behavior. One example of a hysteresis model that considers a trilinear backbone curve and stiffness degradation in the unloading branches of the inner and outer loops is the trilinear Takeda's model [14]. Available in the software Midas-Civil and with the constitutive law represented in Figure 7a, the fitting of the experimental results with this model is based on the values in Table 1, and on the parameters that define the unloading stiffness in the corresponding branches of the cycles through the use of a formula (it was adopted for the input parameters in this formula 0.75 for the exponent in unloading stiffness calculation and 1.00 for the inner loop unloading stiffness calculation).

The trilinear Takeda's model can better predict the experimental results for smaller lateral displacements, compared with the previous model. However, much like the preceding model, it tends to overestimate the post-yielding stiffness, despite presenting a less pronounced increase (Figure 7b).

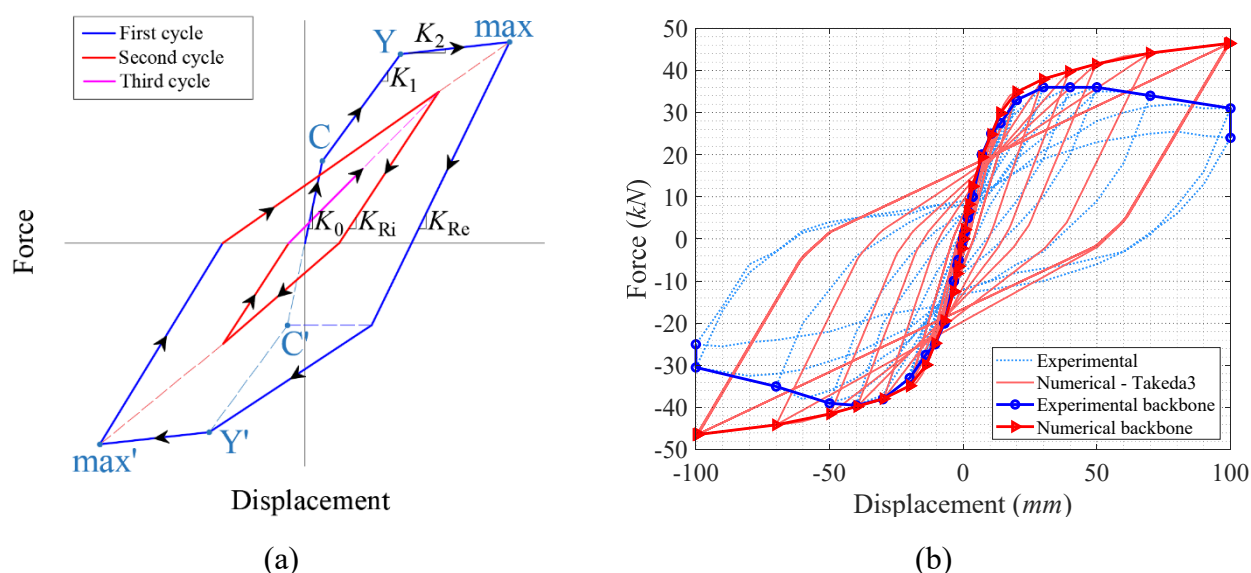


Figure 7. Takeda's trilinear model. (a) Hinge model, (b) comparison of results.

Hence, this model is unable to represent the negative stiffness verified immediately before unloading, resulting in an overestimation of the post-yielding stiffness.

3.4. Tetralinear Takeda's hysteresis model

To account for the drawbacks of the precedent hysteresis models, the tetralinear Takeda's model [15], shows a modification to the previous model that includes a fourth branch related to the negative stiffness on loading.

Following the typical relations presented in the ideal model in Figure 8a and the tabled values in Table 1, a close estimation of the experimental results can be obtained, as can be seen by Figure 8b. The fitting parameters (unloading stiffness parameters) used are the same as the previous version of the model, leading to a better approximation of the real cyclic behavior of the RC frame structure.

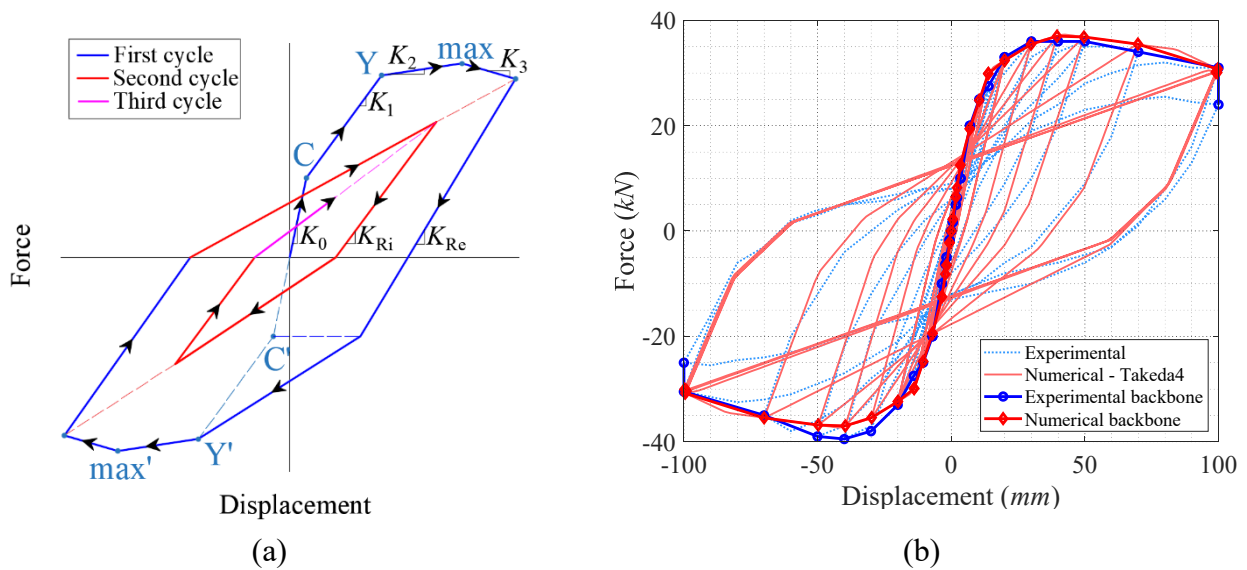


Figure 8. Takeda's tetralinear model. (a) Hinge model, (b) comparison of results.

3.5. Fiber model

The fiber model enables a localized analysis of the non-linear behavior of the frame structure, by discretizing the section of the structural elements into fibers that are associated with each material, whose behavior admits axial deformation only. Thus, the elementary constitutive laws are defined for each material that composes the section instead of a global curve for the entire section [17], further leading to a global constitutive law through the application of an increasing moment/rotation.

The fiber model in Midas-Civil considers that the section remains plane throughout deformation, and perpendicular to the neutral axis, and consequently the reinforcement slippage is not considered.

The state of each fiber is assessed through axial and flexural deformations. The axial forces and bending moments are computed based on the level of stress of each fiber, and thus the properties of the section's non-linear behavior are defined by a stress-strain relation of the fibers that compose the section.

The fiber model can effectively represent the non-linear behavior of frame structures, being able to trace the moment-curvature relation of one section, monitoring the neutral axis depth, obtaining the axial force of each fiber, and computing the spread or extension of the plastic hinge. On the other hand, it may be laborious in the subdivision of the respective sections in fibers to which are associated the elementary constitutive laws, and the constitutive laws have to adequately represent the real behavior of each material, otherwise, the model may not adequately represent the global law of the section under study. In addition, the common uniaxial compressive tests carried out to characterize the behavior of concrete may not adequately estimate the behavior of this material, since concrete's confinement plays an important role in its strength capacity, which becomes more relevant in the case of cyclic loading. Hence, the parameters that characterize the materials were chosen according to the experimental tests, except for concrete, for which different values of confinement were adopted to better fit the experimental results (based on the percentage of transversal reinforcement).

For the current study, the Magenotto-pinto steel model and the Kent and Park concrete model [18], both available in Midas-Civil, were used. The discretization of the beam and columns' critical sections is presented in Figure 9a. It should be referred that the experimental model had a loading device that limited the deformation of the beam during testing, and thus the beam may be in the linear elastic regime. Hence, it is also assumed that the non-linear behavior is lumped at the columns' ends only.

After performing the non-linear analysis of the frame using the fiber model, it was verified that it can provide a good estimate of the experimental model behavior (Figure 9b). The numerical hysteretic curves are found to be very close to the experimental results. However, this model presents some limitations associated with the materials' constitutive laws, being more suitable for moderate lateral displacements and analysis of not only the global behavior but the behavior at the section's level that can calibrate models such as the ones presented in this paper, i.e., polygonal and smooth hysteresis models.

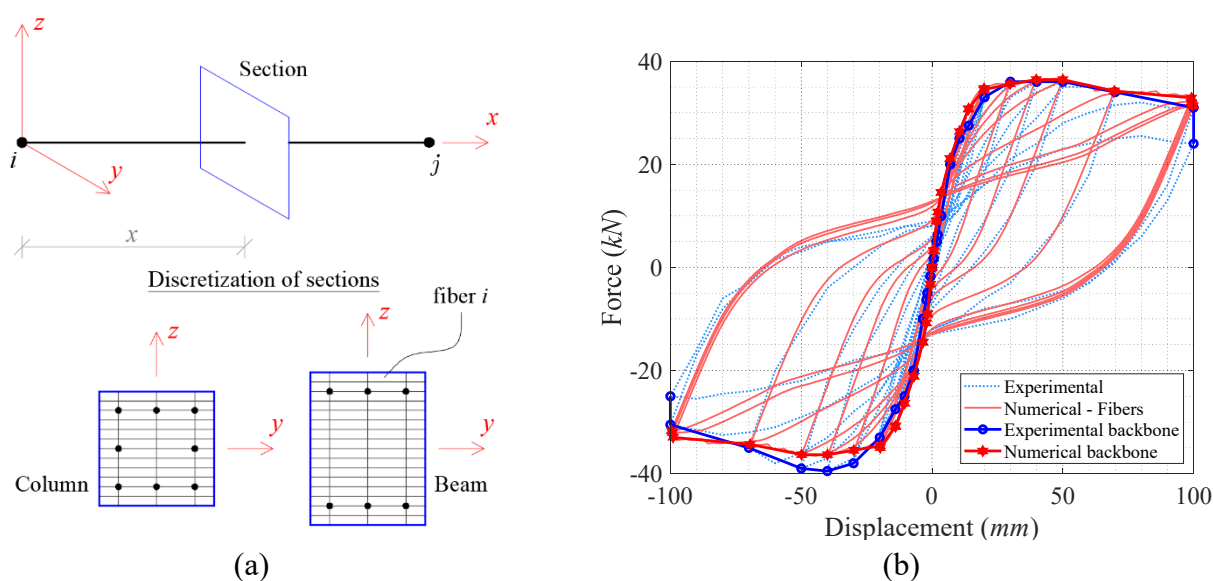


Figure 9. Fiber model. (a) Element's sections discretization used, (b) comparison of results.

3.6. Sivaselvan and Reinhorn's hysteresis model

A different model was used in an attempt to replicate the non-linear behavior of the RC frame structure. This model is a smooth hysteresis model developed by Sivaselvan and Reinhorn [8] and is a subsequent modification of the original Bouc-Wen model [19–21]. A schematic representation of this model is presented in Figure 10a, comprising three springs in combination, being able to simulate different behaviors of a structural system under cyclic loading, viz., strength hardening, the Bauschinger effect, asymmetrical yielding, stiffness and strength degradation, and the pinching effect.

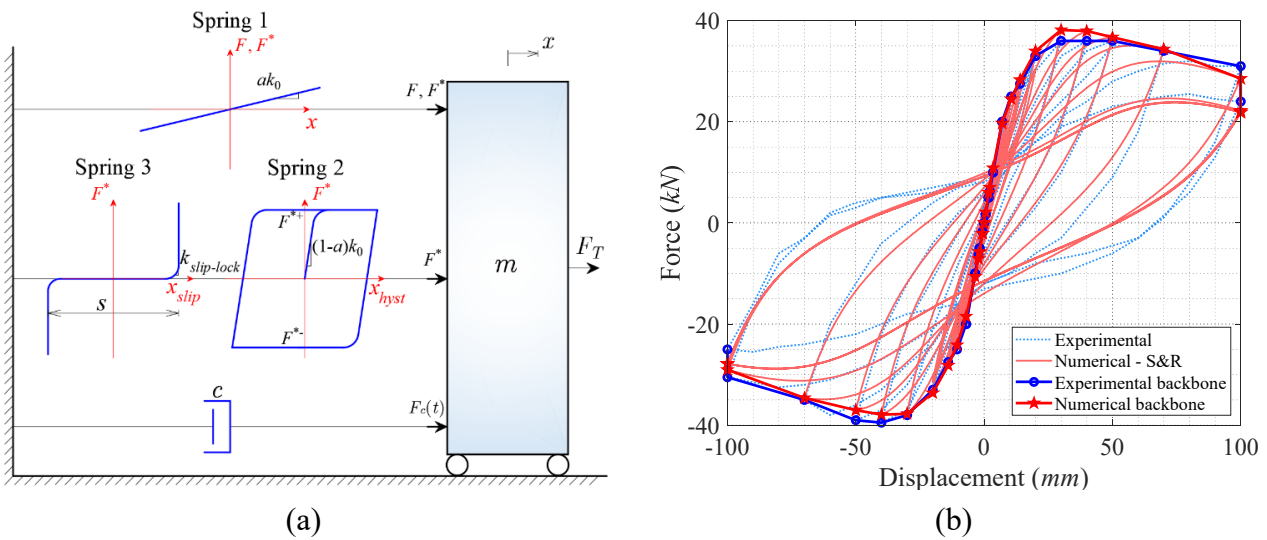


Figure 10. Sivaselvan and Reinhorn’s smooth hysteresis model. (a) Hinge model, (b) comparison of results.

This model presents versatility and thus can represent the non-linear behavior of different structural systems and different materials, provided that the respective parameters are adequately chosen. In the present study, the model is used to simulate the global non-linear behavior of the frame structure as a single-degree-of-freedom (SDOF), rather than considering plastic hinges in the critical locations, thus creating a global stiffness matrix of the structure.

An algorithm was developed in MATLAB to use the aforementioned model in the corresponding conditions. The differential equation that computes the hysteretic force is given in rate-dependent form as follows [8],

$$\dot{F}^* = k_{hyst} \dot{x} \Leftrightarrow \dot{F}^* = \dot{x} (R_K - a) k_0 \left\{ 1 - \left| \frac{F^*}{F_Y^*} \right|^{n_W} \left[\eta_1 \text{sgn}(F^* \dot{x}) + \eta_2 \right] \right\} \quad (1)$$

$$k_H = \frac{k_{hyst} k_{slip-lock}}{k_{hyst} + k_{slip-lock}} \quad (2)$$

where F^* and F_Y^* are the hysteretic force and corresponding yielding force, x is the displacement, k_0 is the initial elastic stiffness, k_H is the total non-linear stiffness, k_{hyst} is the hysteretic stiffness, $k_{slip-lock}$ is the slip-lock stiffness accounting for pinching (related with the pinching parameters: R_s , σ , and λ), R_K is the parameter related with stiffness degradation (that is regulated by parameter α), a is the post-yield to initial stiffness ratio, n_W is the parameter controlling the smoothness transition from the elastic to the post-yielding range, and η_1 and η_2 are parameters controlling the shape of the unloading path (that can be reduced to one parameter, η , in substitution of η_1 , provided that $\eta_1 + \eta_2 = 1 \leftrightarrow \eta_2 = 1 - \eta_1$, for compatibility with plasticity). In addition, strength degradation is controlled by parameters β_1 and β_2 , being the latter related with energy demands and the former with ductility demands, and hence associated with the ultimate ductility factor, μ_{ult} .

Since the experimental model was loaded with displacement control in a quasi-static cyclic manner, the equation of motion only comprises the term related to the restoring force, which is divided into an elastic and inelastic component. In addition, Eq 1 can be rewritten for quasi-statically loaded systems by eliminating dt , since the motion is considered to be independent of time,

$$\begin{aligned} \frac{dF^*}{dt} &= \frac{dx}{dt} (R_K - a) k_0 \left\{ 1 - \left| \frac{F^*}{F_Y^*} \right|^{n_w} \left[\eta_1 \operatorname{sgn} \left(F^* \frac{dx}{dt} \right) + \eta_2 \right] \right\} \\ \Leftrightarrow dF^* &= dx (R_K - a) k_0 \left\{ 1 - \left| \frac{F^*}{F_Y^*} \right|^{n_w} \left[\eta_1 \operatorname{sgn} (F^* dx) + \eta_2 \right] \right\} \end{aligned} \quad (3)$$

and the non-linear equation is solved using a numerical solver such as functions, *fzero*, *fsolve*, or *vpasolve* in MATLAB [16].

The optimization procedure was performed using the function *fminsearch* in MATLAB [16] that follows the Nelder-Mead simplex algorithm. To narrow down a large number of possibilities of an outcome, and find the optimal set of values for the mentioned parameters, a function called *fminsearchbnd* [22] was used. This function is based on the *fminsearch* algorithm, but with bound constraints applied to the variables. This function was used so to minimize the root mean square error (RMSE) between the predicted and experimental values of the restoring force. This error was further normalized (NRMSE) with the range of the experimental restoring force and converted to a percentage. Emphasis was also given to the points in the backbone curve by the calculation of the corresponding NRMSE and by assigning weighting factors to the referred errors. This will aid in the search for an optimal solution that can reasonably estimate the cyclic backbone curve.

To find the optimal values for the parameters defining the smooth hysteresis model that best fit the experimental results a procedure was followed according to the flowchart in Figure 11. The process began with the establishment of the boundary values for the parameters and the choice of an initial guess based on the characteristics of the experimental hysteretic loops. Manual calibration was then performed so to find a reasonable starting point for the use of function *fminsearch/fminsearchbnd*. If the comparison between the numerical and experimental data returns an NRMSE that is less than 5% and meets the stopping criteria (which were set so that the difference between two subsequent iterations on the calculation of the NRMSE is less than 1×10^{-4}), the solution converged. If the solution converges, agreeing with the stopping criteria, the solution is saved and the process terminates.

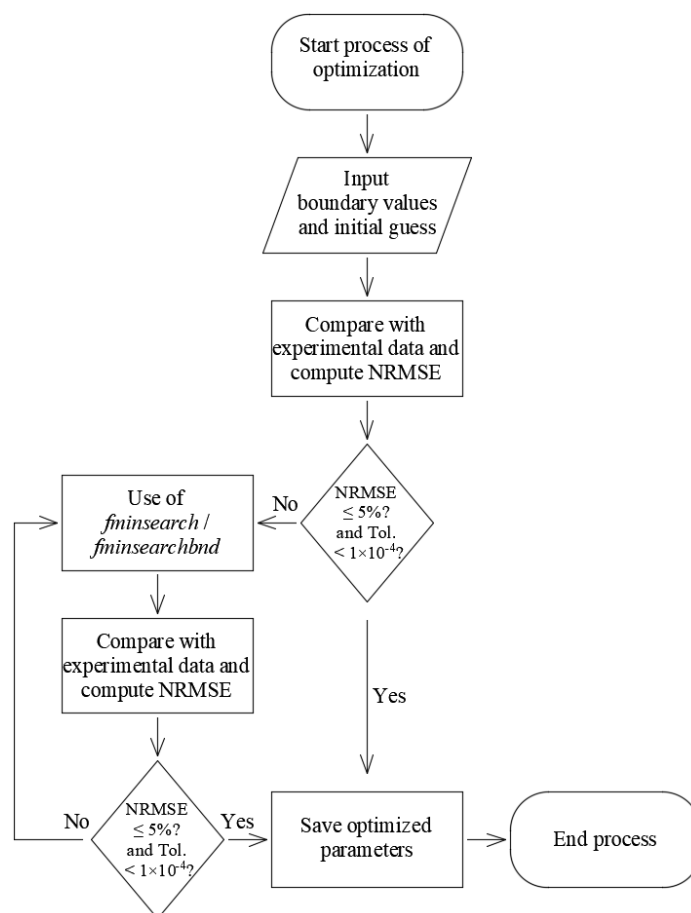


Figure 11. Flowchart of the optimization process used in the MATLAB script.

Within the variables considered, the initial stiffness and the yield force levels in both directions were also considered in the process of optimization, verifying that the optimum values are close to the actual values.

The result of the optimization procedure is presented in Figure 10b and the optimized parameters are: $n_W = 2.07$, $a = -0.061$, $\eta = 0.81$, $\alpha = 1.33$, $\beta_1 = 0.03$, $\beta_2 = 0.005$, $R_s = 0.17$, $\sigma = 0.40$, $\lambda = 0.004$, $k_0 = 3.567 \times 10^6$ N/m, $F_{Y0}^+ = 4.8203 \times 10^4$ N, $F_{Y0}^- = 4.9907 \times 10^4$ N, $\mu_{ult}^+ = 7.85$, $\mu_{ult}^- = 14.85$. The model can satisfactorily predict the cyclic backbone curve as well as the overall hysteretic curves. It is seen that the model presents some deficiencies in predicting the unloading branches, the transitions from pre- to the post-yielding response of the experimental results, and the actual values of strength degradation.

3.7. Comparison of the dissipated hysteresis energy

To further compare the models considered in this study, the cumulated hysteresis energy dissipation is plotted in function of the cumulative displacement (Figure 12).

Clough's model presented the higher accumulated hysteresis energy dissipation, which is clearly due to the bigger hysteresis loops in Figure 6b. The tetralinear Takeda's model presents a better approximation to the experimental energy dissipation compared to Clough's and trilinear Takeda's model. The fiber model and Sivaselvan and Reinhorn's model presented the best approximations to

the cumulative hysteresis energy dissipation, being the latter almost coincident. Although, Sivaselvan and Reinhorn's model was used to represent the non-linear hysteresis behavior of the RC frame in a global/macro way, and the fiber model requires knowledge of the constitutive relations of the materials involved and the parameters tuning may be very difficult.

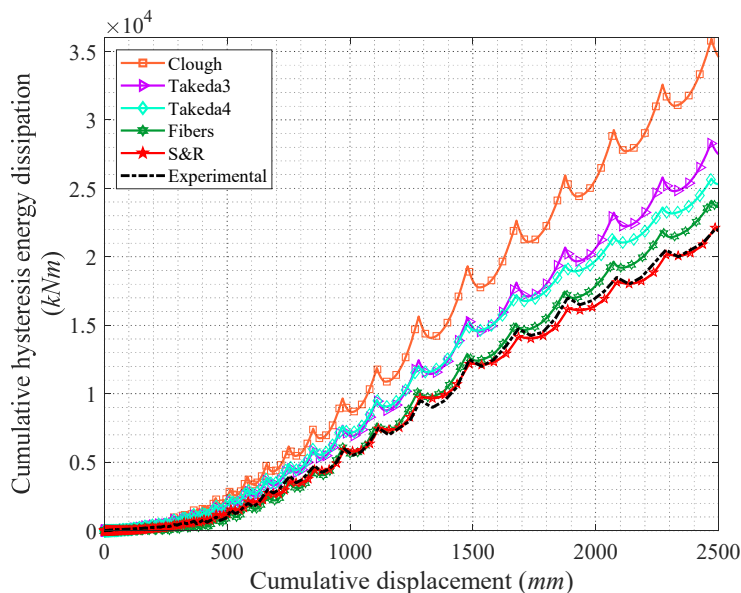


Figure 12. Comparison of the cumulative hysteresis energy dissipation.

4. Conclusions

This study reveals that the degree of numerical precision in replicating experimental results depends on the level of complexity associated with the constitutive models. It is possible to numerically represent the actual non-linear cyclic behavior of RC frame structures by the suitable selection of a hysteresis model.

Simplified analysis using polygonal hysteresis models to simulate the non-linear behavior at critical sections of structural elements may satisfactorily reproduce the experimental results, as was proven with tetralinear Takeda's model in the present case.

More complex and rigorous models, such as the fiber model, can lead to the best results in fitting the desired response. Nevertheless, the number of parameters involved, the required knowledge of the constitutive laws of the constituent materials, and increased computational demands might not compensate for the increased quality of the obtained results.

It was also found that a versatile hysteresis model, emulating the global non-linear cyclic behavior of the RC frame structure, can provide great estimates of the experimental results, provided that a correct selection of the respective parameters is made. Although, this type of model might not be available in commercial software, requiring the prior development and adaptation of algorithms for specific purposes.

Further studies will address validation with more experimental data comprising different hysteretic configurations, and the consideration of non-structural elements within the RC frame structure.

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Conflict of interest

There is no conflict of interests with the authors and the publication of this paper.

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